
Modeling Lobbying Industry with Multi-Agent Multi-Armed Bandit

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Abstract

What is lobbying? To answer this question, I model the lobbying industry with a multi-agent multi-armed bandit problem. Then by simulation, I provide supportive evidence for the argument that explains lobbying as a delegated information acquisition process. First, I show that interest groups can successfully explore a large legislative space by delegating information acquisition process to lobbyists. Second, I show that lobbyists become specialized when lobbyists have expert knowledge on a specific issue area across legislators. These two results provide supportive evidence why interest groups have incentives to delegate information acquisition process to lobbyists and why real-world lobbyists interact with both sides of the legislators in terms of interest of their clients.

1 Introduction

What is lobbying? This simple question has been asked in different literatures but there is no clear answer with agreement among scholars. For example, *vote-buying* theory argues that lobbying is to buy votes in legislation (Grossman and Helpman, 1994). However, this theory doesn't explain why the average price of vote is too cheap compared to the expected return of lobbying. As another example, *persuasion* theory argue that lobbying is to persuade legislators to change their positions on legislation close to the position of the lobbyists (Young and David, 1951; Bauer et al., 2017; Milbrath, 1984). However, this theory doesn't explain why lobbyists lobby already like-minded legislators those who don't need any persuasions.

Apart from these theories, this work provides a supportive evidence that interest groups have incentives to delegate information acquisition process to lobbyists. By modeling the lobbying industry with a multi-agent multi-armed bandit problem, I simulatively show that interest groups can successfully explore a large legislative space by delegating information acquisition process to lobbyists.

In addition, I show that lobbyists become specialized when lobbyists have expert knowledge on a specific issue area across multiple legislators by simulation. This result provides a supportive evidence why lobbyists in real world interact with both sides of the legislators with respect to the interest of their clients.

2 Multi-Agent Multi-Armed Bandit Problem

In lobbying industry, interest groups need to specify which legislator has the biggest power to influence the issue area of their interest. To do so, they need to interact with different legislators to explore the expected rewards from each legislator. However, total chances of exploration is limited because of the cost of interaction. Therefore, interest groups need to decide whether to keep exploring

or exploiting the best legislator they have already explored at each timestep. This situation can be modeled by multi-armed bandit problem which formalizes the exploration–exploitation dilemma.

2.1 Categorical Multi-Armed Bandit Problem

Since legislators have varying level of authorities over different issue areas, I model this situation by categorical multi-armed bandit problem. There exists $|J|$ number of interest groups $\{1, 2, \dots, J\}$, $|K|$ number of legislators $\{1, 2, \dots, K\}$, $|C|$ number of categories of interest $\{1, 2, \dots, C\}$ and $|T|$ number of timesteps $\{1, 2, \dots, T\}$. Then each interest group $j \in J$ has its own category of interest $\phi(j) \in C$ and each legislator $k \in K$ has a categorical distribution $\text{Cat}(C, \mathbf{p}_k)$ where $C = \{1, 2, \dots, C\}$ and $\mathbf{p}_k = [p_k^{(1)}, p_k^{(2)}, \dots, p_k^{(C)}]$. Whenever an interest group j interact with the k -th legislator, they sample $X_k \in C$ from $\text{Cat}(C, \mathbf{p}_k)$. If the sampled category X_k matches with the category of interest of the interest group, i.e. $X_k = \phi(j)$, it gets the reward of $r_j^k = \mathbb{1}(X_k = \phi(j))$.

2.2 Thompson Sampling as a Bandit Strategy

Let's assume that each interest group hold a prior belief over the categorical distribution of each legislator. Since Dirichlet distribution is conjugate prior of categorical distribution, I model the prior belief of each interest group j over the categorical distribution of the k -th legislator as $\text{Dir}(C, \alpha_{jk})$ where $\alpha_{jk} = [\alpha_{jk}^{(1)}, \alpha_{jk}^{(2)}, \dots, \alpha_{jk}^{(C)}]$. At each timestep $t \in T$, interest group j choose the k -th legislator to interact with and sample $X_k^t \in C$ from $\text{Cat}(C, \mathbf{p}_k)$. Then the interest group j updates its prior belief by adding 1 to $\alpha_{jk}^{(X_k^t)}$. After update, the interest group j chooses the next legislator k_{t+1} to interact at time $t + 1$ by sampling $\hat{\mathbf{p}}_{1j}, \hat{\mathbf{p}}_{2j}, \dots, \hat{\mathbf{p}}_{Kj}$ from $\text{Dir}(C, \alpha_{j1}), \text{Dir}(C, \alpha_{j2}), \dots, \text{Dir}(C, \alpha_{jK})$ respectively. Then it choose $k_{t+1} = \arg\max_K \{\hat{p}_{1j}^{(\phi(j))}, \hat{p}_{2j}^{(\phi(j))}, \dots, \hat{p}_{Kj}^{(\phi(j))}\}$. In this way the interest group j can systematically update its prior belief and balance the exploration and exploitation based on the randomness of sampling from Dirichlet distributions. This strategy is called *Thompson sampling* (Thompson, 1933).

3 Large Search Space Problem in Lobbying Industry

In this section, I provide a supportive evidence that interest groups have incentives to delegate information acquisition process to lobbyists to solve the large search space problem. Starting from showing that interest groups fail to find the best legislator to interact in case of large search space, I show that interest groups can solve the large search space problem by hiring lobbyists by simulation.

3.1 Simulation I: Small Search Space

First, I simulate the case of small search space. In this simulation, I used $|K| = 32$, $|C| = 4$, $|J| = 1$ ¹ and $|T| = 2000$. Since I don't have a good representation of the categorical distributions of legislators, I randomly generate a hypothetical categorical distribution of each legislator for simulation.

Besides, I use the normalized cumulative regret as the performance metric in the following simulations. The normalized cumulative regret is defined as $\sum_t^T (p_{\phi(j)}^{\max} - p_{a_t}^j) / \sum_t^T (p_{\phi(j)}^{\max} - p_{\phi(j)}^{\min})$ where $p_{\phi(j)}^{\max} = \arg\max_{k \in K} \{p_1^{(\phi(j))}, p_2^{(\phi(j))}, \dots, p_K^{(\phi(j))}\}$, $p_{\phi(j)}^{\min} = \arg\min_{k \in K} \{p_1^{(\phi(j))}, p_2^{(\phi(j))}, \dots, p_K^{(\phi(j))}\}$ and a_t represent the action (choice of legislator) taken by the interest group at time t . Regret represents how much the agent could have been done better in terms of the reward if it had chosen the best action. Similarly, cumulative normalized regret is sum of regret over the entire time horizon T normalized by the sum of upper bound of regret².

Figure 1a shows that normalized cumulative regrets of interest groups decrease as the number of timesteps increases. The mean of cumulative normalized regrets at time T from 10 different random seeds³ is 0.012 with the variance of $5.98e - 05$. This means that a interest group can successfully find

¹This is single agent setting

²I use this metric to compare the level of regret between different hyperparameter settings.

³Each random seed corresponds to the different set of categorical distributions of legislators.

the best rewarding legislator to interact in case of small search space. In this scenario, search space consists of $|K| \times |C| = 128$ number of parameters which is relatively small compared to the case of actual lobbying industry. Figure 1b shows that prior belief of the interest group precisely estimates the real probabilities of category of interest across legislators. This means that the interest group successfully reconstructed the categorical distributions of legislators by 2000 times of interaction. Therefore, figure 1c shows that the interest group successfully found the best legislator and keep interacting with the best one with the highest ratio.

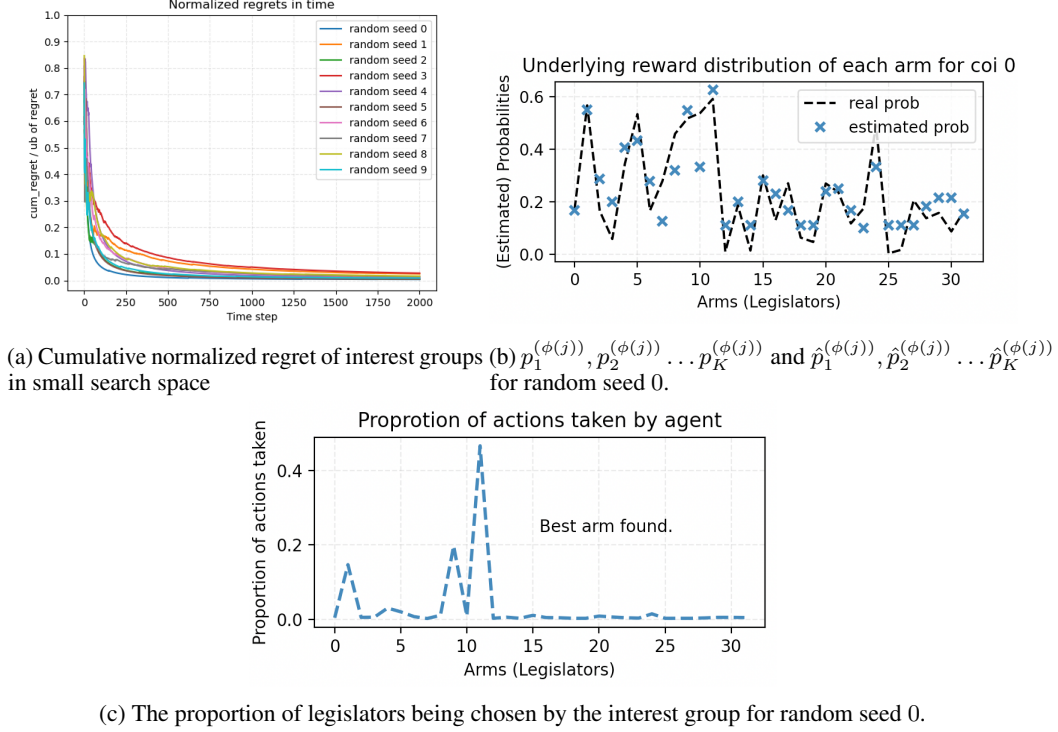


Figure 1: Simulation in case of Small Search Space

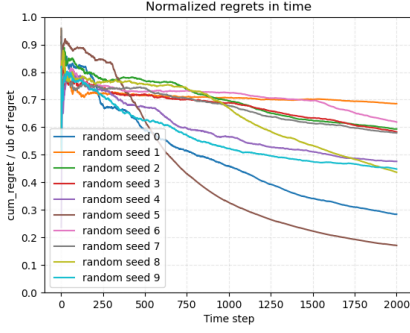
3.2 Simulation II: Large Search Space

In the previous simulation, I used arbitrary small numbers for $|K|$ and $|C|$. Now I use $|K| = 112$, $|C| = 26$, $|J| = 1$ and $|T| = 2000$. $|K| = 112$ is the average number of legislators to whom top 10 lobbying firms campaign contribute and $|C| = 26$ is the average number of issue areas per clients in 2020. Both numbers are obtained from *Lobbying Disclosure Act* data⁴ and I expect this simulation to be more realistic compared to the previous simulation.

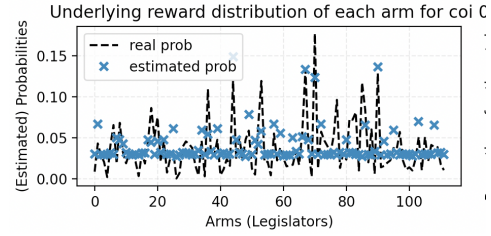
In this simulation, the mean of cumulative normalized regrets at time T from 10 different random seeds⁵ is 0.48 with the variance of 0.023 which is much higher mean regret compared to the previous simulation (See Fig 2a). It implies that interest group fails to find the best rewarding legislator. This is because the search space is almost 23 times larger than the previous simulation. Due to the large search space, within the time constraint of $T = 2000$, it's difficult for the interest group to explore enough and find the best legislator alone. Therefore, prior belief of the interest group fails to estimate the real world probabilities of legislators as shown in Fig 2b. This result in exploiting a worse legislator compared to the best one with the highest ratio as shown in Figure 2c.

⁴Data is available through <https://lda.senate.gov/filings/public/filing/search/>

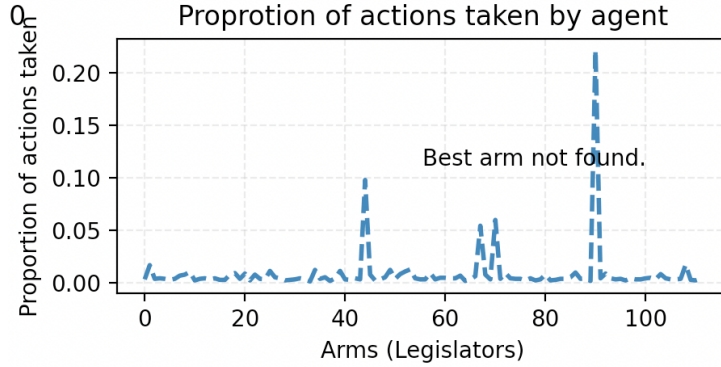
⁵Each random seed corresponds to the different set of categorical distributions of legislators.



(a) Cumulative normalized regret of interest groups in large search space



(b) $p_1^{(\phi(j))}, p_2^{(\phi(j))}, \dots, p_K^{(\phi(j))}$ and $\hat{p}_1^{(\phi(j))}, \hat{p}_2^{(\phi(j))}, \dots, \hat{p}_K^{(\phi(j))}$ for random seed 0.



(c) The proportion of legislators being chosen by the interest group for random seed 0.

Figure 2: Simulation in case of Large Search Space

3.3 Introducing Lobbyists into the Multi-Armed Bandit Setting

I conjecture that interest groups with a shared category of interest can hire the same lobbyist to find the best legislators even in the large search space. To allow interest groups to hire lobbyists, I introduce a lobbyist $l \in L$ with its own set of prior beliefs $\text{Dir}(C, \alpha_{lk}) \quad \forall k \in K$ where $\alpha_{lk} = [\alpha_{lk}^{(1)}, \alpha_{lk}^{(2)}, \dots, \alpha_{lk}^{(C)}]$. At each timestep t , an interest group j choose the next legislator $k_{t+1} = \underset{K}{\text{argmax}} \left[\{\hat{p}_{1j}^{(\phi(j))}, \hat{p}_{2j}^{(\phi(j))}, \dots, \hat{p}_{Kj}^{(\phi(j))}\} \cup \bigcup_{l \in L} \{\hat{p}_{1l}^{(\phi(j))}, \hat{p}_{2l}^{(\phi(j))}, \dots, \hat{p}_{Kl}^{(\phi(j))}\} \right]$. This means that interest group j chooses the best rewarding legislator based on the prior belief of all lobbyists and himself. I assume complete information so that any interest groups can access to any lobbyists' prior belief at any timestep. In this setting, if the interest group j chooses the next legislator k_{t+1} based on the prior belief of lobbyist l , I update the prior belief of lobbyist l with the sampled observation $X_k^{t+1} \sim \text{Cat}(C, \mathbf{p}_k)$ by adding 1 to $\alpha_{lk}^{(X_k^{t+1})}$. However, I don't update the prior belief of interest group j , $\alpha_{jk}^{(X_k^{t+1})}$. This is because if an interest group hires a lobbyist, the lobbyist explore the legislative space on behalf of the interest group. This means that the interest group who is employer of lobbyist barely accumulates any experience what the lobbyist has experienced. In contrast, if an interest group choose the next legislator based on their own prior belief, we update the prior belief of interest group j as usual.

3.3.1 Simulation III: Large Search Space with a Lobbyist

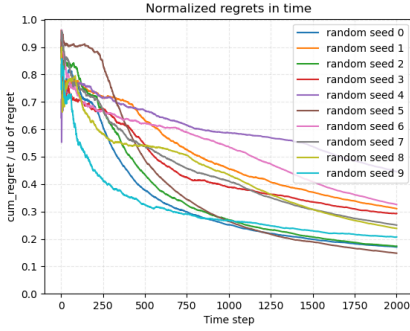
In this simulation, I maintain the size of the search space as same as the previous simulation by configuration of $|K| = 112, |C| = 26, |T| = 2000$ with a lobbyist $|L| = 1$ and $|J| = 5$. I assume that 5 different interest groups share the same category of interest. I choose $|J| = 5$ based on the

average number of clients per each issue code in *Lobbying Disclosure Act* data in 2020 to be more realistic.

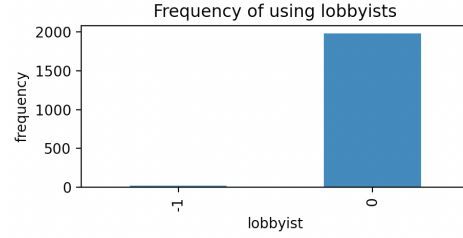
In this simulation, the mean of cumulative normalized regrets at time T from 10 different random seeds⁶ is 0.25 with the variance of 0.007 which is lower than the previous simulation. (See Fig 3a). In addition, with lobbyist and group of interest groups who share the same category of interest, an interest group found the best legislator as shown in Fig 3c. This is possible because multiple agents share the same lobbyist and collectively update the prior belief of the lobbyist by keep selecting the lobbyist. Fig 3b shows that the interest group keep selecting into using the lobbyist's prior rather than its own prior. This is because the lobbyist's prior finds the best rewarding legislator fast compared to its own prior.

3.4 Lobbying as a Delegated Information Acquisition Process

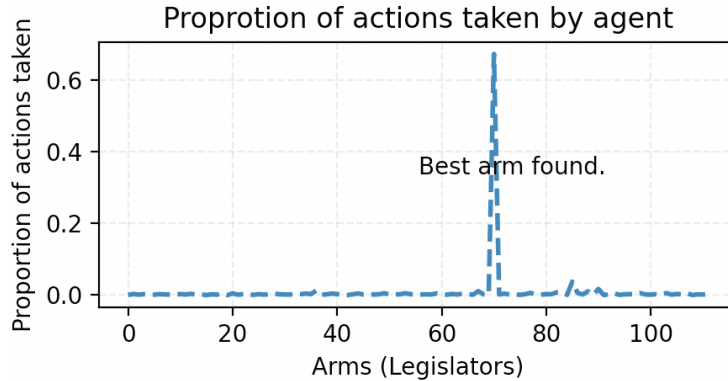
Simulations in this section suggest a supportive evidence that lobbyists can be used to find the best legislators when interest groups face large search space problem. If the interest groups share the same category of interest, they can form a collaborative relationship via lobbyist to find the best legislator for their shared category of interest. This can be interpreted as interest groups are forming a coalition through lobbyist. For example, associations are one of the most common type of registered lobbyists in the United States. With this modeling, we can understand that the associations are coalition between members to successfully explore the legislative space by concentrating their resources.



(a) Cumulative normalized regret of interest groups in large search space with lobbyist



(b) Frequency of using lobbyist. 0 represents using lobbyist and -1 represents not using lobbyist.



(c) The proportion of each legislator taken by an interest group across legislators for random seed 0.

Figure 3: Simulation in case of Large Search Space with Lobbyist

⁶Each random seed corresponds to the different set of categorical distributions of legislators.

4 Specialization of Lobbyists and Expert knowledge

In this section, I explain why lobbyists lobby both sides of legislators in reality by comparing the simulation results from the two different types of expert knowledge of lobbyists. This “both sides lobbying” is regarded as a puzzle⁷ in the literatures because lobbyists don’t need to lobby already supportive legislators according to the existing theories on lobbying.

In the section 3.3, I assumed that each lobbyist l has its own prior belief of legislators $\alpha_{11}, \alpha_{12}, \dots, \alpha_{1K} \in \mathbb{N}^C$ which are K number of concentration parameter of dirichlet distributions. It can be visualized as shown in Fig 4. A lobbyist can either have expert knowledge on a specific issue area across all legislators (blue in Fig 4) or have expert knowledge on a specific legislator (red in Fig 4) across all issue areas.

I conjecture that lobbyists can be specialized in case lobbyists having expert knowledge on a specific issue area across all legislators rather than having expert knowledge on a specific legislator across all issue areas. If this is true, lobbyists will lobby both sides of legislators to acquire the expert knowledge on a specific issue area across all legislators.

Categories (Issue Area) $\{1, 2, \dots, C\}$

Legislator 1						
Legislator 2						
Legislator 3						
Legislator 4						
\vdots						
Legislator K-3						
Legislator K-2						
Legislator K-1						
Legislator K						

Figure 4: Prior belief of lobbyist is parameterized by $K \times C$ matrix.

4.1 Simulation Setting

In this simulation, I use $|K| = 112, |C| = 26, |T| = 2000, |J| = 10$ and $|L| = 2$. Partition 10 number of interest groups into two groups of 5 interest groups each with the shared category of interest c_1 and c_2 for each group respectively. In addition, I also define a category of expert knowledge of a lobbyist l as $\phi(l)$. For the two lobbyist l_1 and l_2 , I assign $\phi(l_1) = c_1$ and $\phi(l_2) = c_2$.

First, to model the expert knowledge of lobbyists on a specific issue area across all legislators, I preset the prior belief of lobbyists to be close to the categorical distribution of legislators. In other words, by setting $\alpha_{lk}^{(c)} = p_k^{(c)} \cdot |C| \cdot \delta$ and $\alpha_{lk}^{(-c)} = (|C| - p_k^{(c)} \cdot |C| \cdot \delta) / (|C| - 1) \forall k$, I introduced expert knowledge on a specific issue area c across all legislators. $\delta \in [0, 1]$ is temperature parameter to control the degree of expert knowledge.

⁷Puzzle refers to the phenomenon that existing theories can’t explain in political science.

Second, to model the expert knowledge of lobbyists on a specific legislator across all issue areas, I set $\alpha_{lk^*}^{(c)} = p_{k^*}^{(c)} \cdot |C| \cdot \delta + \epsilon \forall c$ where $k^* = \operatorname{argmax}_K \{p_1^{(\phi(l))}, p_2^{(\phi(l))}, \dots, p_K^{(\phi(l))}\}$. ϵ is a small number to prevent concentration parameter from being zero by $\delta \in [0, 1]$. By doing this, I can gradually introduce expert knowledge on a specific legislator k^* across all issue areas.

With this setting, I ran the simulation and measure the ratio of hiring “matching” lobbyist. Since each group of interest groups has the shared category of interest c_1, c_2 and two lobbyist l_1, l_2 have the expert knowledge on c_1, c_2 , I expect that the ratio of hiring “matching” lobbyist indicates that how much lobbying industry is specialized and lobbyist can be selected by their clients based on their expert knowledge.

Figure 5 shows the simulation results. The simulation is ran twice for two different types of expert knowledge of lobbyists. The figure shows that only in case of expert knowledge on a specific issue area across all legislators, the ratio of hiring “matching” lobbyist is higher than 0.5 which is a baseline of randomly selecting lobbyist. In case of lobbyist having expert knowledge on a specific issue area across all legislators, ratio of hiring "matching" lobbyist converges to 1 as the temperature parameter δ increases. However, in case of lobbyist having expert knowledge on a specific legislator across all issue areas, ratio of hiring "matching" lobbyist never become clearly higher than 0.5. This result indicates that lobbyists are specialized in lobbying industry toward the direction to acquire expert knowledge on a specific issue area across all legislators. By doing so, lobbyists can be selected by their clients and survive in the market.

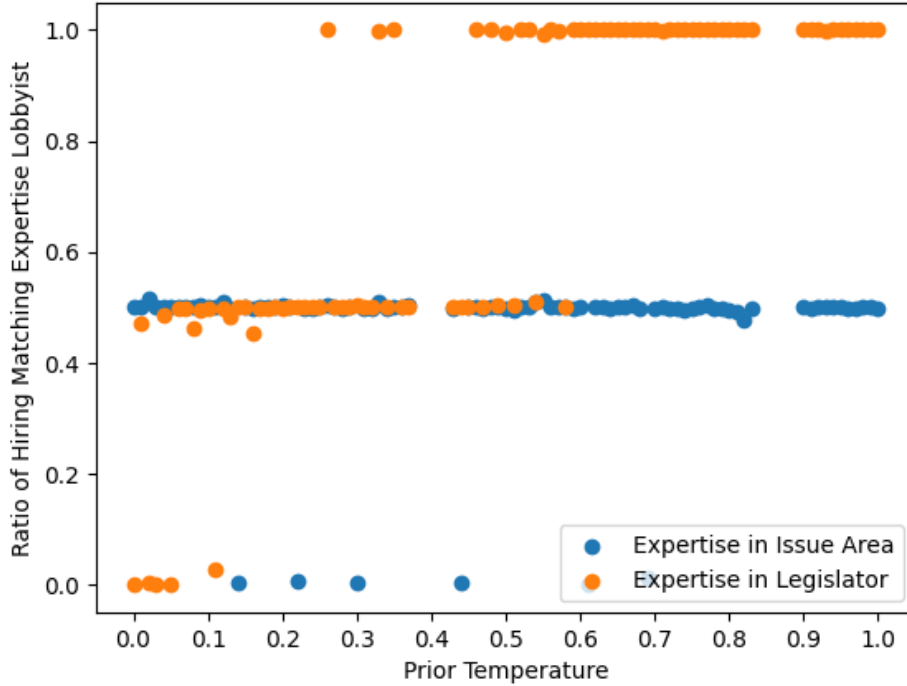


Figure 5: Mean ratio of hiring “matching” lobbyist for two different types of expert knowledge of lobbyist.

5 Conclusion and Limitation

In this work, I modeled the lobbying industry as a multi-armed bandit problem. With this modeling, I found that interest groups are facing large search space problem and they can overcome this problem by using lobbyists by simulations. In addition, I found that lobbyists are specialized in lobbying industry with their expert knowledge on a specific issue area across all legislators. This finding

explains a puzzle in the literatures why lobbyists lobby to both sides of legislators. This is because lobbyists need to equip themselves with expert knowledge on a specific issue area across all legislators. Therefore, they need to keep in touch with supportive legislators as well as opposing legislators.

Although this work shed a light on understanding lobbying industry, there are still many limitations in this work. First, this work uses a hypothetical categorical distribution of legislators' authority over different issue areas. Since this distribution is hypothetical, conclusion from a few random seeds might be limited to extend to the understanding of real world. Second, this work assumes each interest group has a unique category of interest. However, in reality, each interest group has multiple categories of interest. In addition, those categories are not orthogonal to each other as this work assumes. Third, this work provides simulation results with very small number of interest groups and lobbyists. In reality, there are more than 70,000 interest groups and 20,000 lobbyists. Therefore, this work needs to be extended to a larger scale to model the lobbying industry more realistically.

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