6.S091 Problem Set 2

Suyeol Yun
6.S091: Causality

January 26, 2023

Problem 1

* Code is available here https://github.com/syyunn/6.S091/blob/main/pset2/code/problem1/problem1.py

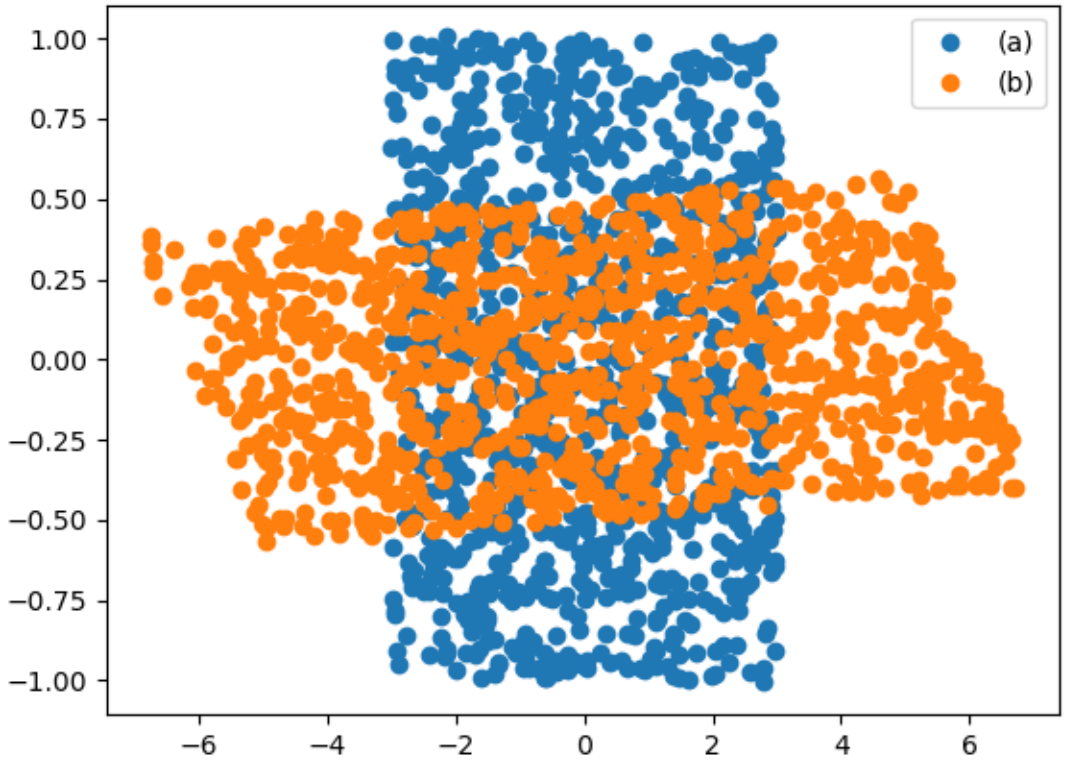
Plotting Regression Residuals [1 point]

(a)

$\hat{\beta}_{12} = 2.0032613606359835$

(b)

$\hat{\beta}_{21} = 0.4853818665988653$



Causal Direction Inference [1 point]

(c)

The plot explains the relationship between ε_1 and ε_2. Since $\varepsilon_1 \perp\!\!\!\perp \varepsilon_2$, we have to choose the SCM which shows such independence. Therefore, the data is more likely to be generated from SCM M^a.

Problem 2

* Code is available here https://github.com/syyunn/6.S091/blob/main/pset2/code/problem2/problem2.py

Partial correlation [2 points]

(a)

$\hat{\rho}\,(X_1, X_4, \varnothing) = 0.18515108160562416$

(b)

$\hat{\rho}\,(X_1, X_4, \{X_2, X_3\}) = 0.0093340865561515$

Fisher's z-transformation [1 point]

(c)

$\hat{z}\,(X_1, X_4, \{X_2, X_3\}) = 0.9332023767104407$

p-values [1 point]

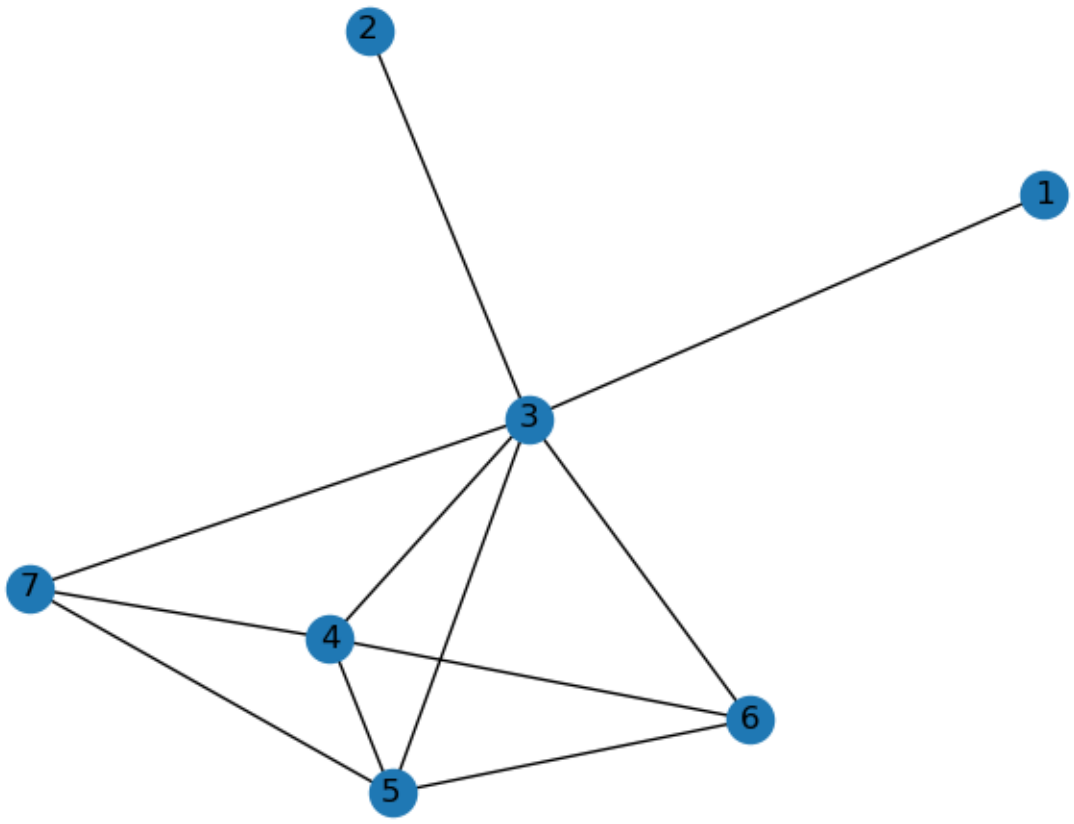
(d)

compute_pvalue (pcalg_samples, 1, 4, [2, 3]) $= 0.35071548783635986$

Skeleton phase [2 points]

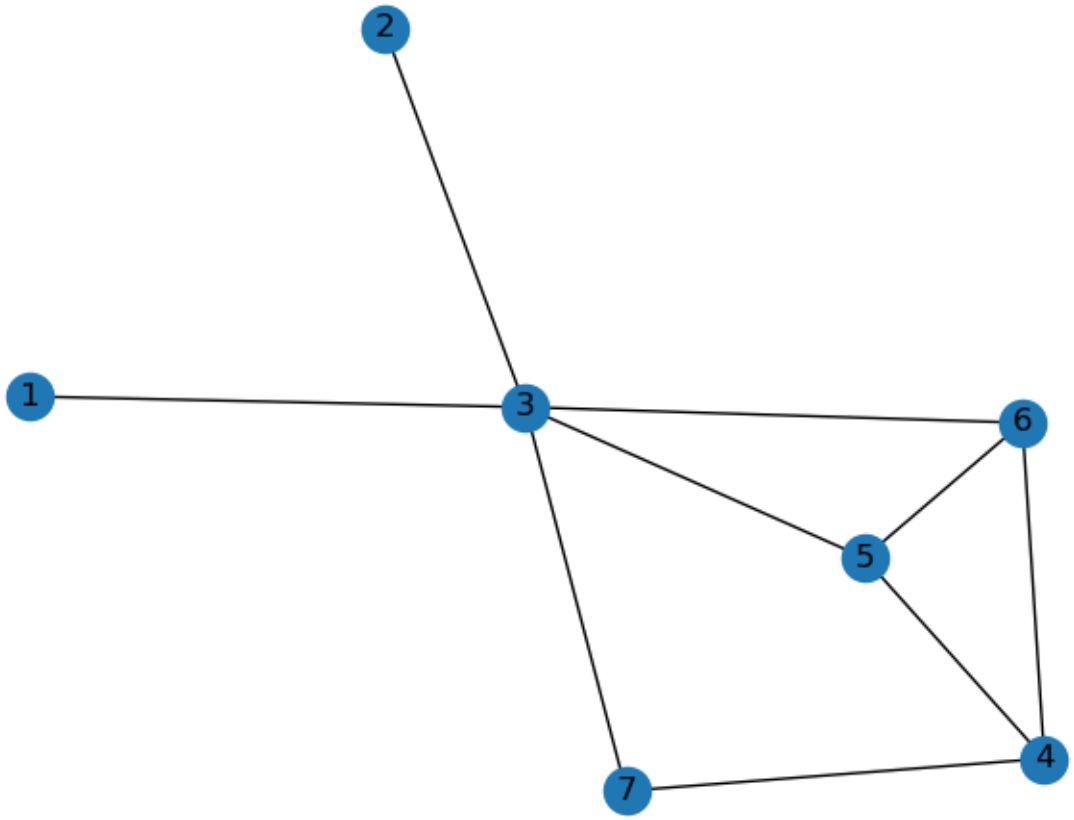
(e)

pcalg_skeleton(samples[: 500], 0.2) = 11



(f)

pcalg_skeleton(samples[: 500], 0.001) = 9



Orientation phase [2 points]

(g)

$X_1 \rightarrow X_3 \leftarrow X_2$ is the only unshileded collider in the output of *pcalg_orient(estimated_skeleton, estimated_separator_function)* .

(h)

The orientations $X_3 \rightarrow X_4, X_3 \rightarrow X_5, X_3 \rightarrow X_6, X_3 \rightarrow X_7$ are added by the Meek's rule #1 (no extra unshileded colliders). However, after the application of Rule 1, no other edges are oriented by the Rule 2, 3, and 4. See the details in the code.