Notation: say, $m = 100$, $n = 20$, $s = 20 \times 2 = 40$, , $r = 18$,

$\tilde{r} = r_1, r_2$.

$r_1 \le r$

A: $m \times n$ E: $s \times m$

$A = U \Sigma V^T$

A ($m \times n$) $= U$ ($m \times r$) Σ ($r \times r$) V^T ($r \times n$)

$EA = \tilde{U} \tilde{\Sigma} \tilde{V}^T$

EA ($s \times n$) $= \tilde{U}$ ($s \times \tilde{r}$) $\tilde{\Sigma}$ ($\tilde{r} \times \tilde{r}$) $\tilde{V}^T$ ($\tilde{r} \times n$)

$EU = U_1 \Sigma_1 V_1^T$

EU ($s \times r$) $= U_1$ ($s \times r_1$) Σ_1 ($r_1 \times r_1$) V_1^T ($r_1 \times r$).

$A \in \mathbb{R}^{m \times n}$, $E \in \mathbb{R}^{s \times m}$, $r = \text{rank}(A)$., $\tilde{r} = \text{rank}(EA)$, $r_1 = \text{rank}(EU)$.

$r_2 = \text{rank}(AN)$.

~~$V \Sigma^{-1} = N$~~ $\tilde{V} \tilde{\Sigma}^{-1} = N$

N ($n \times \tilde{r}$) $= \tilde{V}$ ($n \times \tilde{r}$) $\tilde{\Sigma}^{-1}$ ($\tilde{r} \times \tilde{r}$)

$AN = U_2 \Sigma_2 V_2^T$

AN ($m \times \tilde{r}$) $= U_2$ ($m \times r_2$) Σ_2 ($r_2 \times r_2$) V_2^T ($r_2 \times \tilde{r}$)