Równanie transportu ciepła

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1 Równanie transportu ciepła:

$$-\frac{d}{dx}(k(x)\frac{du(x)}{dx}) = 100x^2$$

$$u(2) = -20$$

$$\frac{du(0)}{dx} + u(0) = 20$$

$$k(x) = \left\{ \begin{array}{ccc} 1 & \text{dla} & x \in [0,1] \\ 2x & \text{dla} & x \in (1,2] \end{array} \right.$$

gdzie u to poszukiwana funkcja

$$[0,2] \ni x \mapsto u(x) \in R$$

2 Rozwiązanie:

Przyjmuję funkcję testującą $v : v \in V, V = [f \in H^1 : f(2) = 0]$

$$-(k(x)u'(x))' = 100x^2 | \cdot v(x)$$

$$-(k(x)u'(x))'v(x) = 100x^2v(x) | \int_\Omega dx$$

$$-\int_0^2 (k(x)u'(x))'v(x)dx = 100\int_0^2 x^2v(x)dx$$

$$-[k(x)u'(x)v(x)]_0^2 + \int_0^2 k(x)u'(x)'v'(x)dx = 100\int_0^2 x^2v(x)dx$$

$$-k(2)u'(2)v(2) + k(0)u'(0)v(0) + \int_0^2 k(x)u'(x)'v'(x)dx = 100\int_0^2 x^2v(x)dx$$

$$\begin{cases} k(0) = 1 \\ v(2) = 0 \\ u'(0)v(0) = 20v(0) - u(0)v(0) \end{cases}$$

$$20v(0) - u(0)v(0) + \int_0^2 k(x)u'(x)'v'(x)dx = 100\int_0^2 x^2 v(x)dx$$

$$\begin{cases} B(u,v) = \int_0^2 k(x)u'(x)'v'(x)dx - u(0)v(0) \\ L(v) = 100\int_0^2 x^2 v(x)dx - 20v(0) \end{cases}$$

$$B(u,v) = L(v)$$