CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 18

Topic: Structural Induction

Let's continue our exploration of Structural Induction by engaging in more proof exercises to further solidify our understanding of this topic. Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1. Consider the following recursively defined Set.
   (i) $A \in S$
   (ii) If $x \in S$, then (x) in S.
   Prove using Structural Induction that every element in S contains equal number of parentheses.

2. Consider the following recursively defined Set.
   (i) $6 \in S$, $15 \in S$
   (ii) If $x, y \in S$, then $x + y \in S$.
   Prove using Structural Induction Show that every element of S is divisible by 3.

3. Let j denote the empty string. Let A be any finite nonempty set. A palindrome over A can be defined as a string that reads the same forward as backward. For example, "mom" and "dad" are palindromes over the set of English alphabets.

1. $j \in S$
2. $\forall a \in A, a \in S$
3. $\forall a \in A \forall x \in S, axa \in S$
4. All the elements in S must be generated by the rules above.

Prove by structural induction that S equals the set of all palindromes over A.