CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 9

Topic: Set Operations

Through this lesson, we will discover set operations such as union, intersection, and complement, and will learn how to manipulate sets using established techniques. Additionally, we will explore different proof methods, including the subset method, membership table, and application of existing identities, to establish the equality of sets. Happy Learning!

Student's Name and ID: ____________________

Instructor's name: ____________________

1 Set Identities:

Identity Name	**Identity**
Identity laws	$A \cap U = A$ $A \cup \emptyset = A$
Domination laws	$A \cup U = U$ $A \cap \emptyset = \emptyset$
Idempotent laws	$A \cup A = A$ $A \cap A = A$
Complementation law	$(A)' = A$
Commutative laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative laws	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's laws	$(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$
Absorption laws	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Complement laws	$A \cup A' = U$ $A \cap A' = \emptyset$

2 Different Proof Methods Involving Sets:

Method	**Description**
Subset method	Show that each side of the identity is a subset of the other side.
Membership table	For each possible combination of the atomic sets, show that an element in exactly these atomic sets must either belong to both sides or belong to neither side.
Apply existing identities	Start with one side, transform it into the other side using a sequence of steps by applying an established identity.

1. Prove the second De Morgan law in Table 1 by showing that if A and B are sets, then $\overline{A \cup B} = \overline{A} \cap \overline{B}$.
   (a) by showing each side is a subset of the other side.

(b) using a membership table

2. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$.

   (a) by showing each side is a subset of the other side.

(b) using a membership table

3. Let A and B be sets. Show that
   (a) $(A \cap B) \subseteq A$

(b) $A \subseteq (A \cup B)$

(c) $A - B \subseteq A$

(d) $A \cap (B - A) = \emptyset$

(e) $A \cup (B - A) = A \cup B$

4. Show that if A and B are sets in a universe U, then $A \subseteq B$ if and only if $A \cup B = U$.