

CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 10

Topic: Functions (Injective, Surjective, And Bijective Functions)

Through this lesson, we will explore three important properties of functions: surjectivity (onto), injectivity (one-to-one), and bijectivity(one to one correspondence). Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1. Prove that the function $F : \mathbb{Z} \rightarrow \mathbb{Z}$ defined as $F(n) = n + 6$ is a bijection.

2. For each of the following functions, prove that the function is 1-1 or find an appropriate pair of points to show that the function is not 1-1:

(a) $F: \mathbb{R} \rightarrow \mathbb{R}$

$$f(n) = \begin{cases} n^2, & \text{for } n \geq 0 \\ -n^2, & \text{for } 0 \geq n \end{cases}$$

(b) $F: R \rightarrow R$

$$f(x) = \begin{cases} X + 1, & \text{for } x \in Q \\ 2x, & \text{for } x \notin Q \end{cases}$$

(c) $F: R \rightarrow R$

$$f(x) = \begin{cases} 3x + 2, & \text{for } x \in Q \\ x^3, & \text{for } x \notin Q \end{cases}$$

(d) $F: R \rightarrow R$

$$f(n) = \begin{cases} n + 1, & \text{for } n \text{ odd} \\ n^3, & \text{for } n \text{ even} \end{cases}$$

3. Show that no $f : \mathbb{R} \rightarrow \mathbb{R}$ is both increasing and strictly decreasing.