# CS113/DISCRETE MATHEMATICS-SPRING 2024

## Worksheet 18

## Topic: Strong Induction

Building on our previous knowledge of Strong Induction, we will now solve some more complex proofs. Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1. Use strong induction to show that every positive integer can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on. [Hint: For the inductive step, separately consider the case where k + 1 is even and where it is odd. When it is even, note that $(k+1)/2$ is an integer.]

2. Suppose you begin with a pile of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute rs. Show that no matter how you split the piles, the sum of the products computed at each step equals $\frac{n(n-1)}{2}$.

3. Prove by strong induction that Breaking a chocolate bar with $n \geq 1$ pieces into individual pieces requires $n - 1$ breaks.