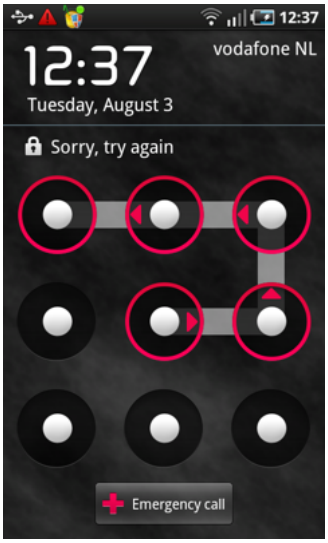
Draw to Unlock
Word Ladders

1. Week 5: Recursion and Permuations

1.1 Draw to Unlock

Rather than remembering passwords or passcodes, many mobile devices now allow the user to draw a pattern on the screen to unlock them.



Here we will explore how many unique patterns are available when drawing such patterns to connect "dots", such as shown in the figure. We assume that people put their finger on one "dot" and then only ever move one position left, right, up or down (but never diagonally) at a time. You are not allowed to return to a "dot" once it has been visited once. If we number the first position in our path as 1, the second as 2 and so on, then beginning in the top left-hand corner, some of the possible patterns of 9 moves are :

```
1 2 3      1 2 3      1 2 3
6 5 4      8 9 4      8 7 4
7 8 9      7 6 5      9 6 5
```

Exercise 1.1 Write a program that computes and outputs all the valid paths. Use **recursion** to achieve this.

- How many different patterns of length 9 are available on a 3×3 grid, if the user begins in the top left corner ?
- How many different patterns of length 9 are available on a 3×3 grid, if the user begins in the middle left ?
- How many different patterns of length 7 are available on a 3×3 grid, if the user begins in the top left corner ?
- How many different patterns of length 25 are available on a 5×5 grid, if the user begins in the top left corner ?

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1.2 Word Ladders

In this game, made famous by the author Lewis Caroll, and investigated by many Computer Scientists including Donald Knuth, you find missing words to complete a sequence. For instance, you might be asked how go from "WILD" to "TAME" by only changing one character at a time:

W I L D
W I L E
T I L E
T A L E
T A M E

A useful concept here is that of the `edit distance`. Here, the edit distance is a count of the number of characters which are different between two words. For words of n characters, the edit distance will be in the range $0 \ldots n$. For 'aboard' and 'canape' the edit distance is 5. An edit distance of zero means that the words are identical.

In a heavily constrained version of this game we make some simplifying assumptions:

- Words are always four letters long.
- We only seek ladders of five words in total.
- Only one letter is changed at a time.
- A letter is only changed from its initial state, to its target state. This is important, since if you decide to change the second letter then you will always know what it's changing from, to what it's changing to.

So, in the example above, it is enough to give the first word, the last word, and the position of the character which changed on each line. On line one, the fourth letter 'D' was changed to an 'E', on the next line the first character 'W' was changed to a 'T' and so on. The whole ladder can be defined by "WILD", "TAME" and the sequence $4, 1, 2, 3$.

W I L D
W I L E
T I L E
T A L E
T A M E

Since each letter changes exactly once, the order in which this happens is a *permutation* of the numbers $1, 2, 3, 4$, which we have looked at in Lectures.

Exercise 1.2 For the constrained version of the game, given a file of valid four letter words, write a program which when given two words on the command line (`argv[1]` and `argv[2]`) outputs the correct solution, if available. Use an exhaustive search over all 24 permutations until one leads to no invalid words being required. Make sure your program works, with amongst others, the following:

C O L D	P O K E	C U B E
C O R D	P O L E	C U B S
C A R D	P O L L	T U B S
W A R D	M O L L	T U N S
W A R M	M A L L	T O N S

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