6.S091 Problem Set 1

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6.S091: Causality

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Preliminaries

(a)

$$\begin{aligned}\mathbb{P}_{\mathcal{X}} &= P(U)P(A|U)P(M|A)P(Y|M,U) \\ &= \text{Ber}(0.5)\,\text{Ber}(U/4)\,\text{Ber}(0.5+0.1A)\,\text{Ber}(M/2+U/4)\end{aligned}$$

(b)

$$\begin{aligned}&\mathbb{P}_{\mathcal{X}}(Y=1) = 0.38125 \\ &\mathbb{P}_{\mathcal{X}}(Y=1 \mid M=0, A=0) = 0.046875 \\ &\mathbb{P}_{\mathcal{X}}(Y=1 \mid M=0, A=1) = 0.0125\end{aligned}$$

Interventional

(c)

$$\begin{aligned}\mathbb{P}_{\mathcal{X}}(U,A,M,Y \mid \text{do}(A=1)) &= P(U)P(A=1)P(M|A=1)P(Y|M,U) \\ &= \text{Ber}(0.5)\cdot 1\cdot \text{Ber}(0.6)\cdot \text{Ber}(M/2+U/4) \\ &= \text{Ber}(0.5)\cdot \text{Ber}(0.6)\cdot \text{Ber}(M/2+U/4)\end{aligned}$$

(d)

$$\mathbb{P}_{\mathcal{X}}(Y \mid \text{do}(A=1)) = \begin{cases}0.93125 & \text{if } Y=1 \\ 0.06875 & \text{if } Y=0\end{cases} \tag{1}$$

$$\mathbb{P}_{\mathcal{X}}(Y \mid \text{do}(A=0)) = \begin{cases}0.3125 & \text{if } Y=1 \\ 0.6875 & \text{if } Y=0\end{cases} \tag{2}$$