1

(a) (Logistic Regression)
We want to find $f^*(x)$ such that

$$
\begin{aligned}
f^*(x) &= argmin_f E_{XY}(L(Y, f(X))) \\
&= argmin_{f(x)} E_{Y|x}[log(1 + e^{-yf(x)})]
\end{aligned}
$$

We take the first derivative of the above equation with respect to $f(x)$, set the resulting expression equal to zero and solve for $f(x)$:

$$
\begin{aligned}
&\frac{\partial E_{Y|x} L(Y, f(x))}{\partial f(x)} \\
&= E_{Y|x}(\frac{\partial L(Y, f(x))}{\partial f(x)}) \\
&= E_{Y|x}[\frac{\partial}{\partial f(x)} log(1 + e^{-Yf(x)})] \\
&= E_{Y|x}(\frac{-Ye^{-Yf(x)}}{1 + e^{-Yf(x)}}) = 0
\end{aligned}
$$

Now evaluating the above expectation with $Y = \pm 1$:

$$
\begin{aligned}
&Pr(Y = -1|x)[\frac{e^{f(x)}}{1 + e^{f(x)}}] + Pr(Y = 1|x)[\frac{-e^{-f(x)}}{1 + e^{-f(x)}}] \\
&= Pr(Y = -1|x)[\frac{e^{f(x)}}{1 + e^{f(x)}}] - Pr(Y = 1|x)[\frac{1}{1 + e^{f(x)}}] = 0
\end{aligned}
$$

Solve the above equation for $f(x)$

$$
\begin{aligned}
\frac{Pr(Y = 1|x)}{Pr(Y = -1|x)} &= e^{f(x)} \\
f^*(x) &= log \frac{Pr(Y = 1|X = x)}{Pr(Y = -1|X = x)}
\end{aligned}
$$

(b) (SVM)
We want to find $f^*(x)$ such that

$$
\begin{aligned}
f^*(x) &= argmin_f E_{XY}(L(Y, f(X))) \\
&= argmin_{f(x)} E_{Y|x}[1 - yf(x)]_+
\end{aligned}
$$

We take the first derivative of the above equation with respect to $f(x)$

$$
\begin{aligned}
&\frac{\partial E_{Y|x} L(Y, f(x))}{\partial f(x)} \\
&= E_{Y|x}(\frac{\partial L(Y, f(x))}{\partial f(x)}) \\
&= E_{Y|x}(sign(-Y))_+ = E_{Y|x} max(sign(-Y), 0) = 0
\end{aligned}
$$

Evaluating the $E_{Y|x}(-Y) = 0$ with $Y = \pm 1$:

$$Pr(Y=-1|x) - Pr(Y=1|x) = 0$$
$$1 - 2Pr(Y=1|x) = 0$$
$$Pr(Y=1|x) - \frac{1}{2} = 0$$

Therefore $f^*(x) = sign(Pr(Y=1|X=x) - \frac{1}{2})$
(c) (Regression)
We want to find $f^*(x)$ such that

$$f^*(x) = argmin_f E_{XY}(L(Y, f(X)))$$
$$= argmin_{f(x)} E_{Y|x}[y - f(x)]^2$$

We take the first derivative of the above equation with respect to $f(x)$

$$\frac{\partial E_{Y|x} L(Y, f(x))}{\partial f(x)}$$
$$= E_{Y|x}(\frac{\partial L(Y, f(x))}{\partial f(x)})$$
$$= E_{Y|x}[-2y + 2f(x)] = 0$$

Evaluating the above equation with $Y = \pm 1$:

$$Pr(Y=-1|X=x)[2+2f(x)] + Pr(Y=1|X=x)[2f(x)-2] = 0$$
$$f(x)[Pr(Y=-1|X=x) + Pr(Y=1|X=x)] = Pr(Y=1|X=x) - Pr(Y=-1|X=x)$$
$$f(x) = 2Pr(Y=1|X=x) - 1$$

Therefore $f^*(x) = 2Pr(Y=1|X=x) - 1$
(d) (Adaboost)
We want to find $f^*(x)$ such that

$$f^*(x) = argmin_f E_{XY}(L(Y, f(X)))$$
$$= argmin_{f(x)} E_{Y|x}[e^{-yf(x)}]$$

We take the first derivative of the above equation with respect to $f(x)$, set the resulting expression equal to zero and solve for $f(x)$:

$$\frac{\partial E_{Y|x} L(Y, f(x))}{\partial f(x)}$$
$$= E_{Y|x}(\frac{\partial L(Y, f(x))}{\partial f(x)})$$
$$= E_{Y|x}(-Ye^{-Yf(x)}) = 0$$

Now evaluating the above expectation with $Y = \pm 1$:

$$Pr(Y=-1|x)(e^{f(x)}) + Pr(Y=1|x)(-e^{-f(x)}) = 0$$
$$e^{2f(x)} Pr(Y=-1|x) - Pr(Y=1|x) = 0$$
$$e^{2f(x)} = \frac{Pr(Y=1|x)}{Pr(Y=-1|x)}$$

Solve the above equation for $f(x)$

$$f^*(x) = \frac{1}{2} log \frac{Pr(Y=1|X=x)}{Pr(Y=-1|X=x)}$$