1.

a) $T = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $R = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $P_c = RTP_w$

$\because RT = \begin{bmatrix} 0 & -1 & 0 & -10 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & -1 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\therefore P_c = \begin{bmatrix} 0 & -1 & 0 & -10 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & -1 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore P_c = \begin{bmatrix} -10 \\ -10 \\ -50 \\ 1 \end{bmatrix}$ (m)

$\therefore$ camera coord: $(-10, -10, -50)$ (m)

(d) $f = 0.01$ m

$M_{proj} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -10 \\ -10 \\ -50 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -0.1 \\ -0.1 \\ -50 \end{bmatrix}$$

$$x = \frac{x'}{z'} = \frac{-0.1}{-50} = 0.002\ (m) = 2mm$$

$$y = \frac{y'}{z'} = \frac{-0.1}{-50} = 0.002\ (m) = 2mm$$

(f) $P_{lower} = (0, -10m, 0m)$

$P_{upper} = (-2m, -10m, 0m)$

$$M_{proj} \times M_{ext} = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -10 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & -1 & -50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

unit : meter

$$M_{proj} \times M_{ext} = \begin{bmatrix} 0 & -0.01 & 0 & -0.1 \\ 0.01 & 0 & 0 & -0.1 \\ 0 & 0 & -1 & -50 \end{bmatrix}$$

$$\begin{bmatrix} x'_{lower} \\ y'_{lower} \\ z'_{lower} \end{bmatrix} = M_{proj} \times M_{ext} \times \begin{bmatrix} 0 \\ -10 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1 \\ -50 \end{bmatrix}$$

$$\begin{bmatrix} x'_{upper} \\ y'_{upper} \\ z'_{upper} \end{bmatrix} = M_{proj} \times M_{ext} \times \begin{bmatrix} -2 \\ -10 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.12 \\ -50 \end{bmatrix}$$

$$y_{lower} = \frac{y'_{lower}}{z'_{lower}} = \frac{-0.1}{-50} (m) \times \frac{1000mm}{1m} = 2mm$$

$$y_{upper} = \frac{y'_{upper}}{z'_{upper}} = \frac{-0.12}{-50} (m) \times \frac{1000mm}{1m} = 2.4mm$$

$\therefore$ Person Image Height $= Y_{upper} - Y_{lower} = 2.4\text{ mm} - 2\text{ mm} = \boxed{0.4\text{ mm}}$

2. model to fit : $ax + by + c = 0$

constraints : $a^2 + b^2 + c^2 = 1$

$d_i = ax_i + by_i + c$

$$E = \left[\sum_{i=1}^{N} (ax_i + by_i + c)^2\right] - \lambda(a^2 + b^2 + c^2 - 1)$$

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{N} 2(ax_i + by_i + c)\cdot(x_i) - 2\lambda a = 0$$

$$\sum_{i=1}^{N} (ax_i^2 + bx_iy_i + cx_i) - \lambda a = 0$$

$$\left(\sum_{i=1}^{N} x_i^2\right)\cdot a + \left(\sum_{i=1}^{N} x_iy_i\right)b + \left(\sum_{i=1}^{N} x_i\right)c = \lambda a$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{N} 2(ax_i + by_i + c)\cdot(y_i) - 2\lambda b = 0$$

$$\sum_{i=1}^{N} (ax_iy_i + by_i^2 + cy_i) - 2\lambda b = 0$$

$$\left(\sum_{i=1}^{N} x_iy_i\right)a + \left(\sum_{i=1}^{N} y_i^2\right)b + \left(\sum_{i=1}^{N} y_i\right)c = \lambda b$$

$$\frac{\partial E}{\partial c} = \sum_{i=1}^{N} 2(ax_i + by_i + c)\cdot(1) - 2\lambda c = 0$$

$$\sum_{i=1}^{N} (ax_i + by_i + c) - \lambda c = 0$$

$$\left(\sum_{i=1}^{N} x_i\right)a + \left(\sum_{i=1}^{N} y_i\right)b + Nc = \lambda c$$

$$\begin{bmatrix} \left(\sum_{i=1}^{N} x_i^2\right) & \sum_{i=1}^{N} x_iy_i & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_iy_i & \left(\sum_{i=1}^{N} y_i^2\right) & \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} y_i & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A \quad h = \lambda h$$

We can solve for [a ; b ; c] by performing SVD to matrix A.
let $A = UDV^T$ and find the column in V that corresponds with eigen values that can be obtained as the diagonal of D. The column extracted from V will contain the eigen vectors of A. Then, we can extract a, b, and c after normalizing these eigen vectors.