

6.S091: Problem Set 3

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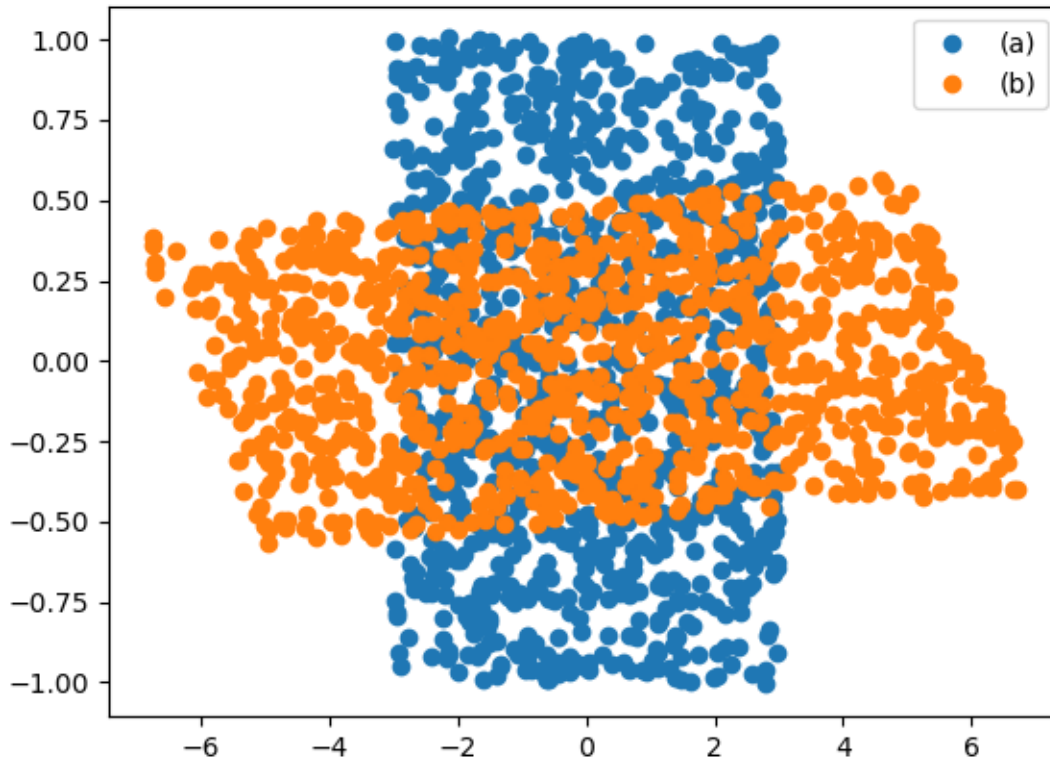
Problem 1: Constructing Minimal I-MAPs [5 points]

(a)

$$\hat{\beta}_{12} = 2.0032613606359835$$

(b)

$$\hat{\beta}_{21} = 0.4853818665988653$$



Causal Direction Inference [1 point]

(c)

The plot explains the relationship between ε_1 and ε_2 . Since $\varepsilon_1 \perp\!\!\!\perp \varepsilon_2$, we have to choose the SCM which shows such independence. Therefore, the data is more likely to be generated from SCM M^a .

Problem 2

* Code is available here <https://github.com/syyunn/6.S091/blob/main/pset2/code/problem2/problem2.py>

Partial correlation [2 points]

(a)

$$\hat{\rho}(X_1, X_4, \emptyset) = 0.18515108160562416$$

(b)

$$\hat{\rho}(X_1, X_4, \{X_2, X_3\}) = 0.0093340865561515$$

Fisher's z-transformation [1 point]

(c)

$$\hat{z}(X_1, X_4, \{X_2, X_3\}) = 0.9332023767104407$$

p-values [1 point]

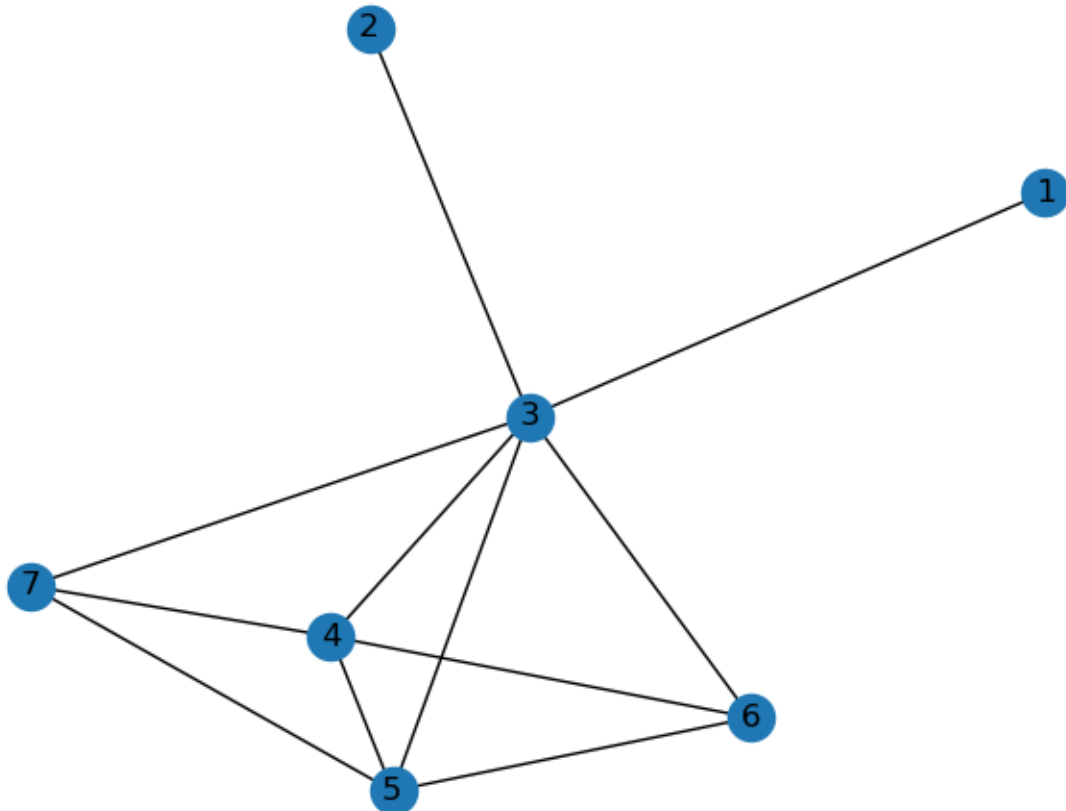
(d)

$$\text{compute_pvalue}(\text{pcalg_samples}, 1, 4, [2, 3]) = 0.35071548783635986$$

Skeleton phase [2 points]

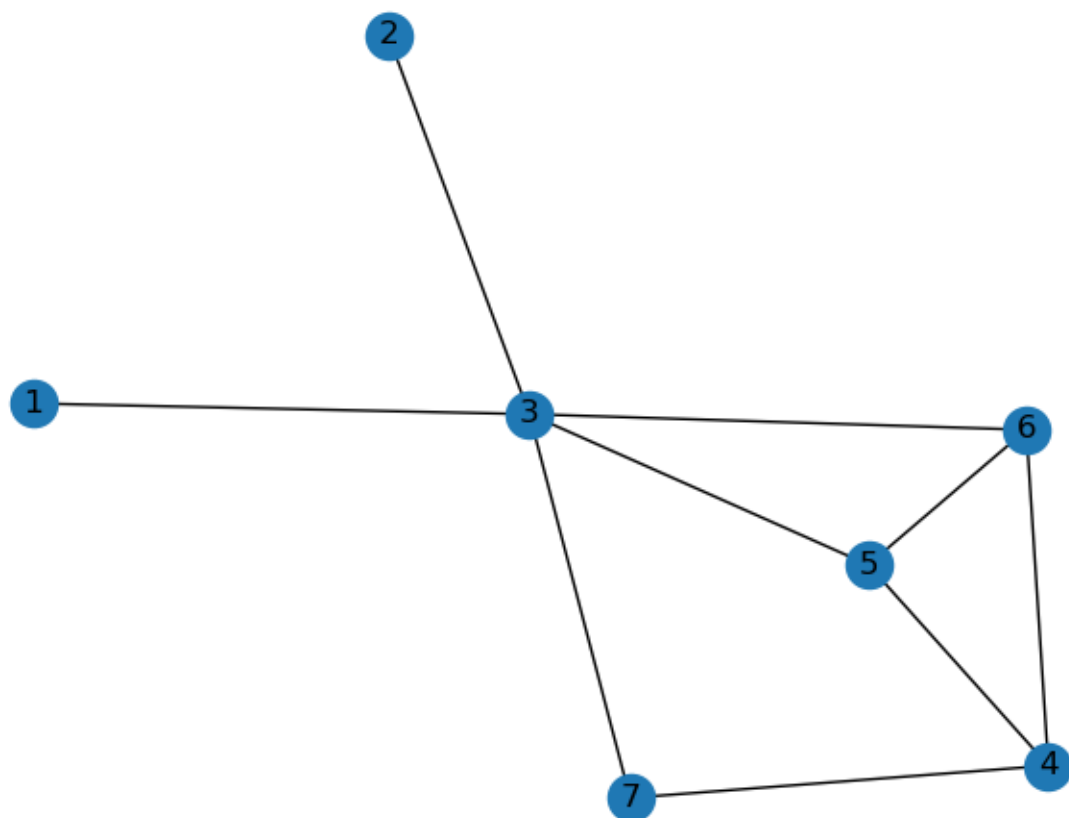
(e)

$$\text{pcalg_skeleton}(\text{samples}[:500], 0.2) = 11$$



(f)

`pcalg_skeleton(samples[: 500], 0.001) = 9`



Orientation phase [2 points]

(g)

$X_1 \rightarrow X_3 \leftarrow X_2$ is the only unshielded collider in the output of `pcalg_orient(estimated_skeleton, estimated_separator_function)`.

(h)

The orientations $X_3 \rightarrow X_4, X_3 \rightarrow X_5, X_3 \rightarrow X_6, X_3 \rightarrow X_7$ are added by the Meek's rule #1 (no extra unshielded colliders). However, after the application of Rule 1, no other edges are oriented by the Rule 2, 3, and 4. See the details in the code.