

CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 15

Topic: Mathematical Induction

Today, we will explore mathematical induction—a powerful tool in mathematics. We will see how it can be used to prove summation formulas and inequalities, providing a systematic and rigorous approach to mathematical reasoning. Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1. Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ whenever n is a nonnegative integer.

2. Prove that $\sum_{j=0}^n \left(\frac{-1}{2}\right)^j = \frac{2n+1+(-1)^n}{3 \cdot 2^n}$ whenever n is a nonnegative integer.

3. Prove that $2n > n^2$ if n is an integer greater than 4.

4. For which nonnegative integers n is $n^2 \leq n!$? Prove your answer.