# Modeling Lobbying Industry with Multi-Agent Multi-Armed Bandit

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## Abstract

What is lobbying? In this work, I model the lobbying industry with a multi-agent multi-armed bandit problem. Under this setting, I provide supportive evidence for the argument that explains lobbying as a delegated information acquisition process. First, I show that interest groups can successfully explore a large legislative space by delegating information acquisition process to lobbyists. Second, I show that lobbyists become specialized when lobbyists have expert knowledge on a specific issue area across multiple legislators. Those resulsts provide supportive evidence why interest groups have incentives to delegate information acquisition process to lobbyists and why lobbyists in real world interact with both sides of the legislators in terms of interest of their clients.

## 1 Introduction

What is lobbying? This simple question has been asked in different literatures but there is no clear answer with agreement among scholars. For example, *vote-buying* theory argues that lobbying is to buy votes in legislation (Grossman and Helpman, 1994). However, this theory doesn't explain why the average price of vote is too cheap compared to the expected return of lobbying. As another example, *persuasion* theory argue that lobbying is to persuade legislators to change their positions on legislation close to the position of the lobbyists (Young and David, 1951; Bauer et al., 2017; Milbrath, 1984). However, this theory doesn't explain why lobbyists lobby already like-minded legislators those who don't need any persuasions.

Apart from these theories, this work provides a supportive evidence that interest groups have incentives to delegate information acquisition process to lobbyists. By modeling the lobbying industry with a multi-agent multi-armed bandit problem, I simulatively show that interest groups can successfully explore a large legislative space by delegating information acquisition process to lobbyists.

In addition, simulation results show that lobbyists become specialized when lobbyists have expert knowledge on a specific issue area across multiple legislators. This result provides a supportive evidence why lobbyists in real world interact with both sides of the legislators with respect to the interest of their clients.

## 2 Multi-Agent Multi-Armed Bandit Problem

In lobbying industry, interest groups need to specify which legislator have a highest potential power to influence the issue area of their interest. To do so, they need to interact with different legislators to explore the potential expected rewards from each legislator. However, total chances of exploration is limited and interest groups need to decide whether to keep exploring or exploiting the best legislator they have already explored at each timestep. This situation can be modeled by multi-armed bandit problem which formalizes the exploration–exploitation dilemma.

### 2.1 Categorical Multi-Armed Bandit Problem

Since legislators have varying level of authorities over different issue areas, I model this situation by categorical multi-armed bandit problem. There exists $|J|$ number of interest groups $\{1, 2, \dots, J\}$, $|K|$ number of legislators $\{1, 2, \dots, K\}$ , $|C|$ number of categories of interset $\{1, 2, \dots, C\}$ and $|T|$ number of timesteps $\{1, 2, \dots, T\}$. Then each interst group $j \in J$ has its own category of interest $\phi(j) \in C$ and each legislator $k \in K$ has a categorical distribution $\mathrm{Cat}(\mathrm{C}, \mathbf{p_k})$ where $C = \{1, 2, \dots, C\}$ and $\mathbf{p_k} = [p_k^{(1)}, p_k^{(2)}, \dots, p_k^{(C)}]$. Whenever an intesrest group $j$ interact with the $k$-th legislator, they sample $X_k \in C$ from $\mathrm{Cat}(\mathrm{C}, \mathbf{p_k})$. If the sampled category $X_k$ matches with the category of interest of the interest group, i.e. $X_k = \phi(j)$, it gets the reward of $r_j^k = \mathbb{1}(x_j^k = \phi(j))$.

### 2.2 Thompson Sampling as a Bandit Strategy

Let's assume that each interest group hold a prior belief over the categorical distribution of each legislator. Since Dirichlet distribution is conjugate prior of categorical distribution, I model the prior belief of each interest group $j$ over the categorical distribution of the $k$-th legislator as $\mathrm{Dir}(C, \alpha_{jk})$ where $\alpha_{\mathbf{jk}} = [\alpha_{jk}^{(1)}, \alpha_{jk}^{(2)}, \dots, \alpha_{jk}^{(C)}]$. At each timestep $t \in T$, interest group $j$ choose the $k$-th legislator to interact with and sample $X_k^t \in C$ from $\mathrm{Cat}(\mathrm{C}, \mathbf{p_k})$. Then the interest group $j$ updates its prior belief over $\mathrm{Cat}(\mathrm{C}, \mathbf{p_k})$ by adding 1 to $\alpha_{jk}^{(X_k^t)}$. After update, the interest group $j$ chooses the next legislator $k_{t+1}$ to interact with by sampling $\hat{\mathbf{p}}_{\mathbf{1j}}, \hat{\mathbf{p}}_{\mathbf{2j}}, \dots \hat{\mathbf{p}}_{\mathbf{Kj}}$ from $\mathrm{Dir}(C, \alpha_{\mathbf{j1}}), \mathrm{Dir}(C, \alpha_{\mathbf{j2}}), \dots, \mathrm{Dir}(C, \alpha_{\mathbf{jK}})$ and choose $k_{t+1} = \underset{K}{\mathrm{argmax}}\{\hat{p}_{1j}^{(\phi(j))}, \hat{p}_{2j}^{(\phi(j))}, \dots, \hat{p}_{Kj}^{(\phi(j))}\}$. In this way the interest group $j$ can systematically update its prior belief and balance the exploration and exploitation by randomness of sampling from Dirichlet distribution. This strategy is called *Thompson sampling* (Thompson, 1933).

## 3 Large Search Space Problem in Lobbying Industry

In this section, I provide a supportive evidence that interest groups have incentives to delegate information acquisition process to lobbyists to solve the large search space problem. Starting from showing that interest groups fail to find the best legislator to interact in case of large search space, I show that interest groups can solve the large search space problem by hiring lobbyists by simulation.

### 3.1 Simulation I: Small Search Space

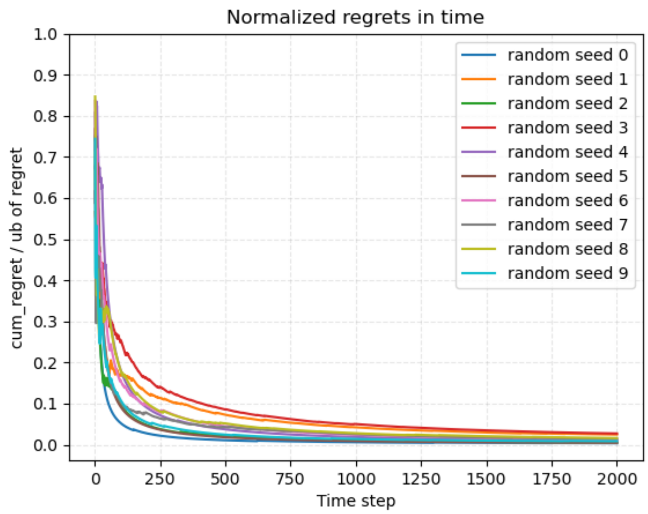
First, I simulate the case of small search space. In this simulation, I used $|K| = 32, |C| = 4, |J| = 1$[1] and $|T| = 2000$. Since I don't have have a good representation of the categorical distributions of legislators in real world, I randomly generate the categorical distribution of each legislator. Therefore, I randomly generate the category of interest of each interest group.

I use the normalized cumulative regret as the performance metric in the following simulations. The normalized cumulative regret is defined as $\sum_t^T (p_{\phi(j)}^{\max} - p_{a_t}^j) / \sum_t^T (p_{\phi(j)}^{\max} - p_{\phi(j)}^{\min})$ where $p_{\phi(j)}^{\max} = \mathrm{argmax}_{k \in K}\{p_1^{(\phi(j))}, p_2^{(\phi(j))}, \dots, p_K^{(\phi(j))}\}$, $p_{\phi(j)}^{\min} = \mathrm{argmin}_{k \in K}\{p_1^{(\phi(j))}, p_2^{(\phi(j))}, \dots, p_K^{(\phi(j))}\}$ and $a_t$ represent the action (choice of legislator) taken by the interest group at time $t$. Regret represents how much the agent could have been done better in terms of the reward if it had chosen the best action. Similarly, cumulative normalized regret is sum of normalized regret over the entire time horizon $T$.

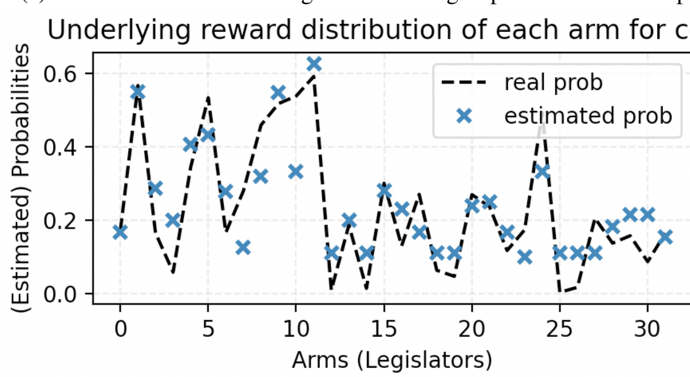
Figure 1a shows that normalized cumulative regrests of interest groups decrease as the number of timesteps increases regardless of random seed which represent different set of categorical distributions of legislators. This means that interest groups successfully find the suitable legislator to interact with in small search space. In this scenario, search space consists of $|K| \times |C| = 128$ number of parameters which are relatively small and interest groups can easily find the best legislator to interact with. Figure fig:smallproba shows that prior belief of the interest group estimates the real probabilities

[1] This is single agent setting

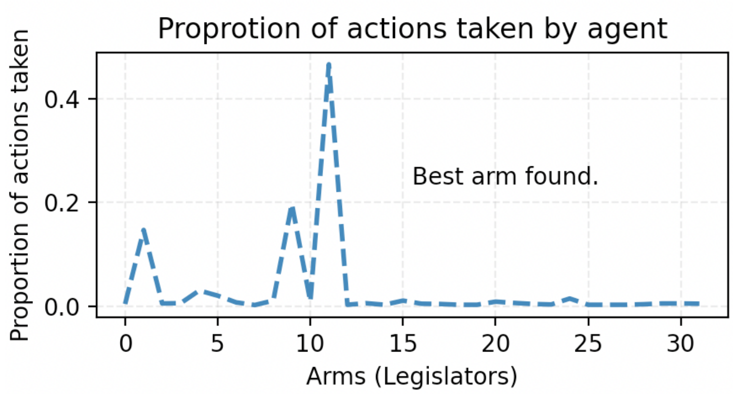
of category of interest across legislators closely. In addition, figure 1c shows that the interest group successfully found the best legislator and keep interacting with the best one with the highest ratio.




(a) Cumulative normalized regret of interest groups in small search space




(b) Real probabilities of category of interest $\phi(j)$ across legislators $p_1^{(\phi(j))}, p_2^{(\phi(j))} \ldots p_K^{(\phi(j))}$ and estimated probabilities of category of interest across legislators $\hat{p}_1^{(\phi(j))}, \hat{p}_2^{(\phi(j))} \ldots \hat{p}_K^{(\phi(j))}$ for random seed 0.




(c) The proportion of each legislator taken by the interest group across legislators for random seed 0.

Figure 1: **Simulation I** Small Search Space

### 3.2 Simulation II: Large Search Space

In the previous simulaton, I used arbitrary hyperparameters for $|K|$ and $|C|$. To closely approximate the real world case, now I use the hyperparameters of $|K| = 112, |C| = 26, |J| = 1$ and $|T| = 2000$. $|K| = 112$ is the average number of legislators to whom top 10 lobbying firms campaign contribute and $|C| = 26$ is the average number of issue areas per clients in 2020. Both numbers are obtained from *Lobbying Disclosure Act* data[2].

In this case, interset groups fail to find the best legislator to interact with regardless of the random seeds as shown in Fig 3b. Compared to the previous simulation, cumulative normalized regret doesn't converge close to 0 within the time horizon $T = 2000$. This is because the search space is almost 23 times larger than the previous simulation. Due to the large search space, within the time constraint of $T = 2000$, it's difficult for interest groups to explore enough to find the best legislator to interact with. Therefore, prior belief of the interest group fails to estimate the real world probabilities of legislators as shown in Fig 3c. This result in exploiting a worse legislator compared to the best one with the highest ratio as shown in Figure $3d$.

### 3.3 Introducing Lobbyists into the Multi-Armed Bandit Setting

I conjecture that interest groups with a shared category of interest can hire the same lobbyist to find the best legislators even in the large search space. To allow interest groups to hire lobbyists, I introduce a lobbyist $l \in L$ with its own set of prior beliefs $\mathrm{Dir}(C, \alpha_{lk})$ where $\alpha_{\mathbf{lk}} = [\alpha_{lk}^{(1)}, \alpha_{lk}^{(2)}, \dots, \alpha_{lk}^{(C)}] \quad \forall k \in K$. At each timestep $t$, an interest group $j$ choose the next legislator $k_{t+1} = \underset{K}{\mathrm{argmax}} \left[ \{\hat{p}_{1j}^{(\phi(j))}, \hat{p}_{2j}^{(\phi(j))}, \dots, \hat{p}_{Kj}^{(\phi(j))}\} \cup \bigcup_{l \in L} \{\hat{p}_{1l}^{(\phi(j))}, \hat{p}_{2l}^{(\phi(j))}, \dots, \hat{p}_{Kl}^{(\phi(j))}\} \right]$. This means that interest group $j$ chooses the best rewarding legislator based on the prior belief of all lobbyists and himself. For this to work, I assume complete information so that any interest groups can access to any lobbyists' prior belief. In this setting, if the interest group $j$ chooses the next legislator $k_{t+1}$ based on the prior belief of lobbyist $l \in L$, I update the prior belief of lobbyist $l$ with the sampled observation $X_k^{t+1} \sim \mathrm{Cat}(\mathrm{C}, \mathbf{p_k})$ by adding $1$ to $\alpha_{lk}^{(X_k^{t+1})}$. However, I don't update the prior belief of interest group $j$, $\alpha_{jk}^{(X_k^{t+1})}$. This is because if an interest group hires a lobbyist, the lobbyist explore the legislative space on behalf of the interest group. This means that the interest group who hired lobbyist barely accumulates any experience what the lobbyist has experienced. In addition, if an interest group choose the next legislator based on their own prior belief, we update the prior belief of interest group $j$ as usual.

#### 3.3.1 Simulation III: Large Search Space with a Lobbyist

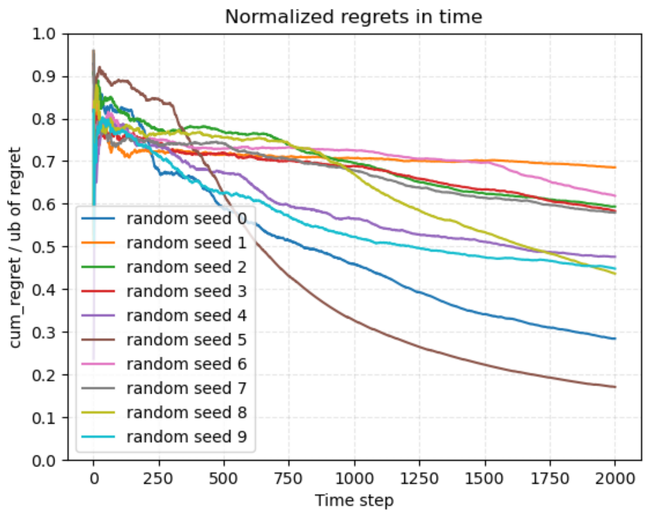
Now, I simulate the configuration of $|K| = 112, |C| = 26$ and $|T| = 2000$ with a lobbyist $|L| = 1$ and $|J| = 5$ number of interest groups with a shared category of interest. This maintains the size of the search space as in the previous simulation. I choose $|J| = 5$ based on the average number of clients per each issue code in *Lobbying Disclosure Act* data in 2020.

With a lobbyist, average lowest

## 4 Specialization of Lobbyists and Expert knowledge

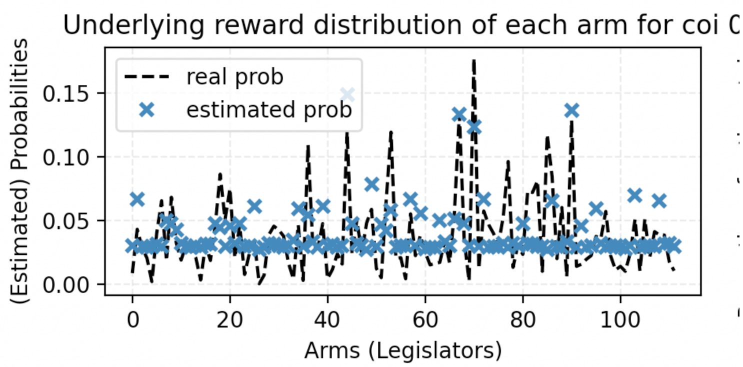
I conjecture that with the existence of lobbyist interest groups can find the best legislator even in the large search space. This is because lobbyists concentrate

[2]Data is available through https://lda.senate.gov/filings/public/filing/search/

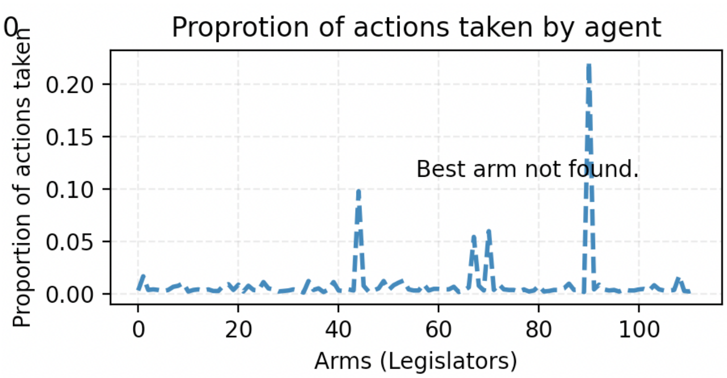



(a)

(b) Cumulative normalized regret of interest groups in small search space




(c) Real probabilities of category of interest $\phi(j)$ across legislators $p_1^{(\phi(j))}, p_2^{(\phi(j))} \dots p_K^{(\phi(j))}$ and estimated probabilities of category of interest across legislators $\hat{p}_1^{(\phi(j))}, \hat{p}_2^{(\phi(j))} \dots \hat{p}_K^{(\phi(j))}$ for random seed 0.




(d) The proportion of each legislator taken by the interest group across legislators for random seed 0.

Figure 2: **Simulation II** Large Search Space

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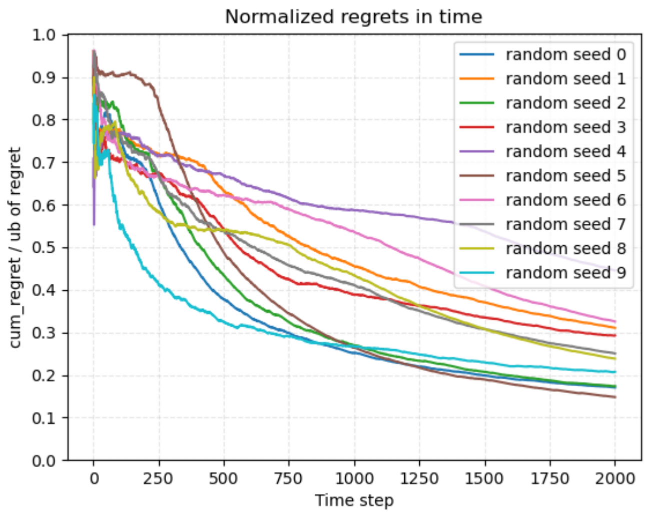


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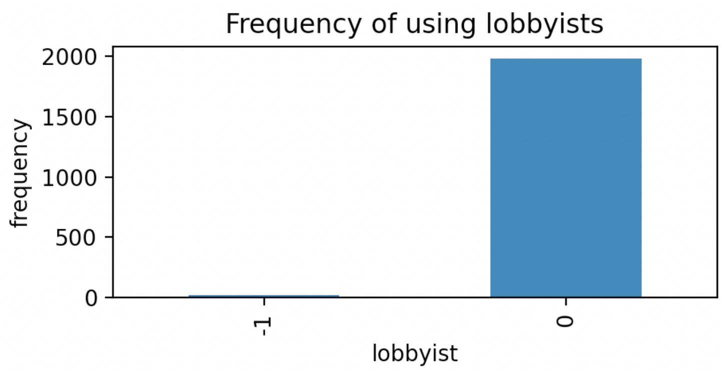
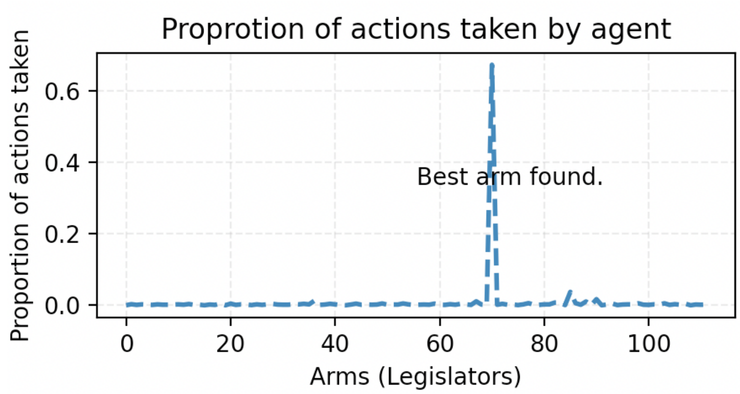



(c) Real probabilities of category of interest $\phi(j)$ across legislators $p_1^{(\phi(j))}, p_2^{(\phi(j))} \ldots p_K^{(\phi(j))}$ and estimated probabilities of category of interest across legislators $\hat{p}_1^{(\phi(j))}, \hat{p}_2^{(\phi(j))} \ldots \hat{p}_K^{(\phi(j))}$ for random seed 0.




(d) The proportion of each legislator taken by the interest group across legislators for random seed 0.

Figure 3: **Simulation II** Large Search Space

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