CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 20

Topic: Structural Induction

Today, we will explore Structural Induction - a powerful proof technique which we use to establish the truth of statements about recursively defined structures or objects. It's an incredibly useful approach, especially when dealing with data structures like lists, trees, and graphs, or any other entities defined recursively. Happy Learning!

Student's Name and ID: ____

Instructor's name: ____

1. Show that the set S, defined by $1 \in S$ and $s + t \in S$ whenever $s \in S$ and $t \in S$, is the set of positive integers.

2. Let S be the set of positive integers defined by Basis step: $1 \in S$. Recursive step: If $n \in S$, then $3n + 2 \in S$ and $n^2 \in S$.

   Show that if $n \in S$, then $n \equiv 1 \pmod 4$.

3. Let S be the subset of the set of ordered pairs of integers defined recursively by: Basis step: $(0, 0) \in S$. Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

   Use strong induction on the number of applications of the recursive step of the definition to show that 5 divides $(a + b)$ when $(a, b) \in S$.