CS113/DISCRETE MATHEMATICS-SPRING 2024

Worksheet 22

Topic: Graph Terminology and Special Types of Graphs

Today, we'll dive into the intriguing world of graphs and explore some special types, with a major focus on bipartite graphs. Happy Learning!

Student's Name and ID: _____

Instructor's name: _____

1 Bipartite Graphs:

A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair (V_1, V_2) a bipartition of the vertex set V of G.

The following Theorem provides a useful criterion for determining whether a graph is bipartite.

2 Theorem:

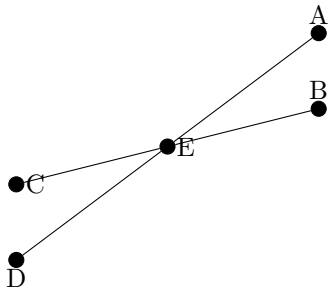
A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

1. Describe a graph model that represents whether each person at a party knows the name of each other person at the party. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

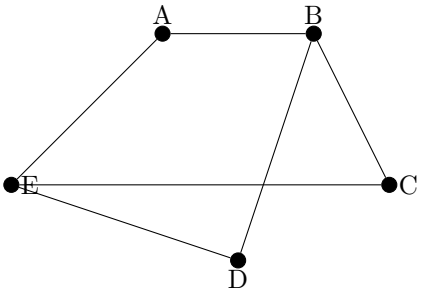
2. Can a simple graph exist with 15 vertices each of degree five?

3. . Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

4. Determine whether the graph is bipartite. You may find it useful to apply Theorem and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

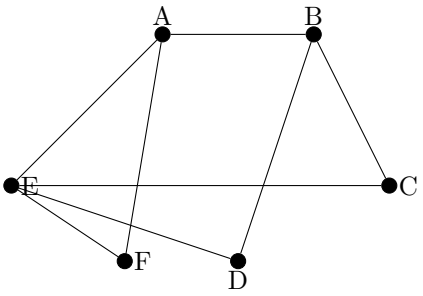


(a)



(b)

(c)



(d)