

6.S091 Problem Set 1

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6.S091: Causality

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Preliminaries

(a)

$$\begin{aligned}\mathbb{P}_X(Y, A, M, U) &= P(U)P(A|U)P(M|A)P(Y|M, U) \\ &= \text{Ber}(0.5) \text{Ber}(U/4) \text{Ber}(0.5 + 0.1A) \text{Ber}(M/2 + U/4)\end{aligned}$$

(b)

$$\begin{aligned}\mathbb{P}_X(Y = 1) &= 0.38125 \\ \mathbb{P}_X(Y = 1 \mid M = 0, A = 0) &= 0.10714285714285714 \\ \mathbb{P}_X(Y = 1 \mid M = 0, A = 1) &= 0.25\end{aligned}$$

Interventional

(c)

$$\begin{aligned}\mathbb{P}_X(U, A, M, Y \mid \text{do}(A = 1)) &= P(U)P(A = 1|A = 1)P(M|A = 1)P(Y|M, U) \\ &= \text{Ber}(0.5) \cdot 1 \cdot \text{Ber}(0.6) \cdot \text{Ber}(M/2 + U/4) \\ &= \text{Ber}(0.5) \cdot 1 \cdot \text{Ber}(0.6) \cdot \text{Ber}(M/2 + U/4)\end{aligned}$$

$$\begin{aligned}\mathbb{P}_X(U, A, M, Y \mid \text{do}(A = 0)) &= P(U)P(A = 0|A = 0)P(M|A = 0)P(Y|M, U) \\ &= \text{Ber}(0.5) \cdot 1 \cdot \text{Ber}(0.5) \cdot \text{Ber}(M/2 + U/4) \\ &= \text{Ber}(0.5) \cdot 1 \cdot \text{Ber}(0.5) \cdot \text{Ber}(M/2 + U/4)\end{aligned}$$

(d)

$$\mathbb{P}_X(Y \mid \text{do}(A = 1)) = \begin{cases} 0.425 & \text{if } Y = 1 \\ 0.575 & \text{if } Y = 0 \end{cases} \quad (1)$$

$$\mathbb{P}_X(Y \mid \text{do}(A = 0)) = \begin{cases} 0.375 & \text{if } Y = 1 \\ 0.625 & \text{if } Y = 0 \end{cases} \quad (2)$$

(e)

If $A = 1, U = 0, P(A = 1, U = 0) = 0$. Therefore, $P(Y|A = 1, U = 0)$ is undefined. So I check only for the case of $\text{do}(A = 0)$ which doesn't involve $P(Y|A = 1, U = 0)$ as a component in its process of computation.

$$\mathbb{P}_X(Y = 1 \mid \text{do}(A = 0)) = 0.375$$

$$\begin{aligned} \sum_{u \in \{0,1\}} \mathbb{P}_X(Y = 1 \mid A = 0, U = u) \mathbb{P}_X(U = u) &= \mathbb{P}_X(Y = 1 \mid A = 0, U = 1) \mathbb{P}_X(U = 1) + \mathbb{P}_X(Y = 1 \mid A = 0, U = 0) \mathbb{P}_X(U = 0) \\ &= 0.5 * 1/2 + 0.25 * 1/2 \\ &= 0.375 \quad (\text{match}) \end{aligned}$$

$$\mathbb{P}_X(Y = 0 \mid \text{do}(A = 0)) = 0.625$$

$$\begin{aligned} \sum_{u \in \{0,1\}} \mathbb{P}_X(Y = 0 \mid A = 0, U = u) \mathbb{P}_X(U = u) &= \mathbb{P}_X(Y = 0 \mid A = 0, U = 1) \mathbb{P}_X(U = 1) + \mathbb{P}_X(Y = 0 \mid A = 0, U = 0) \mathbb{P}_X(U = 0) \\ &= 0.5 * 1/2 + 0.75 * 1/2 \\ &= 0.625 \quad (\text{match}) \end{aligned}$$

(f)

$$\begin{aligned} \sum_{m \in \{0,1\}} \mathbb{P}_X(M = m \mid A = 0) \sum_{a' \in \{0,1\}} (\mathbb{P}_X(Y \mid M = m, A = a') \mathbb{P}_X(A = a')) \\ = \end{aligned}$$

Problem 2

(a)

(Ask whether we start from $\mathbf{V} = \overline{\text{an}}_{\mathcal{G}}(\mathbf{A} \cup \mathbf{B} \cup \mathbf{S})$) By the setting, $\mathbf{V} = \overline{\text{an}}_{\mathcal{G}}(\mathbf{A} \cup \mathbf{B} \cup \mathbf{S})$. Since γ is d-connecting path, all nodes in γ are unblocked. For a node to be unblocked, either 1) it is not collider and not in S or 2) it is collider but it is not in S and so is its descendants. In both cases, the node is not in \mathbf{S} . In addition, since the definition of path doesn't include starting and end point which are in A and B respectively, we can exclude the self $A \cup B \cup S$ from V , i.e. $\mathbf{V} = \text{an}_{\mathcal{G}}(\mathbf{A} \cup \mathbf{B} \cup \mathbf{S})$ ■

(b)

Since there exists a d-connecting path γ , $(\mathbf{A}, \mathbf{B}, \mathbf{S}) \notin \mathcal{I}_{\perp}(\mathcal{G})$. Since all nodes in γ doesn't include any node in S as shown in (a), we can use γ as a path in $\overline{\mathcal{G}[\mathbf{V}]}$ from A to B which doesn't pass through S by replacing the directed edges to undirected ones.

Problem 3

(a)

$$\begin{aligned} \mathbb{P}_X(Y = 1 \mid A = N) &= \sum_{S \in \{L,R\}} \mathbb{P}_X(Y = 1 \mid S, A = N) \mathbb{P}_X(S) \\ &= 0.73 \cdot 0.49 + 0.93 \cdot 0.51 \\ &= 0.832 \\ \mathbb{P}_X(Y = 1 \mid A = O) &= \sum_{S \in \{L,R\}} \mathbb{P}_X(Y = 1 \mid S, A = O) \mathbb{P}_X(S) \\ &= 0.69 \cdot 0.49 + 0.87 \cdot 0.51 \\ &= 0.7818 \end{aligned}$$

(b)

Since S satisfies backdoor criterion, we can say

$$\mathbb{P}_X(Y = 1 \mid \text{do}(A = N)) = \sum_{\mathbf{s}} \mathbb{P}_X(Y \mid A = N, \mathbf{S} = \mathbf{s}) \mathbb{P}_X(\mathbf{S} = \mathbf{s}) = 0.832 \text{ and } \mathbb{P}_X(Y = 1 \mid \text{do}(A = O)) = \sum_{\mathbf{s}} \mathbb{P}_X(Y \mid A = O, \mathbf{S} = \mathbf{s}) \mathbb{P}_X(\mathbf{S} = \mathbf{s}) = 0.7818.$$

Problem 4

β_{aw}	$\hat{\beta}_{aw}$	$\hat{\beta}_{yw}$	$\widehat{\beta}_{yw}/\widehat{\beta}_{aw}$
5	4.52	33.84	7.47
0.5	0.72	5.54	7.60
0.05	0.32	2.49	7.62