CS113/DISCRETE MATHEMATICS-SPRING 2024

Quizzes

August 1, 2023

1. WEEK2 QUIZ
   (a) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

   (b) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent using laws of equivalence.

   (c) Show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

2. WEEK3 QUIZ
   (a) Determine the truth value of the statement $\forall x \exists y(xy = 1)$ if the domain for the variables consists of:
   a)the nonzero real numbers.
   b) the nonzero integers.
   c) the positive real numbers.

   (b) Prove or Disprove that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x(P(x) \wedge Q(x))$ are not logically equivalent.

   (c) Use quantifiers to express:
   a)Associative Law of multiplication and Addition
   b) Commutative Law of Addition and Multiplication
   c) Distributive Law of Multilication Over Addition
   d) The fact that a quadratic polynomial with real number coefficients has at most two real roots.

   (d) Express each of these mathematical statements using predicates, quantifiers, logical connectives, and mathematical operators.Then, find a counterexample, if possible when the domain consists of all integers.
   a) For all values of number x and number Y, if square of x equals square of y, then x equals y.
   b) For every number x there is a number y such that square of x equals y.
   c) The product of any two integers x and y is equal or greater than x.

3. WEEK4 QUIZ
   (a) Professor Enigma left a series of clues regarding the location of a missing artifact. The artifact can only be in one place. If the artifact is in the library, then it is hidden behind a bookshelf. If the artifact is not in the library or it is buried in the garden, then the statue in the backyard is made of bronze and the statue in the front yard is not made of marble. If the artifact is in the attic, then the statue in the front yard is not made of marble. If the artifact is not buried in the garden, then the statue in the backyard is not made of bronze. The artifact is not in the library. Using rules of inference, determine where the missing artifact is located.

(b) Detective Sherlock has to solve the case of a stolen painting. The painting can only be in one place. If the painting is in the art gallery, then it is displayed on the east wall. If the painting is not in the art gallery or it is hidden in the vault, then the security alarm is deactivated and the security cameras are not turned off. If the painting is in the study room, then the security cameras are not turned off. If the painting is not hidden in the vault, then the security alarm is activated. The painting is not in the art gallery. Using rules of inference, determine where the stolen painting is located.

(c) Captain Adventure is on a quest to find a legendary treasure chest. The treasure can only be in one place. If the treasure is on the island, then it is buried in the sand. If the treasure is not on the island or it is hidden in the cave, then the ancient symbol on the rock is of a snake, and the ancient symbol on the tree is not of an eagle. If the treasure is in the cave, then the ancient symbol on the tree is not of an eagle. If the treasure is not hidden in the cave, then the ancient symbol on the rock is not of a snake. The treasure is not on the island. Using rules of inference, determine where the legendary treasure chest is hidden.

(d) Justify the rule of universal transitivity, which states that if $\forall x(P(x) \rightarrow Q(x))$ and $\forall x(Q(x) \rightarrow R(x))$ are true, then $\forall x(P(x) \rightarrow R(x))$ is true, where the domains of all quantifiers are the same.

(e) Justify the rule of universal modus tollens by showing that the premises $\forall x(P(x) \rightarrow Q(x))$ and $\neg Q(a)$ for a particular element a in the domain, imply $\neg P(a)$.

4. WEEK 5 QUIZ

(a) Prove that if $m+n$ and $n+p$ are even integers, where m, n, and p are integers, then $m+p$ is even. What kind of proof did you use?

(b) Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

(c) Use a proof by contraposition to show that if $x+y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

5. WEEK 6 QUIZ

(a) Prove Demorgans law of Intersection $\overline{A \cap B} = \overline{A} \cup \overline{B}$by showing that:
i)Each side is a subset to each other
ii)By using membership table

(b) Prove the second distributive law, which states that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
i)Each side is a subset to each other
ii)By using membership table

6. WEEK 7 QUIZ

(a) Prove that the function F: Z $\rightarrow$ Z defined as $f(n) = n^2$ is neither an injective nor a surjective function. Can you make this function Injective without doing any changes to the original function?

(b) Prove that the inverse of the function F: Z $\rightarrow$ Z defined as $f(n) = n + 1$, is also a function.

(c) A function $f : I \rightarrow R$ is strictly increasing on an interval I if for all $x_1, x_2 \in I$ with $x_1 < x_2$, it holds that $f(x_1) < f(x_2)$. Here "I" represents the interval on which the function is strictly increasing. Given the definition of strictly increasing function, prove that a strictly increasing function is always injective.

(d) A function $f : I \to R$ is strictly decreasing on an interval I if for all $x_1, x_2 \in I$ with $x_1 < x_2$, it holds that $f(x_1) > f(x_2)$. Here "I" represents the interval on which the function is strictly decreasing. Given the definition of strictly decreasing function, prove that a strictly decreasing function is always injective.

(e) Give an explicit formula for a function from the set of integers to the set of positive integers that is:
a) one-to-one, but not onto.
b) onto, but not one-to-one.
c) one-to-one and onto.
d) neither one-to-one nor onto

7. WEEK 8 QUIZ

(a) Derive summation formula for first n natural numbers.

(b) Find the solution to each of these recurrence relations and initial conditions.Hint: Use iterative Approach.
a) $a_n = 3a_{n-1}$, $a_0 = 2$

b) $a_n = a_{n-1} + 2$, $a_0 = 3$

c) $a_n = a_{n-1} + n$, $a_0 = 1$

d) $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$

e) $a_n = 2a_{n-1} - 1$, $a_0 = 1$

f) $a_n = 3a_{n-1} + 1$, $a_0 = 1$

g) $a_n = na_{n-1}$, $a_0 = 5$

h) $a_n = 2na_{n-1}$, $a_0 = 1$

(c) Let $a_n = 2n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$.

a) Find a_0, a_1, a_2, a_3, and a_4.

b) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

8. WEEK 9 QUIZ

(a) Give an example of two uncountable sets A and B such that $A \cap B$ is

a) finite.

b) countably infinite.

c) uncountable.

(b) Prove that the set $A = \{\ln(n) : n \in N\} \subseteq R$ is countably infinite.

(c) Prove that the set of all integers Z is a countable set.

(d) Prove that the set $A = \{(5n, -3n) : n \in Z\}$ is countably infinite.

(e) Prove or disprove: There exists a bijective function $f : Q \to R$.

(f) Show that the two given sets have equal cardinality by describing a bijection from one to the other. Describe your bijection with a formula (not as a table).
a)The set of even integers and the set of odd integers.

(g) If A is any set, then $|A| < |P(A)|$.

9. WEEK 10 QUIZ

   (a) For which nonnegative integers n is $n^2 \leq n!$? Prove your answer.

   (b) Prove that $1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n+1)! - 1$ whenever n is a positive integer.

   (c) Prove that $3n < n!$ if n is an integer greater than 6.

   (d) Prove that $2 - 2 \cdot 7 + 2 \cdot 7^2 - \ldots + 2(-7)^n = \frac{1-(-7)^{n+1}}{4}$ whenever n is a nonnegative integer.

10. WEEK 11 QUIZ

   (a) Use strong induction to show that all dominoes fall in an infinite arrangement of dominoes if you know that the first three dominoes fall, and that when a domino falls, the domino three farther down in the arrangement also falls.

   (b) A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

   (c) Prove that Handshaking theorem is valid.

11. WEEK 12 QUIZ

   (a) An undirected graph has an even number of vertices of odd degree.

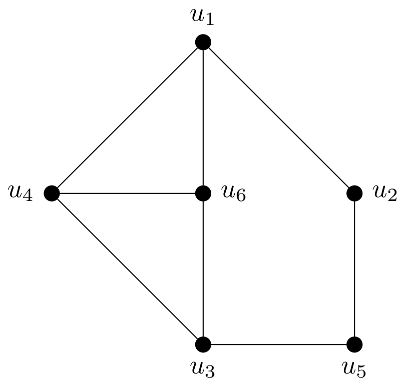
12. WEEK 13 QUIZ

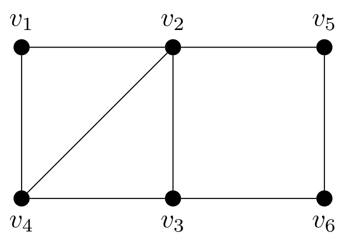
   (a) Prove or disprove K_6 is bipartite.

   (b) Consider a graph G such that at least one of its vertices 'v' is connected to all other vertices. Prove or disprove that G is not bipartite.

13. WEEK 14 QUIZ

   (a) Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.





14. WEEK 15 QUIZ

(a) The given graph represents the classrooms at school and the paths that connect them together. Your teacher asks you to go along all the paths to make sure they are clean and tidy, but you only have a few minutes before the start of class so you can't go over any path twice.

1. What kind of path are you looking for? Euler/Hamilton
2. Can you find a path through the graph?
3. Can you find a circuit? (It might not have one)

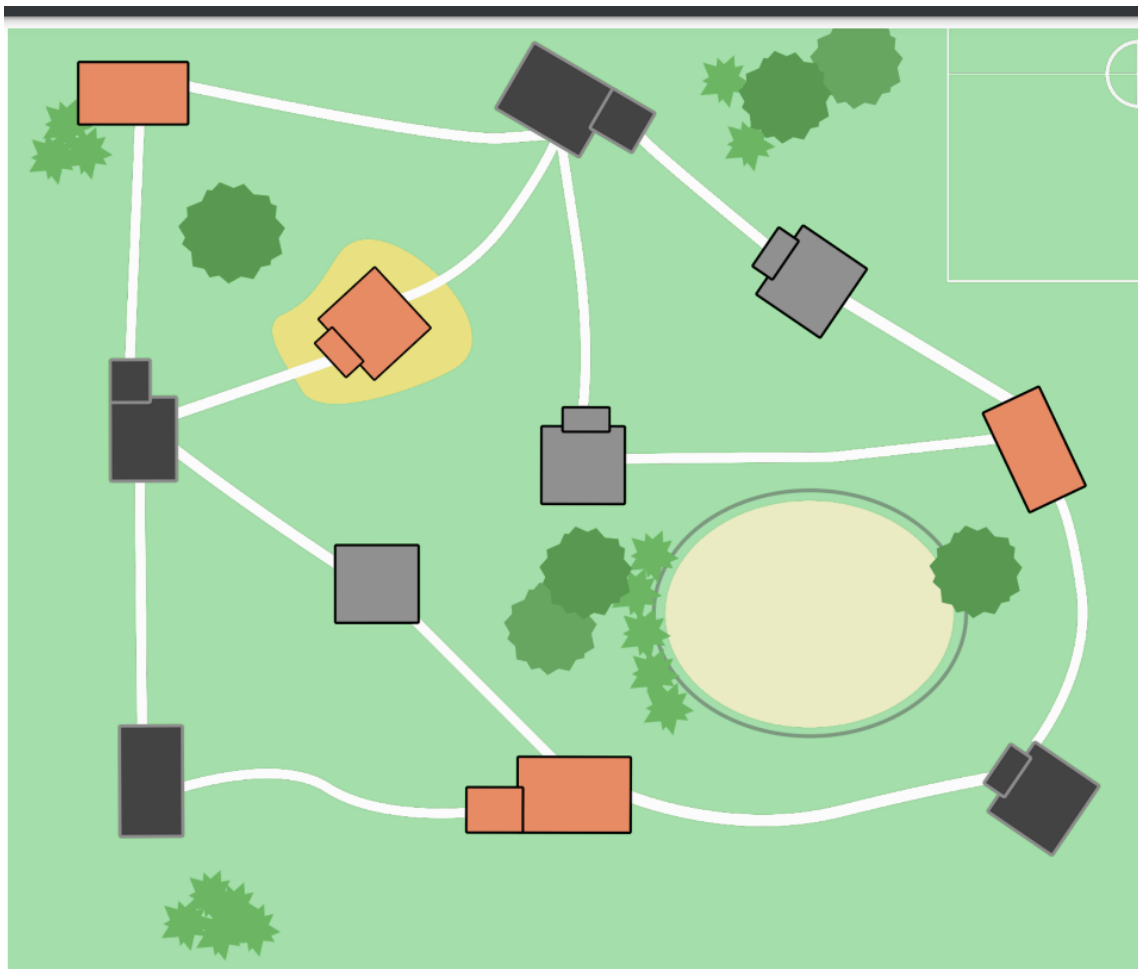


Figure 1: This graph represents the classrooms at school and the paths that connect them together.