

CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 2

Topic: Logic And Proofs

Use the given tables of laws/Truth tables to construct equivalences between compound propositions. Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1 Laws Of Logical Equivalences:

Equivalence	Rule
Identity	$P \wedge T \equiv P$
Domination	$P \vee F \equiv P$
Double Negation	$\neg(\neg P) \equiv P$
Idempotent	$P \vee P \equiv P$
Commutative	$P \vee Q \equiv Q \vee P$
Associative	$(P \vee Q) \vee R \equiv P \vee (Q \vee R)$
Distributive	$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
De Morgan's	$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
Absorption	$P \vee (P \wedge Q) \equiv P$

Equivalence	Rule
Conditional Law 1	$p \rightarrow q \equiv \neg p \vee q$
Conditional Law 2	$p \rightarrow q \equiv \neg q \rightarrow \neg p$
Conditional Law 3	$p \vee q \equiv \neg p \rightarrow q$
Conditional Law 4	$p \wedge q \equiv \neg(p \rightarrow \neg q)$
Conditional Law 5	$\neg(p \rightarrow q) \equiv p \wedge \neg q$
Conditional Law 6	$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
Conditional Law 7	$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
Conditional Law 8	$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
Conditional Law 9	$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Equivalence	Rule
Biconditional Law 1	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
Biconditional Law 2	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
Biconditional Law 3	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
Biconditional Law 4	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

1. Show that following conditional statements are tautologies. (use laws of equivalences and not the truth table to prove it.)

(a)

$$(\neg p \wedge (p \vee q)) \rightarrow q$$

(b)

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

(c)

$$(p \wedge (p \rightarrow q)) \rightarrow q$$

(d)

$$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$$

2. Show using truth table that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

3. Show using truth table that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is tautology.