



Finding the Sufficient Condition for the Formulation of Lobbying Industry

via Simulative Experiment Using Multi-Agent Multi-Armed Bandit
Problem

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- What is lobbying?
 - ▶ This question is answered by different questions in different literatures, but mostly about role of lobbyist or goal of lobbying.
 - ★ What do lobbyists do?
 - ★ Buying vote?
 - ★ Influence policy or legislators?
- Answer this question by
 - ▶ Finding the **sufficient condition** that makes lobbying industry.
 - ▶ Search for the condition that clients hire lobbyists by simulation experiment.



- Yun and Preston (2022) define lobbying as **delegated information acquisition** process.
 - ▶ Interest groups in lobbying industry hires lobbyists to acquire information about legislative space.
 - ▶ Lobbyists are the agents that acquire information about legislative space on behalf of their clients.
 - ▶ This view is different from the most of views that understand lobbying as a process of buying vote or influencing legislators.

How to Acquire Information?



- By **interaction** with legislators.
 - ▶ Campaign contribution
 - ▶ Meeting with congressional staffers, etc.
- How to simulate this interaction?
 - ▶ By using Multi-Armed Bandit (MAB) problem.

What is Multi-Armed Bandit¹ (MAB) problem?



- Formulate the exploration-exploitation dilemma problem.
 - ▶ Formulate the **exploration-exploitation** dilemma problem.
 - ▶ Assume that there are K possible choices (called “arms”) for the agent to make.
 - ▶ Each arm has a reward probability P_k that is unknown to the agent.
 - ▶ Whenever the agent chooses an arm, it receives a reward r_k with probability P_k .
 - ▶ The agent choose one of the arms for T times and tries to maximize the total reward.
 - ▶ As the agent sequentially choose the arms, the agent builds his own estimate of the reward probability P_k of each arm.
 - ▶ In this scenario, the agent keep facing the **exploration-exploitation** dilemma whether to choose the best rewarding arm based on his current estimates (so called “exploitation”) or to try another arm to improve the current estimates because the current estimates can be biased (so called “exploration”).

¹Bandit is pejorative name for slot machine because it empties players' pocket



- Interest groups face the Multi-Armed Bandit problem when they participate information acquisition process in legislative space.
- There are K number of legislators that interest groups can interact with.
- Each legislator has a reward probability P_k that is unknown to interest groups.
- Interest groups has a limited budget and time to interact with legislators. (Model it as T times of interaction)
- How to balance between exploration and exploitation to find the best fit legislator within this limited chances of interactions? (MAB problem)

Formulation of MAB Problem of Interest Groups



- There exists $C \in \mathbb{N}$ number of categories of interest.
- Assume there are $j \in J$ number of interest groups with $\phi(j) : J \rightarrow C$ which represents unique category of interest $\phi(j)$ for each interest group.
- There are K number of legislators with P_k reward distribution.
- P_k is modeled as a **Categorical distribution** with C number of categories. P_k is parameterized by $\mathbf{p}_k = [p_k^{(1)}, p_k^{(2)}, \dots, p_k^{(C)}]$ where $p_k^{(i)} \in [0, 1], \sum p_k^{(i)} = 1$ with the support of $x_k \in \{1, 2, \dots, C\}$. (In other words,)
- Whenever an interest group j interact with a legislator k , they receive $c \in \{1, 2, \dots, C\}$ sampled from P_k .
- Each interest group j gets reward of $r_j^{kn} = \mathbb{1}(x_j^{kn} = \phi(j))$ when j choose legislator k at time t and sampled x_j^{kn} from P_k .
 - ▶ In other words, interest group j with $\phi(j) = c$ gets reward of 1 when they sample c from interaction with legislator k .

How to solve MAB problem?



- Use **Thompson Sampling** algorithm.
 - ▶ Each interest group j has their own prior belief over \mathbf{p}_k .
 - ▶ Keep updating this prior belief over \mathbf{p}_k using the sampled observations from the interaction with legislators.
 - ▶ The Dirichlet distribution is the conjugate prior of the categorical distribution.
 - ★ $f(\mathbf{p}_k|\mathcal{O}) \propto \mathcal{L}(\mathcal{O}|\mathbf{p}_k)f(\mathbf{p}_k)$ where $\mathbf{p}_k \sim \text{Dirichlet}(\mathbf{D}_k)$ with $\mathbf{D}_k = [d_k^{(1)}, d_k^{(2)}, \dots, d_k^{(C)}] \in \mathbb{R}^C$ and \mathcal{O} represents sampled observations from the interaction with legislators, i.e. $\mathcal{O} = [x_{j,t=1}^{k_1}, x_{j,t=2}^{k_2}, x_{j,t=3}^{k_3}, \dots]$ where $x_{j,t}^{k_t}$ is an observation sampled from the interaction with the legislator k at time t .
 - ★ This is a C dimensional generalization of the **Beta conjugate with Bernoulli likelihood**.
 - ★ As we did in Beta conjugate, we can update the prior belief by simply adding 1 to d_c when observe $x_j = c \in C$ at each step $t \in T$.
 - ▶ After update, choose the best rewarding legislator k based on the samples from posterior distributions $\{f(\mathbf{p}_k|\mathcal{O})|k \in \{1, 2, \dots, C\}\}$ for the next legislator to interact with.

Visual Representation of Belief Parameter



Categories (Issue Area) $\{1, 2, \dots, C\}$

Legislator 1					
Legislator 2					
Legislator 3					
Legislator 4					
\vdots					
Legislator K-3					
Legislator K-2					
Legislator K-1					
Legislator K					

- $K \times C$ matrix.
- Starts from flat prior (all entries are 1)
- Add 1 whenever observe c from the interaction with the legislator k .

Validate Thompson Sampling Algorithm

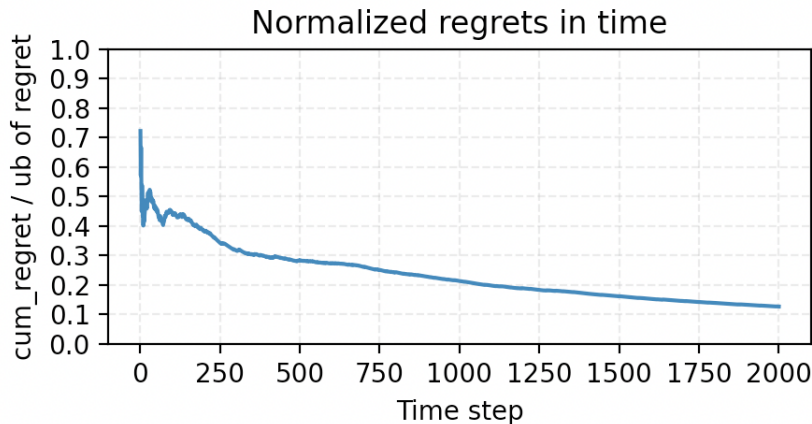


- Let's check whether the Thompson Sampling algorithm can actually find the best rewarding legislator within the limited number of interactions.
- We use the metric called **Regret** to measure the performance of the algorithm for MAB problem.
- $\text{Regret}_t^j \triangleq p_c^* - p_{a_t}^j$ where $c = \phi(j)$ and $p_c^* = \text{argmax}_{k \in K} \{p_1^{(c)}, p_2^{(c)}, \dots, p_K^{(c)}\}$ and $p_{a_t}^j$ is the c th parameter of P_k when the agent choose the legislator k at time t . a_t represent the action (choice of legislator) taken by the agent at time t .
- Regret represents how much the agent could have been done better in terms of the reward if it had chosen the best action.
- Similarly, Cumulative Regret $\text{Regret}_t^c \triangleq \sum_t^T p_c^* - p_{a_t}^j$.
- Agent tries to minimize the cumulative regret for the entire time horizon T .

Simulation Results I: Small Search Space



- $|K| = 32, |C| = 4, |T| = 2000$.

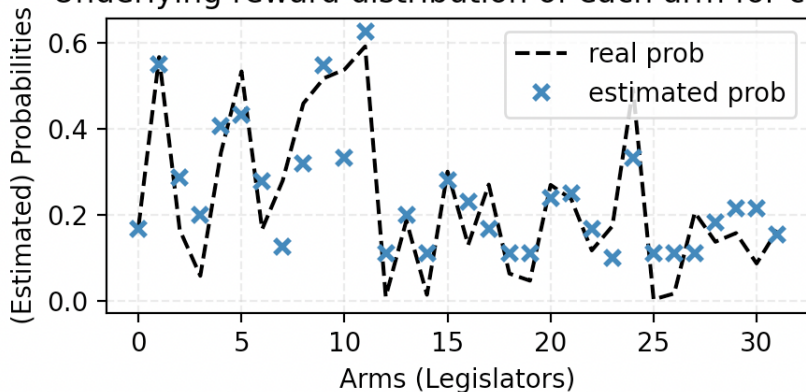


- Total $32 \times 4 = 128$ number of parameters to explore - which is relatively small compared to real world.

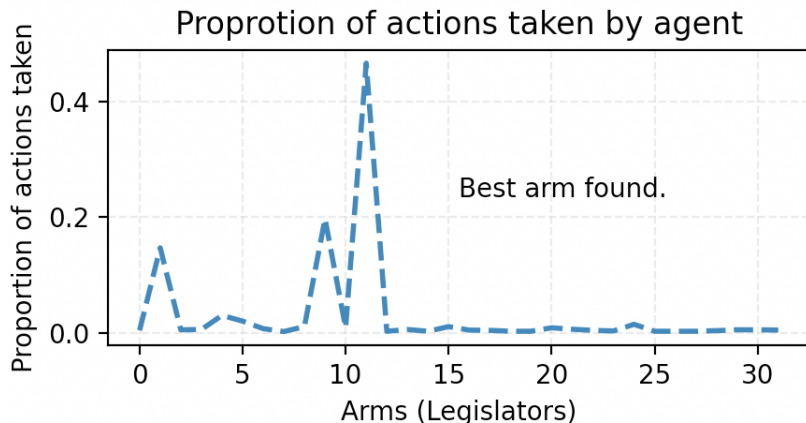
Simulation Results I: Small Search Space



Underlying reward distribution of each arm for coi 0



- Each x-tick corresponds to $p_{k=1}^c, p_{k=2}^c, p_{k=3}^c, \dots, p_{k=32}^c$ where the agent has a category of interest c
- $K = 11$ is the best rewarding legislator for category c .

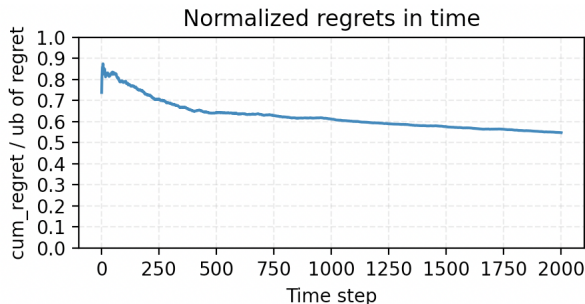


- The agent successfully finds the best rewarding legislator $K = 11$ and exploited it.

Simulation Results II: Large Search Space



- $|K| = 112^2, |C| = 26^3, |T| = 2000$.



- Total $112 \times 26 = 2912$ number of parameters to explore - which is relatively large compared to the previous case. Hard to explore all the parameters with the same time horizon of $T = 2000$.

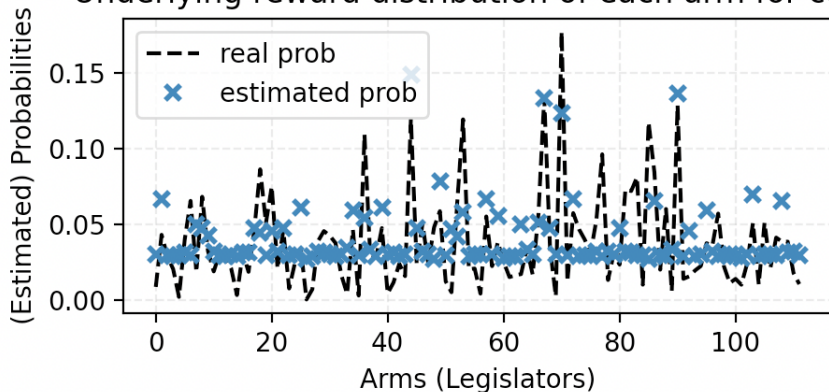
²Average number of legislators to whom top 10 lobbying firms campaign contribute in 2020.

³Average number of issue codes per client.

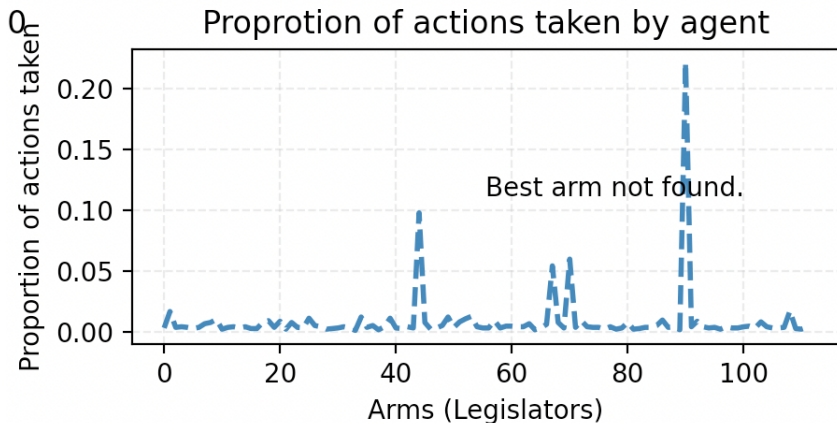
Simulation Results II: Large Search Space



Underlying reward distribution of each arm for category C



- Each x-tick corresponds to $p_{k=1}^c, p_{k=2}^c, p_{k=3}^c, \dots, p_{k=112}^c$ where c is the agent's category of interest.
- $K = 68$ is the best rewarding legislator for category c .
- Most of the legislators are not explored properly.



- The agent failed to find the best rewarding legislator $K = 70$ and exploited the worse rewarding legislator $K = 90$.



- Interest groups fail to find the best rewarding legislator in case of a large search space.
- How to solve this large search space problem?
- **Conejcture:**
 - ▶ Interest groups use lobbyists to successfully explore the large search space.
 - ▶ A lobbyist has multiple interest groups as their clients and the clients concentrates the resources to explore to the lobbyist.
 - ▶ Lobbyist explore legislative space and share the observations to their clients.
- **Hypothesis:**
 - ▶ Interest groups can successfully explore the large search space by concentrating their resources to their lobbyist by delegating the exploration of legislative space.

How to Introduce Lobbyist into MAB formulation?

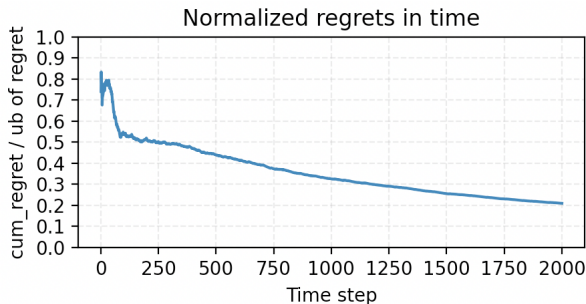


- Introduce new $l \in L$ arms which represent lobbyists.
- Each lobbyist arm l has a set of parameters of Dirichlet distribution for each legislator $\{D_k^l | k \in 1, 2, \dots, K\}$ as same as any other interest groups.
- Allow any interest groups to choose any lobbyists at each step.
- If an interest group choose to use a lobbyist, the interest group determine with which legislator to interact based on the current posterior distributions of the lobbyist.
- After getting observation from the chosen legislator, the interest group update the parameter of the lobbyist's parameter of Dirichlet distribution, not its own.
 - ▶ **Pros:** Interest group can take advantage of the lobbyist's knowledge of the legislators' reward distribution. This distribution could be less biased because many interest groups can collectively update the lobbyist's parameters.
 - ▶ **Cons:** Interest group can not update its own distribution. This means that the interest group can not learn from its own experience.

Simulation III: Large Search Space with Lobbyist



- $|K| = 112^4, |C| = 26^5, |T| = 2000, |IG| = 5^6, |L| = 1$.



- Lowest normalized regret achieved without lobbyist was 0.55.
- With lobbyist, it achieves 0.2 of the lowest normalized regret.

⁴Average number of legislators to whome top 10 lobbying firms campaign contribute in 2020.

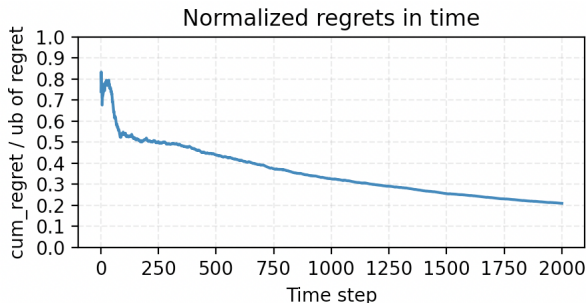
⁵Average number of issue codes for each client in 2020 (from Lobbying Disclosure Act data).

⁶Average number of clients that top 10 lobbying firms has for one issue area in 2020

Simulation III: Large Search Space with Lobbyist



- $|K| = 112^7$, $|C| = 26^8$, $|T| = 2000$, $|IG| = 5^9$, $|L| = 1$.



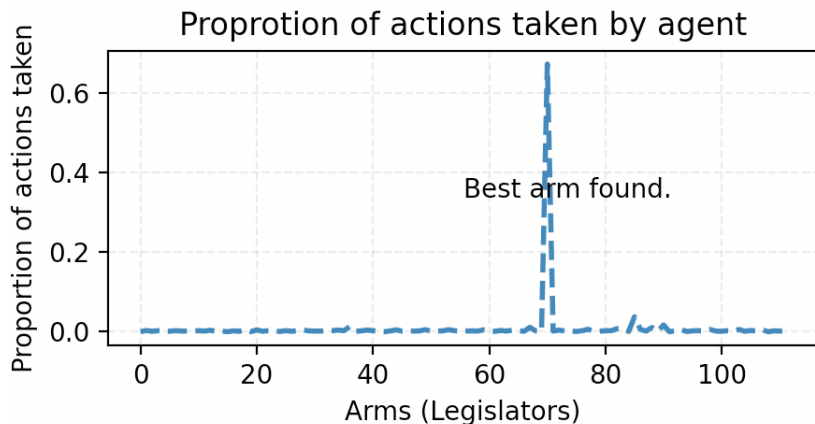
- Lowest normalized regret achieved without lobbyist was 0.55.
- With lobbyist, it achieves 0.2 of the lowest normalized regret.

⁷Average number of legislators to whom top 10 lobbying firms campaign contribute in 2020.

⁸Average number of issue codes appearing in each lobbying report in 2020.

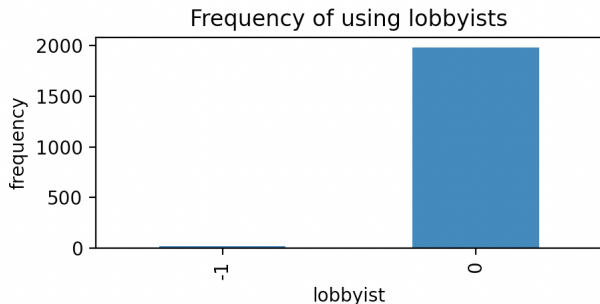
⁹Average number of clients that top 10 lobbying firms has for one issue area in 2020

Simulation III: Large Search Space with Lobbyist



- The agent successfully find the best rewarding legislator $K = 70$ and exploit it.
- This is possible because 5 number of agents (IGs) collaboratively update the lobbyist's parameters.

Simulation III: Large Search Space with Lobbyist

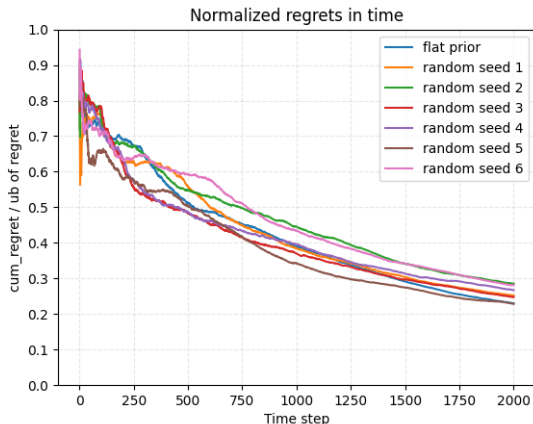


- For this simulation, we used **flat prior (all 1s)** for the lobbyist's parameters.
- This means that they have **no expertise** over the legislators' reward distribution at $t = 1$.
- Even in this situation, agents (IGs) quickly choose to use lobbyist and collaborate using lobbyist.

Simulation III: Large Search Space with Lobbyist



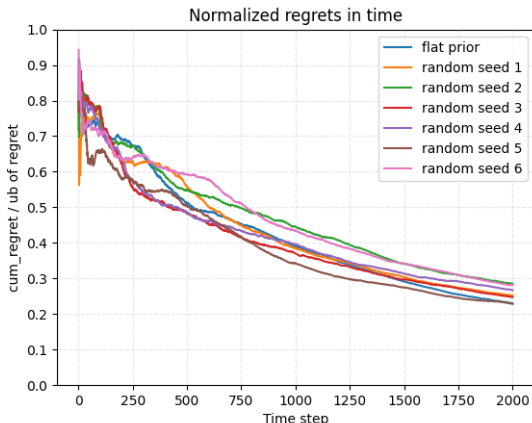
- Regret plot of the simulation with lobbyist with **flat prior** and **random prior**.
- Regardless of the expertise of the lobbyist, the agents (IGs) use lobbyist for collaboration and successfully solve the large search space problem.



Simulation III: Large Search Space with Lobbyist



- This implies that what forms the lobbying industry is not the expert knowledge of the lobbyist, but the ability to concentrate the resource of exploration from multiple clients (IGs).
- In other words, **delegation of exploration** is the main reason for the lobbying industry to be formed.





- Although the lobbyist's expertise is not the main reason for the lobbying industry to be formed, it is still important in terms of specialization.
- In reality, IGs who share the same topic of interest tend to hire the same lobbyist. (**specialization**)
- There are two different types of expertise of lobbyist:
 - ▶ **Expertise in a Legislator:** Knowing the reward distribution of a specific legislator.
 - ▶ **Expertise in an Issue Area:** Knowing the reward distribution of a specific issue area across legislators.
- Which type of expert knowledge is important for the lobbying industry to be *specialized*?

Two Different Types of Expert Knowledge



- **Expertise in a Legislator:** Knowing the reward distribution of a specific legislator. (Red)
- **Expertise in an Issue Area:** Knowing the reward distribution of a specific issue area across legislators. (Blue)

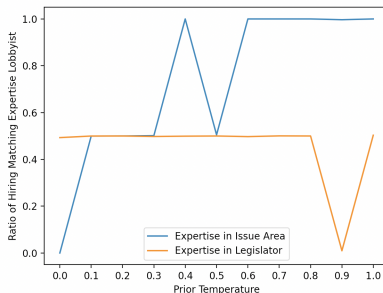
Categories (Issue Area) $\{1, 2, \dots, C\}$

Legislator 1						
Legislator 2						
Legislator 3						
Legislator 4						
\vdots						
Legislator K-3						
Legislator K-2						
Legislator K-1						
Legislator K						

Simulation IV: Condition for Specialization



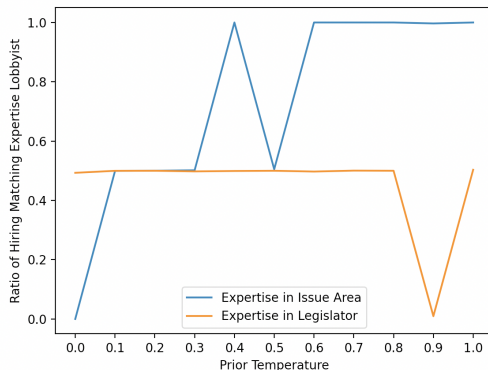
- $|K| = 112, |C| = 26, |T| = 2000, |IG| = 10^{10}, |L| = 2$.



- X-axis: How lobbyist closely knows the reward distribution of a legislator or an issue area.
- Y-axis: How well specialization is achieved .
- Knowing the reward distribution of an issue area across legislators is more important for specialization than knowing the reward distribution of a legislator across issue areas.

¹⁰5 for each category of interest 0 and 1

Simulation IV: Condition for Specialization



- This sheds light on the puzzle that 1) why lobbyist lobby the already like-minded legislators and 2) why lobbyist campaign contribution to legislators of both sides (for/against) of topic of interest.



- Explicitly and formally model the lobbying industry as a multi-agent system.
 - ▶ Thanks to formal modeling, we can plug-in any representations learned from the real world data to approximate the real-world more closely. (e.g. Reward distribution of legislatures)
- Simulatively shown that the lobbying industry is formed because of the large search space problem.
- Provided a simulative ground for a new theory to solve the puzzle why lobbyist tend to lobby both sides.