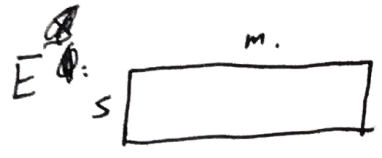
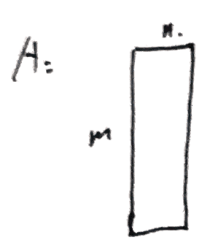


①

Notation: say, $m = 100$, $n = 20$, $s = 20 \times 2 = 40$, $r = 18$.

$$\tilde{r} = r_1, r_2$$

$$r_1 \leq r$$



$$A = U \Sigma V^T$$

Diagram showing the SVD decomposition of matrix A. Matrix A (m by n) is equal to the product of matrix U (m by r), matrix Sigma (r by r), and matrix V^T (r by n). Matrix U is shown as a vertical rectangle with height m and width r. Matrix Sigma is shown as a square with side length r, containing diagonal elements sigma_1, ..., sigma_r. Matrix V^T is shown as a horizontal rectangle with height r and width n.

$$EA = \tilde{U} \tilde{\Sigma} \tilde{V}^T$$

Diagram showing the decomposition of the product EA. Matrix EA (s by n) is equal to the product of matrix U-tilde (s by r-tilde), matrix Sigma-tilde (r-tilde by r-tilde), and matrix V-tilde^T (r-tilde by n). Matrix U-tilde is shown as a vertical rectangle with height s and width r-tilde. Matrix Sigma-tilde is shown as a square with side length r-tilde, containing diagonal elements sigma_1-tilde, ..., sigma_r-tilde-tilde. Matrix V-tilde^T is shown as a horizontal rectangle with height r-tilde and width n.

$$EU = U_1 \Sigma_1 V_1^T$$

Diagram showing the decomposition of the product EU. Matrix EU (s by r) is equal to the product of matrix U_1 (s by r_1), matrix Sigma_1 (r_1 by r_1), and matrix V_1^T (r_1 by r). Matrix U_1 is shown as a vertical rectangle with height s and width r_1. Matrix Sigma_1 is shown as a square with side length r_1, containing diagonal elements sigma_1-tilde, ..., sigma_r_1-tilde-tilde. Matrix V_1^T is shown as a horizontal rectangle with height r_1 and width r.

$$A \in \mathbb{R}^{m \times n}, E \in \mathbb{R}^{s \times m}, r = \text{rank}(A), \tilde{r} = \text{rank}(EA), r_1 = \text{rank}(EU), r_2 = \text{rank}(AN)$$

~~$$V \Sigma^{-1} = N$$~~

$$A N = U_2 \Sigma_2 V_2^T$$

Diagram showing the decomposition of the product AN. Matrix AN (m by n) is equal to the product of matrix U_2 (m by r_2), matrix Sigma_2 (r_2 by r_2), and matrix V_2^T (r_2 by n). Matrix U_2 is shown as a vertical rectangle with height m and width r_2. Matrix Sigma_2 is shown as a square with side length r_2, containing diagonal elements sigma_1-tilde, ..., sigma_r_2-tilde-tilde. Matrix V_2^T is shown as a horizontal rectangle with height r_2 and width n.