

# CS113/DISCRETE MATHEMATICS-SPRING 2024

## Worksheet 20

### Topic: Structural Induction

Let's continue our exploration of Structural Induction by engaging in more proof exercises to further solidify our understanding of this topic. Happy Learning!

Student's Name and ID: \_\_\_\_\_

Instructor's name: \_\_\_\_\_

1. Consider the following recursively defined Set.

(i)  $A \in S$

(ii) If  $x \in S$ , then  $(x)$  in  $S$ .

Prove using Structural Induction that every element in  $S$  contains equal number of parentheses.

2. Consider the following recursively defined Set.

(i)  $6 \in S$ ,  $15 \in S$

(ii) If  $x, y \in S$ , then  $x + y \in S$ .

Prove using Structural Induction Show that every element of  $S$  is divisible by 3.

3. Let  $j$  denote the empty string. Let  $A$  be any finite nonempty set. A palindrome over  $A$  can be defined as a string that reads the same forward as backward. For example, “mom” and “dad” are palindromes over the set of English alphabets.

1.  $j \in S$

2.  $\forall a \in A, a \in S$

3.  $\forall a \in A \forall x \in S, axa \in S$

4. All the elements in  $S$  must be generated by the rules above.

Prove by structural induction that  $S$  equals the set of all palindromes over  $A$ .