

CUDA Toolkit 4.1 CUSPARSE Library

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Chapter 1 Introduction

The CUSPARSE library contains a set of basic linear algebra subroutines used for handling sparse matrices. It is implemented on top of the NVIDIA[®] CUDA[™] runtime (that is part of CUDA Toolkit) and is designed to be called from C and C++. The library routines can be classified into four categories:

- ▶ Level 1: operations between a vector in sparse format and a vector in dense format.
- ▶ Level 2: operations between a matrix in sparse format and a vector in dense format.
- ▶ Level 3: operations between a matrix in sparse format and a set of vectors in dense format (that can also usually be viewed as a dense tall matrix).
- ▶ Conversion: operations that allow conversion between different matrix formats.

The CUSPARSE library allows the user to access the computational resources of NVIDIA Graphics Processing Unit (GPU), but does not auto-parallelize across multiple GPUs. The CUSPARSE API assumes that the input and output data reside in GPU (device) memory, unless specifically indicated otherwise by the string DevHostPtr being part of the parameter name of a function (for example, *resultDevHostPtr in cusparse<t>doti).

It is the responsibility of the user to allocate memory and to copy data between GPU memory and CPU memory using standard CUDA runtime API routines, such as, cudaMalloc(), cudaFree(), cudaMemcpy(), and cudaMemcpyAsync().

The CUSPARSE library requires hardware with compute capability (CC) of at least 1.1 or higher. Please see *NVIDIA CUDA C Programming Guide*, Appendix A for the list of the compute capabilities corresponding to all NVIDIA GPUs.

Note: The CUSPARSE library requires hardware with CC of at least 1.1.

1.1 New and Legacy CUSPARSE API

Starting with version 4.1, the CUSPARSE Library provides a new updated API, in addition to the existing legacy API. This section discusses why a new API is provided, the advantages of using it, and the differences with the existing legacy API.

The new CUSPARSE library API can be used by including the header file "cusparse_v2.h". It has the following features that the legacy CUSPARSE API does not have:

- ▶ the scalars α and β can be passed by reference on the host or the device, instead of only being allowed to be passed by value on the host. This change allows library functions to execute asynchronously using streams even when α and β are generated by a previous kernel.
- ▶ when a library routine returns a scalar result, it can be returned by reference on the host or the device, instead of only being allowed to be returned by value only on the host. This change allows library routines to be called asynchronously when the scalar result is generated and returned by reference on the device resulting in maximum parallelism.
- ▶ the function cusparseSetKernelStream() was renamed cusparseSetStream() to be more consistent with the other CUDA libraries.
- ▶ the enum type cusparseAction_t was introduced to indicate if the routine operates only on indices or on values and indices at the same time.

The legacy CUSPARSE API, explained in more detail in the Appendix A, can be used by including the header file "cusparse.h". Since the legacy API is identical to the previously released CUSPARSE library API, existing applications will work out of the box and automatically use this legacy API without any source code changes. In general, new applications should not use the legacy CUSPARSE API, and existing applications should convert to using the new API if it requires sophisticated and optimal stream parallelism. For the rest of the document, the new CUSPARSE Library API will simply be referred to as the CUSPARSE Library API.

As mentioned earlier the interfaces to the legacy and the CUSPARSE library APIs are the header file "cusparse.h" and "cusparse_v2.h", respectively. In addition, applications using the CUSPARSE library need to link against the dynamic shared object (DSO) cusparse.so on Linux, the dynamic-link library (DLL) cusparse.dll on Windows, or the dynamic library cusparse.dylib on Mac OS X. Note that the same dynamic library implements both the new and legacy CUSPARSE APIs.

1.2 Naming Convention

The CUSPARSE library functions are available for data types float, double, cuComplex, and cuDoubleComplex. The sparse Level 1, Level 2, and Level 3 functions follow the naming convention:

cusparse<t>[<matrix data format>]<operation>[<output matrix data format>]

where <t> can be S, D, C, Z, or X, corresponding to the data types float, double, cuComplex, cuDoubleComplex and generic type, repectively.

The <matrix data format> can be dense, coo, csr, csc and hyb, corresponding to the dense, coordinate, compressed sparse row, compressed sparse column and hybrid storage formats, respectively.

Finally, the <operation> can be axpyi, doti, dotci, gthr, gthrz, roti and sctr, corresponding to the Level 1 functions. Also, it can be mv and sv as well as mm and sm, corresponding pair-wise to the Level 2 and 3 functions, respectively.

All of these functions have the return type cusparseStatus_t and will be explained in more detail in the following chapters.

1.3 Asynchronous Execution

The CUSPARSE library functions are executed asynchronously with respect to the host and may return control to the application on the host before the result is ready. The user can use cudaThreadSynchronize function to ensure that the execution of a particular CUSPARSE library routine has completed.

The application can also use cudaMemcpy routine to copy data from the device to host and vice-versa, using cudaMemcpyDeviceToHost and cudaMemcpyHostToDevice parameters, respectively. In this case there is no need to add a call to cudaThreadSynchronize, because the call to cudaMemcpy routine with the above parameters is blocking and will only complete when the results are ready on the host.

Chapter 2 Using the CUSPARSE API

This section describes how to use the CUSPARSE library API. It does not contain a detailed reference for all API datatypes and functions – those are provided in subsequent chapters. The Legacy CUSPARSE API is also not covered in this section – that is handled in the Appendix A.

2.1 Thread Safety

The library is thread safe and its functions can be called from multiple host threads.

2.2 Scalar Parameters

In the CUSPARSE API the scalar parameters α and β can be passed by reference on the host or the device.

Also, the few functions that return a scalar result, such as doti() and nnz(), return the resulting value by reference on the host or the device. Notice that even though these functions return immediately, similarly to matrix and vector results, the scalar result is ready only when execution of the routine on the GPU completes. This requires proper synchronization in order to read the result from the host.

These changes allow the library functions to execute completely asynchronously using streams even when α and β are generated by a previous kernel. For example, this situation arises when we use the CUSPARSE library to implement iterative methods for the solution of linear systems and eigenvalue problems [3].

2.3 Parallelism with Streams

If the application performs several small independent computations or if it makes data transfers in parallel with the computation, CUDATM streams can be used to overlap these tasks.

The application can conceptually associate each stream with each task. In order to achieve the overlap of computation between the tasks, the user should create CUDATM streams using the function cudaStreamCreate() and set the stream to be used by each individual CUSPARSE library routine by calling cusparseSetStream() just before calling the actual CUSPARSE routine. Then, the computation performed in separate streams would be overlapped automatically when possible on the GPU. This approach is especially useful when the computation performed by a single task is relatively small and is not enough to fill the GPU with work, or when there is a data transfer that can be performed in parallel with the computation.

We recommend using the new CUSPARSE API with scalar parameters and results passed by reference in the device memory to achieve maximum overlap of the computation when using streams.

Although the user can create many streams, in practice it is not possible to have more than 16 concurrent kernels executing at the same time.

Chapter 3 CUSPARSE Indexing and Data Formats

The CUSPARSE library supports the following dense, sparse vector and matrix formats.

3.1 Index Base Format

The library supports zero- and one-based indexing. The index base is selected through the cusparseIndexBase_t type, which is passed as a standalone parameter or as a field in the matrix descriptor cusparseMatDescr_t type.

3.2 Vector Formats

3.2.1 Dense Format

Dense vectors are represented with a single data array that is stored linearly in memory. For example, consider the 7×1 dense vector

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 2.0 & 3.0 & 0.0 & 4.0 \end{bmatrix} \tag{3.1}$$

3.2.2 Sparse Format

Sparse vectors are represented with two arrays.

- ▶ The data array has the non-zero values from the equivalent array in dense format.
- ▶ The integer index array has the positions of the corresponding non-zero values in the equivalent array in dense format.

For example, the dense vector (3.1) can be stored as a sparse vector with one-based idexing

$$\begin{bmatrix} 1.0 & 2.0 & 3.0 & 4.0 \\ 1 & 4 & 5 & 7 \end{bmatrix}$$
 (3.2)

or as a sparse vector with zero-based idexing

$$\begin{bmatrix} 1.0 & 2.0 & 3.0 & 4.0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 4 & 6 \end{bmatrix}$$
(3.3)

where the top row is the data array and the bottom row is the index array.

Note: It is assumed that the indices are provided in an increasing order and that each index appears only once.

3.3 Matrix Formats

3.3.1 Dense Format

The dense matrix X is assumed to be stored in column-major format in memory and is represented by the following parameters:

m (integer) The number of rows in the ma	trix.
--	-------

n (integer) The number of columns in the matrix.

1dX (integer) The leading dimension of X, which must be greater than or equal to m. If 1dX is greater than m, then X represents a sub-matrix of a larger matrix

stored in memory.

X (pointer) Points to the data array containing the matrix elements. It is assumed that enough storage is allocated for X to hold all of the matrix elements and that CUSPARSE library functions may access values outside of the sub-matrix, but will never overwrite them.

For example, $m \times n$ dense matrix X with leading dimension 1dX can be stored as

$$\begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,n} \\ X_{2,1} & X_{2,2} & \dots & X_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{m,1} & X_{m,2} & \dots & X_{m,n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{ldX,1} & X_{ldX,2} & \dots & X_{ldX,n} \end{bmatrix}$$

with one-based indexing. Its elements are arranged linearly in memory as

$$[X_{1,1} \quad X_{2,1} \quad \dots \quad X_{m,1} \quad \dots \quad X_{ldX,1} \quad \dots \quad X_{1,n} \quad X_{2,n} \quad \dots \quad X_{m,n} \quad \dots \quad X_{ldX,n}]$$

Please note that this format and notation is similar to the format and notation used in the NVIDIA CUBLAS library.

Note: Dense matrices are assumed to be stored in column-major format in memory.

3.3.2 Coordinate Format (COO)

The $m \times n$ sparse matrix A is represented in COO format by the following parameters:

nnz	(integer)	The number of non-zero elements in the matrix.
cooValA	(pointer)	Points to the data array of length nnz that holds all non-zero
		values of A in row-major format.
${\tt cooRowIndA}$	(pointer)	Points to the integer array of length nnz that contains the row
		indices of the corresponding elements in array cooValA.
${\tt cooColIndA}$	(pointer)	Points to the integer array of length nnz that contains the column
		indices of the corresponding elements in array cooValA.

Sparse matrices are assumed to be stored in row-major COO format, in other words, the index arrays are first sorted by row indices and then within the same row by column indices. Also it is assumed that each pair of row and column indices appears only once.

For example, consider the following 4×5 matrix A:

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in COO format with zero-based indexing as

and in the COO format with one-based indexing as

Note: Sparse matrices in COO format are assumed to be stored in row-major format in memory. Also, it is assumed that each pair of row and column indices appears only once.

3.3.3 Compressed Sparse Row Format (CSR)

The only difference between the COO and CSR formats is that the array containing the row indices is compressed in CSR format. The $m \times n$ sparse matrix A is represented in CSR format by the following parameters:

nnz (integer) The number of non-zero elements in the matrix. csrValA Points to the data array of length nnz that holds all non-zero (pointer) values of A in row-major format. Points to the integer array of length m + 1 that holds indices csrRowPtrA (pointer) into the arrays csrColIndA and csrValA. The first m entries of this array contain the indices of the first non-zero element in the ith row for i=1,...,m, while the last entry contains nnz + csrRowPtrA(0). In general, csrRowPtrA(0) is 0 or 1 for zeroand one-based indexing, respectively. Points to the integer array of length ${\tt nnz}$ that contains the column csrColIndA (pointer) indices of the corresponding elements in array csrValA.

Sparse matrices are assumed to be stored in row-major CSR format, in other words, the index arrays are first sorted by row indices and then within the same row by column indices. Also it is assumed that each pair of row and column indices appears only once.

For example, consider again the 4×5 matrix A:

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in CSR format with zero-based indexing as

and in CSR format with one-based indexing as

Note: Sparse matrices in CSR format are assumed to be stored in row-major format in memory. Also, it is assumed that each pair of row and column indices appears only once.

3.3.4 Compressed Sparse Column Format (CSC)

The only two differences between the COO and CSC formats is that the matrix is stored in column-major format and that the array containing the column indices is compressed in CSC format. The $\mathtt{m} \times \mathtt{n}$ matrix A is represented in CSC format by the following parameters:

nnz	(integer)	The number of non-zero elements in the matrix.
cscValA	(pointer)	Points to the data array of length nnz that holds all non-zero
		values of A in column-major format.
${\tt cscRowIndA}$	(pointer)	Points to the integer array of length nnz that holds the row indices
		of the corresponding elements in cscValA.
cscColPtrA	(pointer)	Points to the integer array of length $n + 1$ that holds indices
		into the arrays cscRowIndA and cscValA. The first n entries of
		this array contain the indices of the first non-zero element in the
		ith column for i=1,,n, while the last entry contains nnz +
		cscColPtrA(0). In general, cscColPtrA(0) is 0 or 1 for zero- or
		one-based indexing, respectively.

Note: The matrix A in CSR format has exactly the same memory layout as its transpose in CSC format (and vice-versa).

For example, consider once again the 4×5 matrix A:

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in CSC format with zero-based indexing as

and in CSC format with one-based indexing as

Note: Sparse matrices in CSC format are assumed to be stored in column-major format in memory. Also, it is assumed that each pair of row and column indices appears only once.

3.3.5 Ellpack-Itpack Format (ELL)

An $m \times n$ sparse matrix A with at most k non-zero elements per row is stored in the Ellpack-Itpack (ELL) format [2] using two dense arrays of dimension $m \times k$. The first data array contains the values of the non-zero elements in the matrix, while the second integer array contains the corresponding column indices.

For example, consider the 4×5 matrix A:

$$\begin{bmatrix} 1.0 & 4.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 3.0 & 0.0 & 0.0 \\ 5.0 & 0.0 & 0.0 & 7.0 & 8.0 \\ 0.0 & 0.0 & 9.0 & 0.0 & 6.0 \end{bmatrix}$$

It is stored in ELL format with zero-based indexing as

$$\begin{array}{c} \mathtt{data} \ = \ \begin{bmatrix} 1.0 & 4.0 & 0.0 \\ 2.0 & 3.0 & 0.0 \\ 5.0 & 7.0 & 8.0 \\ 9.0 & 6.0 & 0.0 \end{bmatrix} \\ \mathtt{indices} \ = \ \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & -1 \\ 0 & 3 & 4 \\ 2 & 4 & -1 \end{bmatrix} \end{array}$$

and in ELL format with one-based indexing as

$$\begin{array}{ccccc} \mathtt{data} & = & \begin{bmatrix} 1.0 & 4.0 & 0.0 \\ 2.0 & 3.0 & 0.0 \\ 5.0 & 7.0 & 8.0 \\ 9.0 & 6.0 & 0.0 \end{bmatrix} \\ \\ \mathtt{indices} & = & \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 1 & 4 & 5 \\ 3 & 5 & -1 \end{bmatrix} \end{array}$$

In the above example, if there are less than k non-zero elements in a row, we denote the empty spaces in the data array with zero and in the integer array with -1.

This format is not supported directly, but it is used to store the regular part of the matrix in the HYB format that is described in the next section.

Note: Sparse matrices in ELL format are assumed to be stored in column-major format in memory. Also, the rows with less than k non-zero elements per row, are padded in the data and indices arrays with zero and -1, respectively.

3.3.6 Hybrid Format (HYB)

The HYB sparse storage format is composed of a regular part usually stored in ELL format and an irregular part usually stored in COO format [1]. It is implemented as an opaque data format that requires the use of a conversion operation to store a matrix in it. The conversion operation partitions the general matrix into the regular and irregular parts automatically or according to the user specified criteria.

For more information, please refer to the description of cusparseHybPartition_t type, as well as the description of the convertion routines dense2hyb and csr2hyb.

Note: The sparse matrix in HYB format is always stored using zero-based indexing for both ELL and COO parts.

Chapter 4 CUSPARSE Types Reference

4.1 Data types

The float, double, cuComplex, and cuDoubleComplex data types are supported. The first two are standard C data types, while the last two are exported from cuComplex.h.

4.2 cusparseAction_t

This type indicates whether the operation is performed only on indices or on data and indices.

Value	Meaning
CUSPARSE_ACTION_SYMBOLIC	the operation is performed only on indices.
CUSPARSE_ACTION_NUMERIC	the operation is performed on data and indices.

4.3 cusparseDirection_t

This type indicates whether the elements of a dense matrix should be parsed by rows or by columns (assuming column-major storage in memory of the dense matrix).

Value	Meaning
CUSPARSE_DIRECTION_ROW	the matrix should be parsed by rows.
CUSPARSE_DIRECTION_COLUMN	the matrix should be parsed by columns.

4.4 cusparseHandle_t

This is a pointer type to an opaque CUSPARSE context, which the user must initialize by calling <code>cusparseCreate()</code> prior to calling any other library function. The handle created and returned by <code>cusparseCreate()</code> must be passed to every CUSPARSE function.

4.5 cusparseHybMat_t

This is a pointer type to an opaque structure holding the matrix in HYB format, which is created by cusparseCreateHybMat and destroyed by cusparseDestroyHybMat.

4.5.1 cusparseHybPartition_t

This type indicates how to perform the partitioning of the matrix into regular (ELL) and irregular (COO) parts of the HYB format.

The partitioning is performed during the conversion of the matrix from a dense or sparse format into the HYB format and is governed by the following rules. When CUSPARSE_HYB_PARTITION_AUTO is selected, the CUSPARSE library automatically decides how much data to put into the regular and irregular parts of the HYB format. When CUSPARSE_HYB_PARTITION_USER is selected, the width of the regular part of the HYB format should be specified by the caller. When CUSPARSE_HYB_PARTITION_MAX is selected, the width of the regular part of the HYB format equals to the maximum number of non-zero elements per row, in other words, the entire matrix is stored in the regular part of the HYB format.

The default is to let the library automatically decide how to split the data.

Value	Meaning
CUSPARSE_HYB_PARTITION_AUTO	the automatic partitioning is selected (default).
CUSPARSE_HYB_PARTITION_USER	the user specified treshold is used.
CUSPARSE_HYB_PARTITION_MAX	the data is stored in ELL format.

4.6 cusparseMatDescr_t

This structure is used to describe the shape and properties of a matrix.

```
typedef struct {
    cusparseMatrixType_t MatrixType;
    cusparseFillMode_t FillMode;
    cusparseDiagType_t DiagType;
    cusparseIndexBase_t IndexBase;
} cusparseMatDescr_t;
```

4.6.1 cusparseDiagType_t

This type indicates if the matrix diagonal entries are unity. The diagonal elements are always assumed to be present, but if CUSPARSE_DIAG_TYPE_UNIT is passed to an API

routine, then the routine will assume that all diagonal entries are unity and will not read or modify those entries. Note that in this case the routine assumes the diagonal entries are equal to one, regardless of what those entries are actuall set to in memory.

Value	Meaning
CUSPARSE_DIAG_TYPE_NON_UNIT	the matrix diagonal has non-unit elements.
CUSPARSE_DIAG_TYPE_UNIT	the matrix diagonal has unit elements.

4.6.2 cusparseFillMode_t

This type indicates if the lower or upper part of a matrix is stored in sparse storage.

Value	Meaning
CUSPARSE_FILL_MODE_LOWER	the lower triangular part is stored.
CUSPARSE_FILL_MODE_UPPER	the upper triangular part is stored.

4.6.3 cusparseIndexBase_t

This type indicates if the base of the matrix indices is zero or one.

Value	Meaning
CUSPARSE_INDEX_BASE_ZERO	the base index is zero.
CUSPARSE_INDEX_BASE_ONE	the base index is one.

4.6.4 cusparseMatrixType_t

This type indicates the type of matrix stored in sparse storage. Notice that for symmetric, Hermitian and triangular matrices only their lower or upper part is assumed to be stored.

Value	Meaning
CUSPARSE_MATRIX_TYPE_GENERAL	the matrix is general.
CUSPARSE_MATRIX_TYPE_SYMMETRIC	the matrix is symmetric.
CUSPARSE_MATRIX_TYPE_HERMITIAN	the matrix is Hermitian.
CUSPARSE_MATRIX_TYPE_TRIANGULAR	the matrix is triangular.

4.7 cusparseOperation_t

This type indicates which operations need to be performed with the sparse matrix.

Value	Meaning
CUSPARSE_OPERATION_NON_TRANSPOSE	the non-transpose operation is selected.
CUSPARSE_OPERATION_TRANSPOSE	the transpose operation is selected.
CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE	the conjugate transpose operation is selected.

4.8 cusparsePointerMode_t

This type indicates whether the scalar values are passed by reference on the host or device. It is important to point out that if several scalar values are passed by reference in the function call, all of them will conform to the same single pointer mode. The pointer mode can be set and retrieved using cusparseSetPointerMode() and cusparseGetPointerMode() routines, respectively.

Value	Meaning
CUSPARSE_POINTER_MODE_HOST	the scalars are passed by reference on the host.
CUSPARSE_POINTER_MODE_DEVICE	the scalars are passed by reference on the device.

4.9 cusparseSolveAnalysisInfo_t

This is a pointer type to an opaque structure holding the information collected in the analysis phase of the solution of the sparse triangular linear system. It is expected to be passed unchanged to the solution phase of the sparse triangular linear system.

4.10 cusparseStatus_t

This is a status type returned by the library functions and it can have the following values.

CUSPARSE_STATUS_SUCCESS		
The operation completed successfully.		
CUSPARSE_STATUS_NOT_INITIALIZED		
The CUSPARSE library was not initialized. This is usually		
caused by the lack of a prior cusparseCreate() call, an error		
in the CUDA Runtime API called by the CUSPARSE routine, or		
an error in the hardware setup.		
To correct: call cusparseCreate() prior to the function call;		
and check that the hardware, an appropriate version of the driver,		
and the CUSPARSE library are correctly installed.		
CUSPARSE_STATUS_ALLOC_FAILED		
Resource allocation failed inside the CUSPARSE library. This is		
usually caused by a cudaMalloc() failure.		
To correct: prior to the function call, deallocate previously al-		
located memory as much as possible.		
CUSPARSE_STATUS_INVALID_VALUE		
An unsupported value or parameter was passed to the function		
(a negative vector size, for example).		
To correct: ensure that all the parameters being passed have		
valid values.		

CUSPARSE_STATUS_ARCH_MISMATCH

The function requires a feature absent from the device architecture; usually caused by the lack of support for atomic operations or double precision.

To correct: compile and run the application on a device with appropriate compute capability, which is 1.1 for 32-bit atomic operations and 1.3 for double precision.

CUSPARSE_STATUS_MAPPING_ERROR

An access to GPU memory space failed, which is usually caused by a failure to bind a texture.

To correct: prior to the function call, unbind any previously bound textures.

CUSPARSE_STATUS_EXECUTION_FAILED

The GPU program failed to execute. This is often caused by a launch failure of the kernel on the GPU, which can be caused by multiple reasons.

To correct: check that the hardware, an appropriate version of the driver, and the CUSPARSE library are correctly installed.

CUSPARSE_STATUS_INTERNAL_ERROR

An internal CUSPARSE operation failed. This error is usually caused by a cudaMemcpyAsync() failure.

To correct: check that the hardware, an appropriate version of the driver, and the CUSPARSE library are correctly installed. Also, check that the memory passed as a parameter to the routine is not being deallocated prior to the routine's completion.

CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED

The matrix type is not supported by this function. This is usually caused by passing an invalid matrix descriptor to the function.

To correct: check that the fields in cusparseMatDescr_t descrA were set correctly.

Chapter 5 CUSPARSE Helper Function Reference

The CUSPARSE helper functions are described in this section.

5.1 cusparseCreate()

cusparseStatus_t
cusparseCreate(cusparseHandle_t *handle)

This function initializes the CUSPARSE library and creates a handle on the CUSPARSE context. It must be called before any other CUSPARSE API function is invoked. It allocates hardware resources necessary for accessing the GPU.

Output

1		
handle	the pointer to the handle to the CUSPARSE context.	
Status Returned		
CUSPARSE_STATUS_S	CUSPARSE_STATUS_SUCCESS the initialization succeeded.	
CUSPARSE_STATUS_N	OT_INITIALIZED	the CUDA Runtime initialization failed.
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.
CUSPARSE_STATUS_A	RCH_MISMATCH	the device compute capability (CC) is less
		than 1.1. The CC of at least 1.1 is required.

5.2 cusparseCreateHybMat()

cusparseStatus_t
cusparseCreateHybMat(cusparseHybMat_t *hybA)

This function creates and initializes the hybA opaque data structure.

Input

hybA	the pointer to the hybrid format storage structure.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the structure was initialized successfully.		
CUSPARSE_STATUS_A	LLOC_FAILED	the resources could not be allocated.

5.3 cusparseCreateMatDescr()

cusparseStatus_t
cusparseCreateMatDescr(cusparseMatDescr_t *descrA)

This function initializes the matrix descriptor. It sets the fields MatrixType and IndexBase to the *default* values CUSPARSE_MATRIX_TYPE_GENERAL and CUSPARSE_INDEX_BASE_ZERO, respectively, while leaving other fields uninitialized.

Input

descrA	the pointer to the matrix descriptor.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the descriptor was initialized successfully.		
CUSPARSE_STATUS_A	LLOC_FAILED	the resources could not be allocated.

5.4 cusparseCreateSolveAnalysisInfo()

cusparseStatus_t

cusparseCreateSolveAnalysisInfo(cusparseSolveAnalysisInfo_t *info)

This function creates and initializes the solve and analysis info structure to default values.

Input

info	the pointer to the solve and analysis structure.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the structure was initialized successfully.		the structure was initialized successfully.
CUSPARSE_STATUS_A	LLOC_FAILED	the resources could not be allocated.

5.5 cusparseDestroy()

cusparseStatus_t

cusparseDestroy(cusparseHandle_t handle)

This function releases CPU-side resources used by the CUSPARSE library. The release of GPU-side resources may be deferred until the application shuts down.

Input

handle	the handle to the CUSPARSE context.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the shutdown succeeded.		
CUSPARSE_STATUS_N	OT_INITIALIZED	the library was not initialized.

5.6 cusparseDestroyHybMat()

cusparseStatus_t

cusparseDestroyHybMat(cusparseHybMat_t hybA)

This function destroys and releases any memory required by the hybA structure.

Input

hybA	the hybrid format storage structure.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the resources were released successfully.		

5.7 cusparseDestroyMatDescr()

cusparseStatus_t

cusparseDestroyMatDescr(cusparseMatDescr_t descrA)

This function releases the memory allocated for the matrix descriptor.

Input

descrA	the matrix descriptor.	
Status Returned		
CUSPARSE_STATUS_S	UCCESS	the resources were released successfully.

5.8 cusparseDestroySolveAnalysisInfo()

cusparseStatus_t

cusparseDestroySolveAnalysisInfo(cusparseSolveAnalysisInfo_t info)

This function destroys and releases any memory required by the info structure.

Input

info	the solve and analysis structure.	
Status Returned		
CUSPARSE_STATUS_S	UCCESS	the resources were released successfully.

5.9 cusparseGetMatDiagType()

cusparseDiagType_t
cusparseGetMatDiagType(const cusparseMatDescr_t descrA)

This function returns the DiagType field of the matrix descriptor descrA.

Input

P	
descrA	the matrix descriptor.
Returned	
	One of the enumerated diagType types.

5.10 cusparseGetMatFillMode()

cusparseFillMode_t

cusparseGetMatFillMode(const cusparseMatDescr_t descrA)

This function returns the FillMode field of the matrix descriptor descrA.

Input

descrA	the matrix descriptor.
Returned	
	One of the enumerated fillMode types.

5.11 cusparseGetMatIndexBase()

cusparseIndexBase_t

cusparseGetMatIndexBase(const cusparseMatDescr_t descrA)

This function returns the IndexBase field of the matrix descriptor descrA.

Input

descrA	the matrix descriptor.		
Returned			
	One of the enumerated indexBase types.		

5.12 cusparseGetMatType()

cusparseMatrixType_t

cusparseGetMatType(const cusparseMatDescr_t descrA)

This function returns the MatrixType field of the matrix descriptor descrA.

Input

descrA	the matrix descriptor.	
Returned		
	One of the enumerated matrix types.	

5.13 cusparseGetPointerMode()

cusparseStatus_t

cusparseGetPointerMode(cusparseHandle_t handle, cusparsePointerMode_t *mode)

This function obtains the pointer mode used by the CUSPARSE library. Please see the section on the cusparsePointerMode_t type for more details.

${\bf Input}$		
handle	the handle to the CUSPARSE context.	
Output		
mode	One of the enumerated pointer mode types.	
Status Returned		
CUSPARSE_STATUS_SUCCESS		the pointer mode was returned successfully.
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.

5.14 cusparseGetVersion()

cusparseStatus_t

cusparseGetVersion(cusparseHandle_t handle, int *version)

This function returns the version number of the CUSPARSE library.

Input			
handle	the handle to the CUSPARSE context.		
Output			
version	the version number of the library.		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the version was returned successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	

5.15 cusparseSetMatDiagType()

cusparseStatus_t

cusparseSetMatDiagType(cusparseMatDescr_t descrA, cusparseDiagType_t diagType)

This function sets the DiagType field of the matrix descriptor descrA.

Input	
diagType	One of the enumerated diagType types.
Output	
descrA	the matrix descriptor.

Status Returned

CUSPARSE_STATUS_SUCCESS	the DiagType field was set successfully.
CUSPARSE_STATUS_INVALID_VALUE	An invalid diagType parameter was passed.

5.16 cusparseSetMatFillMode()

cusparseStatus_t

cusparseSetMatFillMode(cusparseMatDescr_t descrA, cusparseFillMode_t fillMode)

This function sets the FillMode field of the matrix descriptor descrA.

Input

fillMode	One of the enumerated fillMode types.	
Output		
descrA	the matrix descriptor.	
Status Returned		
CUSPARSE_STATUS_SUCCESS		the FillMode field was set successfully.
CUSPARSE_STATUS_INVALID_VALUE		An invalid fillMode parameter was passed.

5.17 cusparseSetMatIndexBase()

cusparseStatus_t

cusparseSetMatIndexBase(cusparseMatDescr_t descrA, cusparseIndexBase_t base)

This function sets the IndexBase field of the matrix descriptor descrA.

Input

base	One of the enumerated indexBase types.		
Output			
descrA	the matrix descriptor	the matrix descriptor.	
Status Returned			
CUSPARSE_STATUS_SUCCESS		the IndexBase field was set successfully.	
CUSPARSE_STATUS_INVALID_VALUE		An invalid base parameter was passed.	

5.18 cusparseSetMatType()

cusparseStatus_t

cusparseSetMatType(cusparseMatDescr_t descrA, cusparseMatrixType_t type)

This function sets the MatrixType field of the matrix descriptor descrA.

Input			
type	One of the enumerated matrix types.		
Output	Output		
descrA	the matrix descriptor.		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the MatrixType field was set successfully.	
CUSPARSE_STATUS_INVALID_VALUE		An invalid type parameter was passed.	

5.19 cusparseSetPointerMode()

cusparseStatus_t

cusparseSetPointerMode(cusparseHandle_t handle, cusparsePointerMode_t mode)

This function sets the pointer mode used by the CUSPARSE library. The *default* is for the values to be passed by reference on the host. Please see the section on the cublasPointerMode_t type for more details.

Input

handle	the handle to the CUSPARSE context.		
mode	One of the enumerated pointer mode types.		
Status Returned			
CUSPARSE STATUS SUCCESS		the pointer mode was set successfully.	

the library was not initialized.

5.20 cusparseSetStream()

CUSPARSE_STATUS_NOT_INITIALIZED

cusparseStatus_t

cusparseSetStream(cusparseHandle_t handle, cudaStream_t streamId)

This function sets the stream to be used by the CUSPARSE library to execute its routines.

Input

handle	the handle to the CUSPARSE context.
streamId	the stream to be used by the library.

Status Returned

CUSPARSE_STATUS_SUCCESS	the stream was set successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.

Chapter 6 CUSPARSE Level 1 Function Reference

This chapter describes sparse linear algebra functions that perform operations between dense and sparse vectors.

6.1 cusparse<t>axpyi

```
cusparseStatus_t
cusparseSaxpyi(cusparseHandle_t handle, int nnz, const float
                                                                        *alpha,
               const float
                                     *xVal, const int *xInd,
                               *y, cusparseIndexBase_t idxBase)
               float
cusparseStatus_t
cusparseDaxpyi(cusparseHandle_t handle, int nnz, const double
                                                                        *alpha,
               const double
                                     *xVal, const int *xInd,
               double
                               *y, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseCaxpyi(cusparseHandle_t handle, int nnz, const cuComplex
                                                                        *alpha,
               const cuComplex
                                     *xVal, const int *xInd,
               cuComplex
                               *y, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseZaxpyi(cusparseHandle_t handle, int nnz, const cuDoubleComplex *alpha,
               const cuDoubleComplex *xVal, const int *xInd,
               cuDoubleComplex *y, cusparseIndexBase_t idxBase)
```

This function multiplies the vector \mathbf{x} in sparse format by the constant α and adds the result to the vector \mathbf{y} in dense format. This operation can be written as

$$\mathbf{y} = \mathbf{y} + \alpha * \mathbf{x},$$

in other words,

```
for i=0 to nnz-1
    y[xInd[i]-idxBase] = y[xInd[i]-idxBase] + alpha*xVal[i]
```

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handle	handle to the CUSPARSE library context.
nnz	number of elements in vector x.
alpha	<type> scalar used for multiplication.</type>
xVal	<type> vector with nnz non-zero values of vector x.</type>
xInd	integer vector with nnz indices of the non-zero values of vector x.
У	<type> vector in dense format.</type>
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
у	<type> updated vector in dense format (that is unchanged if nnz</type>
	o)

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_	
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	

6.2 cusparse<t>doti

```
cusparseStatus_t
cusparseSdoti(cusparseHandle_t handle, int nnz, const float
                                                                      *xVal,
             const int *xInd, const float
                              *resultDevHostPtr, cusparseIndexBase_t idxBase)
             float
cusparseStatus_t
cusparseDdoti(cusparseHandle_t handle, int nnz, const double
                                                                      *xVal,
              const int *xInd, const double
                              *resultDevHostPtr, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseCdoti(cusparseHandle_t handle, int nnz, const cuComplex
                                                                      *xVal,
              const int *xInd, const cuComplex
              cuComplex
                              *resultDevHostPtr, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseZdoti(cusparseHandle_t handle, int nnz, const cuDoubleComplex *xVal,
              const int *xInd, const cuDoubleComplex *y,
              cuDoubleComplex *resultDevHostPtr, cusparseIndexBase_t idxBase)
```

This function returns the dot product of a vector \mathbf{x} in sparse format and vector \mathbf{y} in dense format. This operation can be written as

$$\mathtt{result} = \mathbf{y}^T\mathbf{x},$$

in other words,

```
for i=0 to nnz-1
    resultDevHostPtr += xVal[i]*y[xInd[i-idxBase]]
```

This function requires some temporary extra storage that is allocated internally. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

Input

handle	handle to the CUSPARSE library context.
nnz	number of elements of vector x.
xVal	<type> vector with nnz non-zero values of vector x.</type>
xInd	integer vector with nnz indices of the non-zero values of vector x.
У	<type> vector in dense format.</type>
resultDevHostPtr	pointer to the location of the result in the device or host memory.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

Output

${\tt resultDevHostPtr}$	scalar result in the device or host memory	(that is zero if $nnz == 0$)).
--------------------------	--	-----------------------------	-----

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_	
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.	

6.3 cusparse<t>dotci

```
cusparseStatus_t
```

This function returns the dot product of a complex conjugate of vector \mathbf{x} in sparse format and vector \mathbf{y} in dense format. This operation can be written as

$$\mathtt{result} = \mathbf{y}^H \mathbf{x},$$

```
in other words,
```

```
for i=0 to nnz-1
    resultDevHostPtr += xVal[i]*y[xInd[i-idxBase]]
```

This function requires some temporary extra storage that is allocated internally. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

Input

	1 II GUADA DAD
handle	handle to a CUSPARSE context.
nnz	number of elements of vector x.
xVal	<type> vector with nnz non-zero values of vector x.</type>
xInd	integer vector with nnz indices of the non-zero values of vector x.
У	<type> vector in dense format.</type>
resultDevHostPtr	pointer to the location of the result in the device or host memory.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

Output

${\tt resultDevHostPtr}$	scalar result in the device or host memory	y (that is zero if $nnz == 0$)).
--------------------------	--	-------------------------------	-----

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED	the reduction buffer could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_	
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.	

6.4 cusparse<t>gthr

This function gathers the elements of the vector y listed in the index array xInd into the data array xVal.

This function requires no extra storage. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.
nnz	number of elements of vector x.
У	$<$ type $>$ vector in dense format (of size $\ge \max(xInd)$ -idxBase+1).
xInd	integer vector with nnz indices of the non-zero values of vector x.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
xVal	<type> vector with nnz non-zero values that were gathered from</type>
	vector y (that is unchanged if nnz == 0).

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

6.5 cusparse<t>gthrz

```
cusparseStatus_t
cusparseSgthrz(cusparseHandle_t handle, int nnz, float
                               *xVal, const int *xInd, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseDgthrz(cusparseHandle_t handle, int nnz, double
                                                                  *y,
               double
                               *xVal, const int *xInd, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseCgthrz(cusparseHandle_t handle, int nnz, cuComplex
                                                                  *y,
               cuComplex
                               *xVal, const int *xInd, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseZgthrz(cusparseHandle_t handle, int nnz, cuDoubleComplex *y,
               cuDoubleComplex *xVal, const int *xInd, cusparseIndexBase_t idxBase)
```

This function gathers the elements of the vector y listed in the index array xInd into the data array xVal. Also, it zeroes out the gathered elements in the vector y.

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handle	handle to the CUSPARSE library context.
nnz	number of elements of vector x.
у	$<$ type $>$ vector in dense format (of size $\ge \max(xInd)$ -idxBase+1).
xInd	integer vector with nnz indices of non-zero values of vector x
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
xVal	<type> vector with nnz non-zero values that were gathered from</type>
	vector y (that is unchanged if nnz == 0).
У	<type> vector in dense format with elements indexed by xInd set to</type>
	zero (it is unchanged if nnz == 0).

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

6.6 cusparse<t>roti

```
cusparseStatus_t
```

cusparseStatus_t

This function applies Givens rotation matrix

$$G = \left(\begin{array}{cc} c & s \\ -s & c \end{array}\right)$$

to sparse \mathbf{x} and dense \mathbf{y} vectors. In other words,

for i=0 to nnz-1

```
y[xInd[i]-idxBase] = c * y[xInd[i]-idxBase] - s*xVal[i]
x[i] = c * xVal[i] + s * y[xInd[i]-idxBase]
```

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handle	handle to the CUSPARSE library context.
nnz	number of elements of vector x.
xVal	<type> vector with nnz non-zero values of vector x.</type>
xInd	integer vector with nnz indices of the non-zero values of vector x.
у	<type> vector in dense format.</type>
С	cosine element of the rotation matrix.
S	sine element of the rotation matrix.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
xVal	<type> updated vector in sparse fomat (that is unchanged if</type>
	nnz==0).
у	<type> updated vector in dense format (that is unchanged if</type>
	nnz==0).

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

6.7 cusparse<t>sctr

```
cusparseStatus_t
cusparseSsctr(cusparseHandle_t handle, int nnz, const float
                                                                       *xVal,
              const int *xInd, float
                                               *y, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseDsctr(cusparseHandle_t handle, int nnz, const double
                                                                       *xVal,
              const int *xInd, double
                                               *y, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseCsctr(cusparseHandle_t handle, int nnz, const cuComplex
              const int *xInd, cuComplex
                                               *y, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseZsctr(cusparseHandle_t handle, int nnz, const cuDoubleComplex *xVal,
              const int *xInd, cuDoubleComplex *y, cusparseIndexBase_t idxBase)
```

This function scatters the elements of the vector \mathbf{x} in sparse format into the vector \mathbf{y} in dense format. It modifies only the elements of \mathbf{y} whose indices are listed in the array \mathbf{xInd} .

Input

1		
handle	handle to the CUSPARSE library context.	
nnz	number of elements of the vector x.	
xVal	<type> vector with nnz non-zero values of vector x.</type>	
xInd	integer vector with nnz indices of the non-zero values of vector x.	
у	$<$ type $>$ dense vector (of size $\ge \max(xInd)$ -idxBase+1).	
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.	
Output		
У	<pre><type> vector with nnz non-zero values that were scattered from</type></pre>	
	vector x (that is unchanged if nnz == 0).	

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the idxBase is neither CUSPARSE_INDEX_
	BASE_ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

Chapter 7 CUSPARSE Level 2 Function Reference

This chapter describes the sparse linear algebra functions that perform operations between sparse matrices and dense vectors.

In particular, the solution of sparse triangular linear systems is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate <code>csrsv_analysis()</code> function. The analysis is specific to the sparsity pattern of the given matrix and to the selected <code>cusparseOperation_t</code> type. The information from the analysis phase is stored in the parameter of type <code>cusparseSolveAnalysisInfo_t</code> that has been initialized previously with a call to <code>cusparseCreateSolveAnalysisInfo()</code>.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the cusparseSolveAnalysisInfo_t parameter by calling the appropriate csrsv_solve() function. The solve phase may be performed multiple times with different right-hand-sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for a set of different right-hand-sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the cusparseSolveAnalysisInfo_t parameter can be released by calling cusparseDestroySolveAnalysisInfo(). For more information please refer to [3].

7.1 cusparse<t>csrmv

```
int n, int nnz, const double
                                                     *alpha,
               const cusparseMatDescr_t descrA, const double
                                                                       *csrValA,
               const int *csrRowPtrA, const int *csrColIndA,
               const double
                                     *x, const double
                                                                *beta,
               double
                               *y)
cusparseStatus_t
cusparseCcsrmv(cusparseHandle_t handle, cusparseOperation_t transA, int m,
               int n, int nnz, const cuComplex
                                                     *alpha,
               const cusparseMatDescr_t descrA, const cuComplex
                                                                      *csrValA,
               const int *csrRowPtrA, const int *csrColIndA,
               const cuComplex
                                     *x, const cuComplex
                                                                *beta,
               cuComplex
                               *y)
cusparseStatus_t
cusparseZcsrmv(cusparseHandle_t handle, cusparseOperation_t transA, int m,
               int n, int nnz, const cuDoubleComplex *alpha,
               const cusparseMatDescr_t descrA, const cuDoubleComplex *csrValA,
               const int *csrRowPtrA, const int *csrColIndA,
               const cuDoubleComplex *x, const cuDoubleComplex *beta,
               cuDoubleComplex *v)
```

This function performs the matrix-vector operation

$$\mathbf{y} = \alpha * \mathsf{op}(A) * \mathbf{x} + \beta * \mathbf{y}$$

where A is $m \times n$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), x and y are vectors, α and β are scalars, and

$$\mathsf{op}(A) = \begin{cases} A & \text{if transA} == \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA} == \texttt{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA} == \texttt{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

When using the (conjugate) transpose of a general matrix or a Hermitian/symmetric matrix, this routine may produce slightly different results during different runs of this function with the same input parameters. For these matrix types it uses atomic operations to compute the final result, consequently many threads may be adding floating point numbers to the same memory location without any specific ordering, which may produce slightly different results for each run.

If exactly the same output is required for any input when multiplying by the transpose of a general matrix, the following procedure can be used:

1. Convert the matrix from CSR to CSC format using one of the csr2csc() functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.

2. Call the csrmv() function with the cusparseOperation_t parameter set to CUSPARSE_OPERATION_NON_TRANSPOSE and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

This function requires no extra storage for the general matrices when operation CUSPARSE_OPERATION_NON_TRANSPOSE is selected. It requires some extra storage for Hermitian/symmetric matrices and for the general matrices when operation different than CUSPARSE_OPERATION_NON_TRANSPOSE is selected. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.		
transA	the operation $op(A)$.		
m	number of rows of matrix A .		
n	number of columns of matrix A .		
nnz	number of non-zero elements of matrix A .		
alpha	<type> scalar used for multiplication.</type>		
descrA	the descriptor of matrix A. The supported matrix types are CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_HERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.		
csrValA	<type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero elements of matrix A .		
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.		
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column indices of the non-zero elements of matrix A.		
х	<pre><type> vector of n elements if $op(A) = A$, and m elements if $op(A) = A^T$ or $op(A) = A^H$.</type></pre>		
beta	<type> scalar used for multiplication. If beta is zero, y does not have to be a valid input.</type>		
У	<pre><type> vector of m elements if $op(A) = A$ and n elements if $op(A) = A^T$ or $op(A) = A^H$.</type></pre>		
Output			
У	<type> updated vector.</type>		

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	
	the matrix type is not supported.

7.2 cusparse<t>csrsv_analysis

```
cusparseStatus_t
```

cusparseStatus_t

 $\verb| cusparseDcsrsv_analysis| (\verb| cusparseHandle_t| | handle, | cusparseOperation_t| | transA, | left | cusparseOperation_t| | transA, | left | cusparseOperation_t| | transA, | left | cusparseOperation_t| | left | cu$

int m, int nnz, const cusparseMatDescr_t descrA,
const double *csrValA, const int *csrRowPtrA,
const int *csrColIndA, cusparseSolveAnalysisInfo_t info)

cusparseStatus_t

 $\verb| cusparseCcsrsv_analysis| (cusparseHandle_t handle, cusparseOperation_t transA, \\$

int m, int nnz, const cusparseMatDescr_t descrA,
const cuComplex *csrValA, const int *csrRowPtrA,
const int *csrColIndA, cusparseSolveAnalysisInfo_t info)

cusparseStatus_t

const cuDoubleComplex *csrValA, const int *csrRowPtrA,
const int *csrColIndA, cusparseSolveAnalysisInfo_t info)

This function performs the analysis phase of the solution of a sparse triangular linear system

$$op(A) * y = \alpha * x$$

where A is $m \times m$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), x and y are the right-hand-side and the solution

vectors, α is a scalar, and

$$\mathsf{op}(A) = \begin{cases} A & \text{if transA} == \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA} == \texttt{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA} == \texttt{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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1			
handle	handle to the CUSPARSE library context.		
transA	the operation $op(A)$.		
m	number of rows of matrix A .		
nnz	number of non-zero e	lements of matrix A .	
descrA		rix A. The supported matrix type is CUSPARSE_	
	MATRIX_TYPE_TRIANGULAR and diagonal types CUSPARSE_DIAG_TYPE_		
	UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.		
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero</type></pre>		
	elements of matrix A .		
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row		
	and the end of the last row plus one.		
csrColIndA		(= csrRowPtrA(m) - csrRowPtrA(0)) column	
	indices of the non-zero elements of matrix A .		
Output			
info	structure with information collected during the analysis phase (that		
	should be passed to the solve phase unchanged).		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m,nnz<0).	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR		an internal operation failed.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED			
		the matrix type is not supported.	

7.3 cusparse<t>csrsv_solve

```
cusparseStatus_t
cusparseScsrsv_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m,
                     const float
                                         *alpha, const cusparseMatDescr_t descrA,
                                         *csrValA, const int *csrRowPtrA,
                     const float
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const float
                                          *x, float
                                                               *y)
cusparseStatus_t
cusparseDcsrsv_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m,
                     const double
                                          *alpha, const cusparseMatDescr_t descrA,
                     const double
                                          *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const double
                                          *x, double
cusparseStatus_t
cusparseCcsrsv_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m,
                     const cuComplex
                                          *alpha, const cusparseMatDescr_t descrA,
                     const cuComplex
                                         *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const cuComplex
                                          *x, cuComplex
cusparseStatus_t
cusparseZcsrsv_solve(cusparseHandle_t handle, cusparseOperation_t transA, int m,
                     const cuDoubleComplex *alpha, const cusparseMatDescr_t descrA,
                     const cuDoubleComplex *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const cuDoubleComplex *x, cuDoubleComplex *y)
```

This function performs the solve phase of the solution of a sparse triangular linear system

$$op(A) * y = \alpha * x$$

where A is $m \times m$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), x and y are the right-hand-side and the solution vectors, α is a scalar, and

$$\mathsf{op}(A) = \begin{cases} A & \text{if transA} == \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA} == \texttt{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA} == \texttt{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

This function may be executed multiple times for a given matrix and a particular operation type.

handle to the CUSPARSE library context.	
the operation $op(A)$.	
number of rows and columns of matrix A .	
<type> scalar used for multiplication.</type>	
the descriptor of matrix A . The supported matrix type is CUSPARS:	
MATRIX_TYPE_TRIANGULAR and diagonal types CUSPARSE_DIAG_TYPE_	
UNIT and CUSPARSE_DIAG_TYPE_NON_UNIT.	
<type $>$ array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero	
elements of matrix A .	
integer array of $m + 1$ elements that contains the start of every row	
and the end of the last row plus one.	
integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column	
indices of the non-zero elements of matrix A .	
structure with information collected during the analysis phase (that	
should have been passed to the solve phase unchanged).	
<type> right-hand-side vector of size m.</type>	
<type> solution vector of size m.</type>	

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m< 0).	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED		

the matrix type is not supported.

7.4 cusparse<t>gtsvStridedBatch

cusparseStatus_t

cusparseSgtsvStridedBatch(cusparseHandle_t handle, int m,

const float *dl, const float *d, *du, float const float *x, int batchCount, int batchStride)

cusparseStatus_t

cusparseDgtsvStridedBatch(cusparseHandle_t handle, int m,

const double *dl, const double *d, const double *du, double *x,

int batchCount, int batchStride)

cusparseStatus_t

cusparseCgtsvStridedBatch(cusparseHandle_t handle, int m,

const cuComplex *dl, const cuComplex *d,
const cuComplex *du, cuComplex *x,
int batchCount, int batchStride)

cusparseStatus_t

cusparseZgtsvStridedBatch(cusparseHandle_t handle, int m,

const cuDoubleComplex *dl, const cuDoubleComplex *d
const cuDoubleComplex *du, cuDoubleComplex *x,
int batchCount, int batchStride)

This function computes the solution of multiple tridiagonal linear systems

$$A^{(i)} * \mathbf{y}^{(i)} = \alpha * \mathbf{x}^{(i)}$$

for $i = 0, \ldots$, batchCount.

The coefficient matrix A of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\mathbf{ld}), main (\mathbf{d}) and upper (\mathbf{ud}) matrix diagonals, while the right-hand-side is stored in the vector \mathbf{x} . Notice that the solution \mathbf{y} overwrites the right-hand-side \mathbf{x} on exit. The different matrices are assumed to be of the same size and are stored with a fixed batchStride in memory.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when m is a power of 2.

This routine requires significant amount of temporary extra storage (batchCount $\times (4 \times m + 2048) \times \text{sizeof}(\langle \text{type} \rangle)$). It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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inpat			
handle	handle to the CUSPARSE library context.		
m	the size of the linear system (must be ≥ 3).		
dl		containing the lower diagonal of the tri-diagonal	
	linear system. The lo	ower diagonal $\mathbf{dl}^{(i)}$ that corresponds to the <i>i</i> th	
	linear system starts at location $dl + batchStride \times i$ in memory		
	Also, the first elemen	t of each lower diagonal must be zero.	
d	<type> dense array containing the main diagonal of the tri-diagonal</type>		
	linear system. The r	nain diagonal $\mathbf{d}^{(i)}$ that corresponds to the <i>i</i> th	
	linear system starts a	t location $d + batchStride \times i$ in memory.	
du	<type> dense array containing the upper diagonal of the tri-diagonal</type>		
		pper diagonal $\mathbf{du}^{(i)}$ that corresponds to the <i>i</i> th	
	linear system starts	at location $du + batchStride \times i$ in memory.	
	Also, the last element of each upper diagonal must be zero.		
X	<type> dense array that contains the right-hand-side of the tri-</type>		
		m. The right-hand-side $\mathbf{x}^{(i)}$ that corresponds	
	to the <i>i</i> th linear system starts at location $x + batchStride \times i$ in		
	memory.		
batchCount	Number of systems to solve.		
batchStride	stride (number of elements) that separates the vectors of every system		
	(must be at least m).		
Output			
X	0.2	that contains the solution of the tri-diagonal	
	· ·	solution $\mathbf{x}^{(i)}$ that corresponds to the <i>i</i> th linear	
	system starts at location $x + batchStride \times i$ in memory.		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m<3,	
		$\mathtt{batchCount} \leq 0$, $\mathtt{batchStride} < \mathtt{m}$).	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.	

7.5 cusparse<t>hybmv

CUSPARSE_STATUS_INTERNAL_ERROR

```
cusparseStatus_t
```

an internal operation failed.

```
const float
                                     *beta, float
                                                             *y)
cusparseStatus_t
cusparseDhybmv(cusparseHandle_t handle, cusparseOperation_t transA,
               const double
                                     *alpha, const cusparseMatDescr_t descrA,
               const cusparseHybMat_t hybA, const double
                                                                   *x,
               const double
                                     *beta, double
                                                             *y)
cusparseStatus_t
cusparseChybmv(cusparseHandle_t handle, cusparseOperation_t transA,
               const cuComplex
                                     *alpha, const cusparseMatDescr_t descrA,
               const cusparseHybMat_t hybA, const cuComplex
                                                                   *x,
               const cuComplex
                                     *beta, cuComplex
cusparseStatus_t
cusparseZhybmv(cusparseHandle_t handle, cusparseOperation_t transA,
               const cuDoubleComplex *alpha, const cusparseMatDescr_t descrA,
               const cusparseHybMat_t hybA, const cuDoubleComplex *x,
               const cuDoubleComplex *beta, cuDoubleComplex *y)
```

This function performs the matrix-vector operation

$$\mathbf{y} = \alpha * \mathsf{op}(A) * \mathbf{x} + \beta * \mathbf{y}$$

where A is an $m \times n$ sparse matrix (that is defined in the HYB storage format by an opaque data structure hybA), \mathbf{x} and \mathbf{y} are vectors, α and β are scalars, and

$$\mathsf{op}(A) = \Big\{ A \quad \text{if transA == CUSPARSE_OPERATION_NON_TRANSPOSE} \\$$

Notice that currently only op(A) = A is supported.

Input	
handle	handle to the CUSPARSE library context.
transA	the operation $op(A)$ (currently only $op(A) = A$ is supported).
m	number of rows of matrix A.
n	number of columns of matrix A.
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix A. The supported matrix type is CUSPARSE_
	MATRIX_TYPE_GENERAL.
hybA	the matrix A in HYB storage format.
Х	<type> vector of n elements.</type>
beta	<type> scalar used for multiplication. If beta is zero, y does not</type>
	have to be a valid input.
У	<type> vector of m elements.</type>
Output	
У	<type> updated vector.</type>

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored hyb format parameters
	are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPP	ORTED
	the matrix type is not supported.

7.6 cusparse<t>hybsv_analysis

cusparseStatus_t

cusparseStatus_t

cusparseStatus_t

cusparseStatus_t

This function performs the analysis phase of the solution of a sparse triangular linear system

$$op(A) * y = \alpha * x$$

where A is $m \times m$ sparse matrix (that is defined in HYB storage format by an opaque data structure hybA), x and y are the right-hand-side and the solution vectors, α is a scalar, and

$$\mathtt{op}(A) = \Big\{ A \quad \text{if transA == CUSPARSE_OPERATION_NON_TRANSPOSE} \\$$

Notice that currently only op(A) = A is supported.

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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$_{ m input}$			
handle	handle to the CUSPARSE library context.		
transA	the operation $op(A)$ (currently only $op(A) = A$ is supported).		
descrA	the descriptor of mate	rix A. The supported matrix type is CUSPARSE_	
	MATRIX_TYPE_TRIANG	ULAR and diagonal type CUSPARSE_DIAG_TYPE_	
	NON_UNIT.		
hybA	the matrix A in HYB	storage format.	
Output	Output		
info	structure with information collected during the analysis phase (that		
	should be passed to the solve phase unchanged).		
Status Returned	Status Returned		
CUSPARSE_STATUS_S	SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE		the internally stored hyb format parameters	
		are invalid.	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR		an internal operation failed.	
CUSPARSE_STATUS_N	MATRIX_TYPE_NOT_SUPP	ORTED	
		the matrix type is not supported.	

7.7 cusparse<t>hybsv_solve

```
cusparseStatus_t
cusparseShybsv_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                                           *alpha, const cusparseMatDescr_t descrA,
                     const float
                     cusparseHybMat_t hybA, cusparseSolveAnalysisInfo_t info,
                     const float
                                           *x, float
                                                                *y)
cusparseStatus_t
cusparseDhybsv_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                     const double
                                           *alpha, const cusparseMatDescr_t descrA,
                     cusparseHybMat_t hybA, cusparseSolveAnalysisInfo_t info,
                     const double
                                                                *y)
                                           *x, double
cusparseStatus_t
cusparseChybsv_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                                           *alpha, const cusparseMatDescr_t descrA,
                     const cuComplex
                     cusparseHybMat_t hybA, cusparseSolveAnalysisInfo_t info,
```

const cuComplex *x, cuComplex *y)

cusparseStatus_t

This function performs the solve phase of the solution of a sparse triangular linear system

$$\operatorname{op}(A) * \mathbf{y} = \alpha * \mathbf{x}$$

where A is $m \times m$ sparse matrix (that is defined in HYB storage format by an opaque data structure hybA), \mathbf{x} and \mathbf{y} are the right-hand-side and the solution vectors, α is a scalar, and

$$\mathtt{op}(A) = \Big\{ A \quad \text{if transA == CUSPARSE_OPERATION_NON_TRANSPOSE} \\$$

Notice that currently only op(A) = A is supported.

This function may be executed multiple times for a given matrix and a particular operation type.

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handle	handle to the CUSPARSE library context.
transA	the operation $op(A)$ (currently only $op(A) = A$ is supported).
alpha	<type> scalar used for multiplication.</type>
descrA	the descriptor of matrix A . The supported matrix type is CUSPARSE_
	MATRIX_TYPE_TRIANGULAR and diagonal type CUSPARSE_DIAG_TYPE_
	NON_UNIT.
hybA	the matrix A in HYB storage format.
info	structure with information collected during the analysis phase (that
	should have been passed to the solve phase unchanged).
X	<type> right-hand-side vector of size m.</type>
Output	
У	<type> solution vector of size m.</type>

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored hyb format parameters
	are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR	the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	
	the matrix type is not supported.

Chapter 8 CUSPARSE Level 3 Function Reference

This chapter describes sparse linear algebra functions that perform operations between sparse and (usually tall) dense matrices.

In particular, the solution of sparse triangular linear systems with multiple right-hand-sides is implemented in two phases. First, during the analysis phase, the sparse triangular matrix is analyzed to determine the dependencies between its elements by calling the appropriate <code>csrsm_analysis()</code> function. The analysis is specific to the sparsity pattern of the given matrix and to the selected <code>cusparseOperation_t</code> type. The information from the analysis phase is stored in the parameter of type <code>cusparseSolveAnalysisInfo_t</code> that has been initialized previously with a call to <code>cusparseCreateSolveAnalysisInfo()</code>.

Second, during the solve phase, the given sparse triangular linear system is solved using the information stored in the cusparseSolveAnalysisInfo_t parameter by calling the appropriate csrsm_solve() function. The solve phase may be performed multiple times with different multiple right-hand-sides, while the analysis phase needs to be performed only once. This is especially useful when a sparse triangular linear system must be solved for different sets of multiple right-hand-sides one at a time, while its coefficient matrix remains the same.

Finally, once all the solves have completed, the opaque data structure pointed to by the cusparseSolveAnalysisInfo_t parameter can be released by calling cusparseDestroySolveAnalysisInfo(). For more information please refer to [3].

8.1 cusparse<t>csrmm

```
cusparseDcsrmm(cusparseHandle_t handle, cusparseOperation_t transA,
              int m, int n, int k, int nnz, const double
              const cusparseMatDescr_t descrA, const double
                                                                     *csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
              const double
                                     *B, int ldb,
              const double
                                     *beta, double
                                                           *C, int ldc)
cusparseStatus_t
cusparseCcsrmm(cusparseHandle_t handle, cusparseOperation_t transA,
                                                              *alpha,
              int m, int n, int k, int nnz, const cuComplex
              const cusparseMatDescr_t descrA, const cuComplex
                                                                     *csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
                                     *B, int ldb,
              const cuComplex
              const cuComplex *beta, cuComplex *C, int ldc)
cusparseStatus_t
cusparseZcsrmm(cusparseHandle_t handle, cusparseOperation_t transA,
              int m, int n, int k, int nnz, const cuDoubleComplex *alpha,
              const cusparseMatDescr_t descrA, const cuDoubleComplex *csrValA,
              const int *csrRowPtrA, const int *csrColIndA,
              const cuDoubleComplex *B, int ldb,
              const cuDoubleComplex *beta, cuDoubleComplex *C, int ldc)
```

This function performs one of the following matrix-matrix operation

$$C = \alpha * \mathsf{op}(A) * B + \beta * C$$

where A is $m \times n$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), B and C are dense matrices, α and β are scalars, and

$$\mathsf{op}(A) = \begin{cases} A & \text{if transA} == \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA} == \texttt{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA} == \texttt{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

When using the (conjugate) transpose of a general matrix or a Hermitian/symmetric matrix, this routine may produce slightly different results during different runs of this function with the same input parameters. For these matrix types it uses atomic operations to compute the final result, consequently many threads may be adding floating point numbers to the same memory location without any specific ordering, which may produce slightly different results for each run.

If exactly the same output is required for any input when multiplying by the transpose of a general matrix, the following procedure can be used:

1. Convert the matrix from CSR to CSC format using one of the csr2csc() functions. Notice that by interchanging the rows and columns of the result you are implicitly transposing the matrix.

2. Call the csrmm() function with the cusparseOperation_t parameter set to CUSPARSE_OPERATION_NON_TRANSPOSE and with the interchanged rows and columns of the matrix stored in CSC format. This (implicitly) multiplies the vector by the transpose of the matrix in the original CSR format.

This function requires no extra storage for the general matrices when operation CUSPARSE_OPERATION_NON_TRANSPOSE is selected. It requires some extra storage for Hermitian/symmetric matrices and for the general matrices when operation different than CUSPARSE_OPERATION_NON_TRANSPOSE is selected. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.	
transA	the operation $op(A)$.	
m	number of rows of sparse matrix A.	
n	number of columns of dense matrices B and C.	
k	number of columns of sparse matrix A.	
nnz	number of non-zero elements of sparse matrix A.	
alpha	<type> scalar used for multiplication.</type>	
descrA	the descriptor of matrix A. The supported matrix types are CUSPARSE_MATRIX_TYPE_GENERAL, CUSPARSE_MATRIX_TYPE_SYMMETRIC, and CUSPARSE_MATRIX_TYPE_HERMITIAN. Also, the supported index bases are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.	
csrValA	<type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero elements of matrix A .	
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row and the end of the last row plus one.	
csrColIndA	integer array of nnz (= $csrRowPtrA(m) - csrRowPtrA(0)$) column indices of the non-zero elements of matrix A .	
В	array of dimensions (ldb, n).	
ldb	leading dimension of B. It must be at least max (1, k) if $op(A) = A$, and at least max (1, m) otherwise.	
beta	<pre><type> scalar used for multiplication. If beta is zero, C does not have to be a valid input.</type></pre>	
C	array of dimensions (ldc, n).	
ldc	leading dimension of C. It must be at least max $(1, m)$ if $op(A) = A$ and at least max $(1, k)$ otherwise.	
Output		
С	<type> updated array of dimensions (ldc, n).</type>	

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,k,nnz <
	0 or 1db and 1dc are incorrect).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPP	ORTED
	the matrix type is not supported.

8.2 cusparse<t>csrsm_analysis

```
cusparseStatus_t
```

cusparseStatus_t

cusparseDcsrsm_analysis(cusparseHandle_t handle, cusparseOperation_t transA,

int m, int nnz, const cusparseMatDescr_t descrA,
const double *csrValA, const int *csrRowPtrA,
const int *csrColIndA, cusparseSolveAnalysisInfo_t info)

cusparseStatus_t

cusparseStatus_t

This function performs the analysis phase of the solution of a sparse triangular linear system

$$\operatorname{op}(A) * Y = \alpha * X$$

with multiple right-hand-sides, where A is $m \times m$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), X and Y are

the right-hand-side and the solution dense matrices, α is a scalar, and

$$\mathtt{op}(A) = \begin{cases} A & \text{if transA == CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA == CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA == CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

It is expected that this function will be executed only once for a given matrix and a particular operation type.

This function requires significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.		
transA	the operation $op(A)$.		
m	number of rows of matrix A .		
nnz	number of non-zero e	lements of matrix A .	
descrA	_	rix A. The supported matrix type is CUSPARSE_	
		ULAR and diagonal types CUSPARSE_DIAG_TYPE_	
	UNIT and CUSPARSE_I		
csrValA		(= csrRowPtrA(m) - csrRowPtrA(0)) non-zero	
	elements of matrix A		
csrRowPtrA		elements that contains the start of every row	
	and the end of the las	-	
${\tt csrColIndA}$		(= csrRowPtrA(m) - csrRowPtrA(0)) column	
	indices of the non-zero elements of matrix A .		
Output			
info	structure with information collected during the analysis phase (that		
	should be passed to the solve phase unchanged).		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m,nnz<0).	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR		an internal operation failed.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED		ORTED	
		the matrix type is not supported.	

8.3 cusparse<t>csrsm_solve

```
cusparseStatus_t
cusparseScsrsm_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                     int m, int n, const float *alpha,
                     const cusparseMatDescr_t descrA,
                     const float
                                           *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const float
                                           *X, int ldx,
                                     *Y, int ldy)
                     float
cusparseStatus_t
cusparseDcsrsm_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                     int m, int n, const double *alpha,
                     const cusparseMatDescr_t descrA,
                     const double
                                           *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const double
                                           *X, int ldx,
                     double
                                     *Y, int ldy)
cusparseStatus_t
cusparseCcsrsm_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                     int m, int n, const cuComplex *alpha,
                     const cusparseMatDescr_t descrA,
                     const cuComplex
                                          *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const cuComplex
                                           *X, int ldx,
                     cuComplex
                                   *Y, int ldy)
cusparseStatus_t
cusparseZcsrsm_solve(cusparseHandle_t handle, cusparseOperation_t transA,
                     int m, int n, const cuDoubleComplex *alpha,
                     const cusparseMatDescr_t descrA,
                     const cuDoubleComplex *csrValA, const int *csrRowPtrA,
                     const int *csrColIndA, cusparseSolveAnalysisInfo_t info,
                     const cuDoubleComplex *X, int ldx,
                     cuDoubleComplex *Y, int ldy)
```

This function performs the solve phase of the solution of a sparse triangular linear system

$$\operatorname{op}(A) * Y = \alpha * X$$

with multiple right-hand-sides, where A is $m \times m$ sparse matrix (that is defined in CSR storage format by the three arrays csrValA, csrRowPtrA, and csrColIndA), X and Y are

the right-hand-side and the solution dense matrices, α is a scalar, and

$$\mathsf{op}(A) = \begin{cases} A & \text{if transA} == \texttt{CUSPARSE_OPERATION_NON_TRANSPOSE} \\ A^T & \text{if transA} == \texttt{CUSPARSE_OPERATION_TRANSPOSE} \\ A^H & \text{if transA} == \texttt{CUSPARSE_OPERATION_CONJUGATE_TRANSPOSE} \end{cases}$$

This function may be executed multiple times for a given matrix and a particular operation type.

Input		
handle	handle to the CUSPARSE library context.	
transA	the operation $op(A)$.	
m	number of rows and o	columns of matrix A.
n	number of columns of	f matrix X and Y .
alpha	<type> scalar used f</type>	or multiplication.
descrA	_	rix A . The supported matrix type is CUSPARSE_
		ULAR and diagonal types CUSPARSE_DIAG_TYPE_
	UNIT and CUSPARSE_I	
csrValA	0 2	(= csrRowPtrA(m) - csrRowPtrA(0)) non-zero
	elements of matrix A	
csrRowPtrA		l elements that contains the start of every row
	and the end of the las	-
${\tt csrColIndA}$		(= csrRowPtrA(m) - csrRowPtrA(0)) column
		o elements of matrix A.
info		nation collected during the analysis phase (that
	_	sed to the solve phase unchanged).
X		ide array of dimensions (ldx, n).
ldx	leading dimension of	$X \text{ (that is } \geq \max(1, m)).$
Output		
Y		y of dimensions (ldy, n).
ldy	leading dimension of	$Y \text{ (that is } \geq \max(1, m)).$
Status Returned		
CUSPARSE_STATUS_S	UCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m< 0).
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.
CUSPARSE_STATUS_MAPPING_ERROR		the texture binding failed.
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.
CUSPARSE_STATUS_I	NTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_M	MATRIX_TYPE_NOT_SUPP	
		the matrix type is not supported.

8.4 cusparse<t>gtsv

```
cusparseStatus_t
cusparseSgtsv(cusparseHandle_t handle, int m, int n,
              const float
                                    *dl, const float
                                                                 *d,
                                    *du, float *B, int ldb)
              const float
cusparseStatus_t
cusparseDgtsv(cusparseHandle_t handle, int m, int n,
              const double
                                    *dl, const double
                                                                 *d,
              const double
                                    *du, double *B, int ldb)
cusparseStatus_t
cusparseCgtsv(cusparseHandle_t handle, int m, int n,
                                    *dl, const cuComplex
              const cuComplex
                                                                 *d,
              const cuComplex
                                    *du, cuComplex
                                                          *B, int ldb)
cusparseStatus_t
cusparseZgtsv(cusparseHandle_t handle, int m, int n,
              const cuDoubleComplex *dl, const cuDoubleComplex *d,
              const cuDoubleComplex *du, cuDoubleComplex *B, int ldb)
```

This function computes the solution of a tridiagonal linear system

$$A * Y = \alpha * X$$

with multiple right-hand-sides.

The coefficient matrix A of each of these tri-diagonal linear system is defined with three vectors corresponding to its lower (\mathbf{ld}), main (\mathbf{d}) and upper (\mathbf{ud}) matrix diagonals, while the right-hand-sides are stored in the dense matrix X. Notice that the solutions Y overwrite the right-hand-sides X on exit.

The routine does not perform any pivoting and uses a combination of the Cyclic Reduction (CR) and Parallel Cyclic Reduction (PCR) algorithms to find the solution. It achieves better performance when m is a power of 2.

This routine requires significant amount of temporary extra storage $(m \times (3 + n) \times sizeof(<type>))$. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

Input

handle	handle to the CUSPARSE library context.
m	the size of the linear system (must be ≥ 3).
n	number of right-hand-sides, columns of matrix B.
dl	<type> dense array containing the lower diagonal of the tri-diagonal</type>
	linear system. The first element of each lower diagonal must be zero.
d	<type> dense array containing the main diagonal of the tri-diagonal</type>
	linear system.
du	<type> dense array containing the upper diagonal of the tri-diagonal</type>
	linear system. The last element of each upper diagonal must be zero.
В	<pre><type> dense right-hand-side array of dimensions (ldb, m).</type></pre>
ldb	leading dimension of B (that is $\geq \max(1, m)$).
Output	

B Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m< 3, n< 0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

<type> dense solution array of dimensions (ldb, m).

Chapter 9 CUSPARSE Format Conversion Reference

This chapter describes the conversion routines between different sparse and dense storage formats.

9.1 cusparse<t>coo2csr

This function converts the array containing the uncompressed row indices (corresponding to COO format) into an array of compressed row pointers (corresponding to CSR format).

It can also be used to convert the array containing the uncompressed column indices (corresponding to COO format) into an array of column pointers (corresponding to CSC format).

Input	
handle	handle to the CUSPARSE library context.
cooRowInd	integer array of nnz uncompressed row indices.
nnz	number of non-zeros of the sparse matrix (that is also the length of
	array cooRowInd).
m	number of rows of matrix A .
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
csrRowPtr	integer array of $m + 1$ elements that contains the start of every row
	and the end of the last row plus one.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	idxBase is neither CUSPARSE_INDEX_BASE_
	ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

9.2 cusparse<t>csc2dense

```
cusparseStatus_t
cusparseScsc2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const float
                                                                        *cscValA.
                const int *cscRowIndA, const int *cscColPtrA,
                float
                                *A. int lda)
cusparseStatus_t
cusparseDcsc2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const double
                                                                        *cscValA,
                const int *cscRowIndA, const int *cscColPtrA,
                double
                                *A, int lda)
cusparseStatus_t
cusparseCcsc2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuComplex
                                                                        *cscValA,
                const int *cscRowIndA, const int *cscColPtrA,
                cuComplex
                                *A, int lda)
cusparseStatus_t
cusparseZcsc2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuDoubleComplex *cscValA,
                const int *cscRowIndA, const int *cscColPtrA,
                cuDoubleComplex *A, int lda)
```

This function converts the sparse matrix in CSC format (that is defined by the three arrays cscValA, cscColPtrA and cscRowIndA) into the matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

the function failed to launch on the GPU.

the matrix type is not supported.

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-		
handle	handle to the CUSPARSE library context.	
m	number of rows of ma	atrix A .
n	number of columns of	f matrix A .
descrA	CUSPARSE_MATRIX_TY are CUSPARSE_INDEX_	natrix A. The supported matrix type is PE_GENERAL. Also, the supported index bases BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
cscValA	elements of matrix A	
cscRowIndA		(= cscRowPtrA(m) - cscRowPtrA(0)) column o elements of matrix A .
cscColPtrA	integer array of $n+1$ and the end of the las	elements that contains the start of every column st column plus one.
lda	leading dimension of	dense array A.
Output		
A	array of dimensions (lda, n) that is filled in with the values of the sparse matrix.	
Status Returned		
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.
CUSPARSE_STATUS_N	NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_	INVALID_VALUE	invalid parameters were passed $(m,n<0)$.
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.

9.3 cusparse<t>csr2coo

CUSPARSE_STATUS_EXECUTION_FAILED

CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED

```
cusparseStatus_t
```

This function converts the array containing the compressed row pointers (corresponding to CSR format) into an array of uncompressed row indices (corresponding to COO format).

It can also be used to convert the array containing the compressed column indices (corresponding to CSC format) into an array of uncompressed column indices (corresponding to COO format).

Input

handle	handle to the CUSPARSE library context.
csrRowPtr	integer array of $m + 1$ elements that contains the start of every row
	and the end of the last row plus one.
nnz	number of non-zeros of the sparse matrix (that is also the length of
	array cooRowInd).
m	number of rows of matrix A .
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.

Output

cooRowInd	integer array of nnz uncompressed row indices.

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	idxBase is neither CUSPARSE_INDEX_BASE_
	ZERO nor CUSPARSE_INDEX_BASE_ONE.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.

9.4 cusparse<t>csr2csc

```
cusparseStatus_t
cusparseScsr2csc(cusparseHandle_t handle, int m, int n, int nnz,
                 const float *csrVal, const int *csrRowPtr,
                 const int *csrColInd, float
                                                       *cscVal,
                 int *cscRowInd, int *cscColPtr,
                 cusparseAction_t copyValues, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseDcsr2csc(cusparseHandle_t handle, int m, int n, int nnz,
                 const double *csrVal, const int *csrRowPtr,
                 const int *csrColInd, double
                 int *cscRowInd, int *cscColPtr,
                 cusparseAction_t copyValues, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseCcsr2csc(cusparseHandle_t handle, int m, int n, int nnz,
                 const cuComplex *csrVal, const int *csrRowPtr,
                 const int *csrColInd, cuComplex
                                                       *cscVal,
                 int *cscRowInd, int *cscColPtr,
                 cusparseAction_t copyValues, cusparseIndexBase_t idxBase)
cusparseStatus_t
cusparseZcsr2csc(cusparseHandle_t handle, int m, int n, int nnz,
                 const cuDoubleComplex *csrVal, const int *csrRowPtr,
                 const int *csrColInd, cuDoubleComplex *cscVal,
```

int *cscRowInd, int *cscColPtr,
cusparseAction_t copyValues, cusparseIndexBase_t idxBase)

This function converts a sparse matrix in CSR format (that is defined by the three arrays csrValA, csrRowPtrA and csrColIndA) into a sparse matrix in CSC format (that is defined by arrays cscVal, cscRowInd, and cscColPtr). The resulting matrix can also be seen as the transpose of the original sparse matrix. Notice that this routine can also be used to convert a matrix in CSC format into a matrix in CSR format.

This function requires significant amount of extra storage that is proportional to the matrix size. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.
m	number of rows of matrix A.
n	number of columns of matrix A.
nnz	number of non-zero elements of matrix A.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero</type></pre>
	elements of matrix A .
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row
	and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column
	indices of the non-zero elements of matrix A .
copyValues	CUSPARSE_ACTION_SYMBOLIC or CUSPARSE_ACTION_NUMERIC.
idxBase	CUSPARSE_INDEX_BASE_ZERO or CUSPARSE_INDEX_BASE_ONE.
Output	
cscValA	<pre><type> array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) non-</type></pre>
	zero elements of matrix A. It is only filled-in if copyValues is set
	to CUSPARSE_ACTION_NUMERIC.
cscRowIndA	integer array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) column
	indices of the non-zero elements of matrix A .
cscColPtrA	integer array of $n+1$ elements that contains the start of every column
	and the end of the last column plus one.

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n,nnz<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.

9.5 cusparse<t>csr2dense

```
cusparseStatus_t
cusparseScsr2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const float
                                                                        *csrValA,
                const int *csrRowPtrA, const int *csrColIndA,
                float
                                *A, int lda)
cusparseStatus_t
cusparseDcsr2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const double
                                                                        *csrValA.
                const int *csrRowPtrA, const int *csrColIndA,
                double
                                *A, int lda)
cusparseStatus_t
cusparseCcsr2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuComplex
                                                                        *csrValA,
                const int *csrRowPtrA, const int *csrColIndA,
                cuComplex
                                *A, int lda)
cusparseStatus_t
cusparseZcsr2dense(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuDoubleComplex *csrValA,
                const int *csrRowPtrA, const int *csrColIndA,
                cuDoubleComplex *A, int lda)
```

This function converts the sparse matrix in CSR format (that is defined by the three arrays csrValA, csrRowPtrA and csrColIndA) into the matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

Input	
handle	handle to the CUSPARSE library context.
m	number of rows of matrix A .
n	number of columns of matrix A .
descrA	the descriptor of matrix A. The supported matrix type is
	CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases
	are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero</type></pre>
	elements of matrix A .
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row
	and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column
	indices of the non-zero elements of matrix A .
lda	leading dimension of array matrix A.

Output

A	array of dimensions (lda, n) that is filled in with the values of the	
	sparse matrix.	
Status Returned		
CUSPARSE_STATUS_SUCCESS the operation completed successfully.		
CUSPARSE_STATUS_NOT_INITIALIZED the		the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m,n<0).
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED 1		the function failed to launch on the GPU.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED		
the matrix type is not supported.		the matrix type is not supported.

9.6 cusparse<t>csr2hyb

```
cusparseStatus_t
cusparseScsr2hyb(cusparseHandle_t handle, int m, int n,
            const cusparseMatDescr_t descrA, const float
                                                                    *csrValA,
            const int *csrRowPtrA, const int *csrColIndA, cusparseHybMat_t hybA,
            int userEllWidth, cusparseHybPartition_t partitionType)
cusparseStatus_t
cusparseDcsr2hyb(cusparseHandle_t handle, int m, int n,
            const cusparseMatDescr_t descrA, const double
                                                                    *csrValA.
            const int *csrRowPtrA, const int *csrColIndA, cusparseHybMat_t hybA,
            int userEllWidth, cusparseHybPartition_t partitionType)
cusparseStatus_t
cusparseCcsr2hyb(cusparseHandle_t handle, int m, int n,
            const cusparseMatDescr_t descrA, const cuComplex
                                                                    *csrValA,
            const int *csrRowPtrA, const int *csrColIndA, cusparseHybMat_t hybA,
            int userEllWidth, cusparseHybPartition_t partitionType)
cusparseStatus_t
cusparseZcsr2hyb(cusparseHandle_t handle, int m, int n,
            const cusparseMatDescr_t descrA, const cuDoubleComplex *csrValA,
            const int *csrRowPtrA, const int *csrColIndA, cusparseHybMat_t hybA,
            int userEllWidth, cusparseHybPartition_t partitionType)
```

This function converts a sparse matrix in CSR format into a sparse matrix in HYB format. It assumes that the hybA parameter has been initialized with cusparseCreateHybMat routine before calling this function.

This function requires some amount of temporary storage and a significant amount of storage for the matrix in HYB format. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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•	
handle	handle to the CUSPARSE library context.
m	number of rows of matrix A .
n	number of columns of matrix A .
descrA	the descriptor of matrix A in CSR format. The supported matrix type
	is CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases
	are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero</type></pre>
	elements of matrix A .
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row
	and the end of the last row plus one.
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column
	indices of the non-zero elements of matrix A .
userEllWidth	width of the regular (ELL) part of the matrix in HYB format, which
	should be less than maximum number of non-zeros per row and is only
	required if partitionType == CUSPARSE_HYB_PARTITION_USER.
partitionType	partitioning method to be used in the conversion (please refer to
	cusparseHybPartition_t on page 15 for details).

Output

hybA

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0).	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.	
CUSPARSE STATUS MATRIX TYPE NOT SUPPORTED		

the matrix A in HYB storage format.

the matrix type is not supported.

9.7 cusparse<t>dense2csc

```
cusparseStatus_t
cusparseSdense2csc(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const float
                                                                       *A,
                int lda, const int *nnzPerCol, float
                                                             *cscValA,
                int *cscRowIndA, int *cscColPtrA)
cusparseStatus_t
cusparseDdense2csc(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const double
                                                                       *A,
```

This function converts the matrix A in dense format into a sparse matrix in CSC format. All the parameters are assumed to have been pre-allocated by the user and the arrays are filled in based on nnzPerCol, which can be pre-computed with cusparse<t>nnz().

Input	
handle	handle to the CUSPARSE library context.
m	number of rows of matrix A .
n	number of columns of matrix A .
descrA	the descriptor of matrix A . The supported matrix type is
	CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases
	are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.
A	array of dimensions (lda, n).
lda	leading dimension of dense array A.
nnzPerCol	array of size n containing the number of non-zero elements per col-
	umn.
Output	
cscValA	<pre><type> array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) non-zero</type></pre>
	elements of matrix A .
cscRowIndA	integer array of nnz (= cscRowPtrA(m) - cscRowPtrA(0)) column
	indices of the non-zero elements of matrix A .
cscColPtrA	integer array of $n+1$ elements that contains the start of every column
	and the end of the last column plus one.

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.	
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0).	
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED		
	the matrix type is not supported.	

9.8 cusparse<t>dense2csr

```
cusparseStatus_t
cusparseSdense2csr(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const float
                                                                        *A,
                int lda, const int *nnzPerRow, float
                                                                *csrValA,
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus_t
cusparseDdense2csr(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const double
                                                                        *A,
                int lda, const int *nnzPerRow, double
                                                                *csrValA,
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus_t
cusparseCdense2csr(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuComplex
                                                                        *A,
                int lda, const int *nnzPerRow, cuComplex
                                                                *csrValA.
                int *csrRowPtrA, int *csrColIndA)
cusparseStatus_t
cusparseZdense2csr(cusparseHandle_t handle, int m, int n,
                const cusparseMatDescr_t descrA, const cuDoubleComplex *A,
                int lda, const int *nnzPerRow, cuDoubleComplex *csrValA,
                int *csrRowPtrA, int *csrColIndA)
```

This function converts the matrix A in dense format into a sparse matrix in CSR format. All the parameters are assumed to have been pre-allocated by the user and the arrays are filled in based on the nnzPerRow, which can be pre-computed with cusparse<t>nnz().

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handle	handle to the CUSPARSE library context.		
m	number of rows of matrix A .		
n	number of columns of matrix A .		
descrA	the descriptor of n	natrix A. The supported matrix type is	
	CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases		
	are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.		
A	array of dimensions (lda, n).	
lda	leading dimension of dense array A.		
nnzPerRow	array of size m containing the number of non-zero elements per row.		
Output			
csrValA	<pre><type> array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) non-zero</type></pre>		
	elements of matrix A .		
csrRowPtrA	integer array of $m + 1$ elements that contains the start of every row		
	and the end of the last row plus one.		
csrColIndA	integer array of nnz (= csrRowPtrA(m) - csrRowPtrA(0)) column		
	indices of the non-zero elements of matrix A .		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_I	NVALID_VALUE	invalid parameters were passed (m,n<0).	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	

CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED

CUSPARSE_STATUS_EXECUTION_FAILED

CUSPARSE_STATUS_INTERNAL_ERROR

the matrix type is not supported.

an internal operation failed.

the function failed to launch on the GPU.

9.9 cusparse<t>dense2hyb

This function converts the matrix A in dense format into a sparse matrix in HYB format. It assumes that the routine cusparseCreateHybMat was used to initialize the opaque structure hybA and that the array nnzPerRow was pre-computed with cusparse<t>nnz().

int userEllWidth, cusparseHybPartition_t partitionType)

This function requires some amount of temporary storage and a significant amount of storage for the matrix in HYB format. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.		
m	number of rows of matrix A .		
n	number of columns of matrix A .		
descrA	the descriptor of the dense matrix A . The supported matrix type is		
	CUSPARSE_MATRIX_TYPE_GENERAL.		
A	array of dimensions (lda, n).		
lda	leading dimension of dense array A.		
nnzPerRow	array of size m containing the number of non-zero elements per row.		
userEllWidth	width of the regular (ELL) part of the matrix in HYB format, which		
	should be less than maximum number of non-zeros per row and is only		
	required if partitionType == CUSPARSE_HYB_PARTITION_USER.		
partitionType	partitioning method to be used in the conversion (please refer to		
	cusparseHybPartition_t on page 15 for details).		
Output			
hybA	the matrix A in HYB storage format.		
Status Returned			
CUSPARSE_STATUS_SUCCESS		the operation completed successfully.	
CUSPARSE_STATUS_NOT_INITIALIZED		the library was not initialized.	
CUSPARSE_STATUS_ALLOC_FAILED		the resources could not be allocated.	
CUSPARSE_STATUS_INVALID_VALUE		invalid parameters were passed (m,n<0).	
CUSPARSE_STATUS_ARCH_MISMATCH		the device does not support double precision.	
CUSPARSE_STATUS_EXECUTION_FAILED		the function failed to launch on the GPU.	
CUSPARSE_STATUS_INTERNAL_ERROR		an internal operation failed.	
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED			

the matrix type is not supported.

9.10 cusparse<t>hyb2dense

cusparseStatus_t

cusparseShyb2dense(cusparseHandle_t handle, const cusparseMatDescr_t descrA, const cusparseHybMat_t hybA, float *A, int lda)

cusparseStatus_t

cusparseDhyb2dense(cusparseHandle_t handle, const cusparseMatDescr_t descrA, const cusparseHybMat_t hybA, double *A, int lda)

cusparseStatus_t

cusparseChyb2dense(cusparseHandle_t handle, const cusparseMatDescr_t descrA, const cusparseHybMat_t hybA, cuComplex *A, int lda)

cusparseStatus_t

cusparseZhyb2dense(cusparseHandle_t handle, const cusparseMatDescr_t descrA, const cusparseHybMat_t hybA, cuDoubleComplex *A, int lda)

This function converts a sparse matrix in HYB format (contained in the opaque structure hybA) into a matrix A in dense format. The dense matrix A is filled in with the values of the sparse matrix and with zeros elsewhere.

This function requires no extra storage. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

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handle	handle to the CUSPARSE library context.
descrA	the descriptor of the matrix A in Hyb format. The supported matrix
	type is CUSPARSE_MATRIX_TYPE_GENERAL.
hybA	the matrix A in HYB storage format.
lda	leading dimension of dense array A.

Output

A	array of dimensions (lda, n) that is filled in with the values of the
	sparse matrix.

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_INVALID_VALUE	the internally stored hyb format parameters
	are invalid.
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE STATUS MATRIX TYPE NOT SUPPORTED	

the matrix type is not supported.

9.11 cusparse<t>nnz

```
cusparseStatus_t
cusparseSnnz(cusparseHandle_t handle, cusparseDirection_t dirA, int m, int n,
             const cusparseMatDescr_t descrA, const float
             int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus_t
cusparseDnnz(cusparseHandle_t handle, cusparseDirection_t dirA, int m, int n,
             const cusparseMatDescr_t descrA, const double
             int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus_t
cusparseCnnz(cusparseHandle_t handle, cusparseDirection_t dirA, int m, int n,
             const cusparseMatDescr_t descrA, const cuComplex
             int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
cusparseStatus_t
cusparseZnnz(cusparseHandle_t handle, cusparseDirection_t dirA, int m, int n,
             const cusparseMatDescr_t descrA, const cuDoubleComplex *A,
             int lda, int *nnzPerRowColumn, int *nnzTotalDevHostPtr)
```

This function computes the number of non-zero elements per row or column and the total number of non-zero elements in a dense matrix.

This function requires no extra storage. It is executed asynchronously with respect to the host and it may return control to the application on the host before the result is ready.

Input		
handle	handle to the CUSPARSE library context.	
dirA	direction that specifies whether to count non-zero elements by	
	CUSPARSE_DIRECTION_ROW or CUSPARSE_DIRECTION_COLUMN.	
m	number of rows of matrix A .	
n	number of columns of matrix A .	
descrA	the descriptor of matrix A. The supported matrix type is	
	CUSPARSE_MATRIX_TYPE_GENERAL. Also, the supported index bases	
	are CUSPARSE_INDEX_BASE_ZERO and CUSPARSE_INDEX_BASE_ONE.	
A	array of dimensions (lda, n).	
lda	leading dimension of dense array A.	
Output		
nnzPerRowColumn	array of size m or n containing the number of non-zero elements per	
	row or column, respectively.	
nnzTotalDevHostPtr	total number of non-zero elements in device or host memory.	

Status Returned

CUSPARSE_STATUS_SUCCESS	the operation completed successfully.
CUSPARSE_STATUS_NOT_INITIALIZED	the library was not initialized.
CUSPARSE_STATUS_ALLOC_FAILED	the resources could not be allocated.
CUSPARSE_STATUS_INVALID_VALUE	invalid parameters were passed (m,n<0).
CUSPARSE_STATUS_ARCH_MISMATCH	the device does not support double precision.
CUSPARSE_STATUS_EXECUTION_FAILED	the function failed to launch on the GPU.
CUSPARSE_STATUS_INTERNAL_ERROR	an internal operation failed.
CUSPARSE_STATUS_MATRIX_TYPE_NOT_SUPPORTED	
	the matrix type is not supported.

Chapter 10

Appendix A: Using the CUSPARSE Legacy API

This appendix does not provide a full reference of each Legacy API datatype and entry point. Instead, it describes how to use the API, especially where this is different from the regular CUSPARSE API.

Note that in this section, all references to the "CUSPARSE Library" refer to the Legacy CUSPARSE API only.

10.1 Thread Safety

The legacy API is also thread safe when used with multiple host threads and devices.

10.2 Scalar Parameters

In the legacy CUSPARSE API, scalar parameters are passed by value from the host. Also, the few functions that do return a scalar result, such as doti() and nnz(), return the resulting value on the host, and hence these routines will wait for kernel execution on the device to complete before returning, which makes parallelism with streams impractical. However, the majority of functions do not return any value, in order to be more compatible with Fortran and the existing sparse libraries.

10.3 Helper Functions

In this section we list the helper functions provided by the legacy CUSPARSE API and their functionality. For the exact prototypes of these functions please refer to the legacy CUSPARSE API header file "cusparse.h".

Helper function	Meaning
<pre>cusparseSetKernelStream()</pre>	sets the stream to be used by the library

10.4 Level-1,2,3 Functions

The Level-1,2,3 CUSPARSE functions (also called core functions) have the same name and behavior as the ones listed in the chapters 6, 7 and 8 in this document. Notice that not all of the routines are available in the legacy API. Please refer to the legacy CUSPARSE API header file "cusparse.h" for their exact prototype. Also, the next section talks a bit more about the differences between the legacy and the CUSPARSE API prototypes, and more specifically how to convert the function calls from one API to another.

10.5 Converting Legacy to the CUSPARSE API

There are a few general rules that can be used to convert from legacy to the CUSPARSE API.

- 1. Exchange the header file "cusparse.h" for "cusparse v2.h".
- 2. Exchange the function cusparseSetKernelStream() for cusparseSetStream().
- 3. Change the scalar parameters to be passed by reference, instead of by value (usually simply adding "&" symbol in C/C++ is enough, because the parameters are passed by reference on the host by default). However, note that if the routine is running asynchronously, then the variable holding the scalar parameter cannot be changed until the kernels that the routine dispatches are completed. In order to improve parallelism with streams, please refer to the sections 2.2 and 2.3 of this document. Also, see the NVIDIA CUDA C Programming Guide for a detailed discussion of how to use streams.
- 4. Change the 10th parameter to the function csr2csc from int 0 or 1 to enum CUSPARSE_ACTION_SYMBOLIC or CUSPARSE_ACTION_NUMERIC, respectively.

Finally, please use the function prototypes in the header files "cusparse.h" and "cusparse v2.h" to check the code for correctness.

Chapter 11

Appendix B: CUSPARSE Library C++ Example

For sample code reference please see the example code below. It shows an application written in C++ using the CUSPARSE library API. The code performs the following actions:

- 1. Creates a sparse test matrix in COO format.
- 2. Creates a sparse and dense vector.
- 3. Allocates GPU memory and copies the matrix and vectors into it.
- 4. Initializes the CUSPARSE library.
- 5. Creates and sets up the matrix descriptor.
- 6. Converts the matrix from COO to CSR format.
- 7. Exercises Level 1 routines.
- 8. Exercises Level 2 routines.
- 9. Exercises Level 3 routines.
- 10. Destroys the matrix descriptor.
- 11. Releases resources allocated for the CUSPARSE library.

```
//Example: Application using C++ and the CUSPARSE library
#include <stdio.h>
#include <stdlib.h>
#include <cuda_runtime.h>
#include "cusparse_v2.h"
#define CLEANUP(s)
    printf ("%s \n", s);
    if (yHostPtr)
                               free(yHostPtr);
                              free(zHostPtr);
    if (zHostPtr)
    if (xIndHostPtr) free(xIndHostPtr);
if (xValHostPtr) free(xValHostPtr);
    if (cooRowIndexHostPtr) free(cooRowIndexHostPtr);
    if (cooColIndexHostPtr) free(cooColIndexHostPtr);
    if (cooValHostPtr)
                            free(cooValHostPtr);
    if(y)
                             cudaFree(y);
```

```
cudaFree(z);
cudaFree(xInd);
cudaFree(xVal);
cudaFree(csrRowPtr);
cudaFree(cooRowIndex);
cudaFree(cooColIndex);
cudaFree(cooVal);
    if(z)
    if (xInd)
    if (xVal)
    if (csrRowPtr)
    if (cooRowIndex)
    if (cooColIndex)
    if (cooVal)
    if (descr)
                            {\tt cusparseDestroyMatDescr(descr);} \setminus
    if (handle)
                             cusparseDestroy(handle);
    fflush (stdout);
} while (0)
int main(){
    cudaError_t cudaStat1 , cudaStat2 , cudaStat3 , cudaStat4 , cudaStat5 , cudaStat6 ;
    cusparseStatus_t status;
    \verb|cusparseHandle_t| \verb|handle=0|;
    cusparseMatDescr_t descr=0;
    int * cooRowIndexHostPtr=0;
            \verb|cooColIndexHostPtr| = 0;
    int *
    double * cooValHostPtr=0;
    int * cooRowIndex=0;
int * cooColIndex=0;
    double * cooVal=0;
    int * xIndHostPtr=0;
    double * xValHostPtr=0;
    double * yHostPtr=0;
    int * xInd=0;
    double * xVal=0;
    double * y=0;
    int * csrRowPtr=0;
    double * zHostPtr=0;
    double * z=0;
    int n, nnz, nnz_vector;
    double dzero =0.0;
    double dtwo =2.0;
    double dthree=3.0;
    double dfive =5.0;
    printf("testing example \n");
    /* create the following sparse test matrix in COO format */
               2.0 3.0
    /* |1.0
        |5.0 	 6.0 	 7.0|
                  9.0 | */
           8.0
    n=4; nnz=9;
    cooRowIndexHostPtr = (int *) malloc(nnz*sizeof(cooRowIndexHostPtr[0]));
    cooColIndexHostPtr = (int *) malloc(nnz*sizeof(cooColIndexHostPtr[0]));
    = (double *) malloc(nnz*sizeof(cooValHostPtr[0]));
    if \ ((! {\tt cooRowIndexHostPtr}) \ || \ (! {\tt cooColIndexHostPtr}) \ || \ (! {\tt cooValHostPtr})) \{
        CLEANUP ("Host malloc failed (matrix)");
        return 1;
    \verb|cooRowIndexHostPtr|[0]=0; | \verb|cooColIndexHostPtr|[0]=0; | \verb|cooValHostPtr|[0]=1.0; |
    cooRowIndexHostPtr[1] = 0; cooColIndexHostPtr[1] = 2; cooValHostPtr[1] = 2.0;
    cooRowIndexHostPtr[2]=0; cooColIndexHostPtr[2]=3; cooValHostPtr[2]=3.0;
    \verb|cooRowIndexHostPtr|[3]=1; | \verb|cooColIndexHostPtr|[3]=1; | \verb|cooValHostPtr|[3]=4.0; | \\
    cooRowIndexHostPtr[4] = 2; cooColIndexHostPtr[4] = 0; cooValHostPtr[4] = 5.0;
    cooRowIndexHostPtr[6] = 2; cooColIndexHostPtr[6] = 3; cooValHostPtr[6] = 7.0;
    cooRowIndexHostPtr[7] = 3; cooColIndexHostPtr[7] = 1; cooValHostPtr[7] = 8.0;
    cooRowIndexHostPtr[8]=3; cooColIndexHostPtr[8]=3; cooValHostPtr[8]=9.0;
```

```
//print the matrix
printf("Input data:\n");
for (int i=0; i < nnz; i++){
     printf("cooRowIndexHostPtr[%d]=%d ",i,cooRowIndexHostPtr[i]);
     printf("cooColIndexHostPtr[%d]=%d ",i,cooColIndexHostPtr[i]);
     printf("cooValHostPtr[%d]=%f \n",i,cooValHostPtr[i]);
/* create a sparse and dense vector */
/* xVal= [100.0 200.0 400.0] (sparse)
xInd= [0 1 3]
    y = [10.0 \ 20.0 \ 30.0 \ 40.0 \ | \ 50.0 \ 60.0 \ 70.0 \ 80.0] \ (dense) */
nnz\_vector = 3;
xIndHostPtr = (int *) malloc(nnz_vector*sizeof(xIndHostPtr[0]));
xValHostPtr = (double *) malloc(nnz_vector*sizeof(xValHostPtr[0]));
\begin{array}{lll} {\tt yHostPtr} &= ({\tt double} \ *){\tt malloc}(2*{\tt n} & *{\tt sizeof}({\tt yHostPtr} \ [0])); \\ {\tt zHostPtr} &= ({\tt double} \ *){\tt malloc}(2*(n+1)) & *{\tt sizeof}({\tt zHostPtr} \ [0])); \\ \end{array}
if((!xIndHostPtr) || (!xValHostPtr) || (!yHostPtr) || (!zHostPtr)){}
     CLEANUP("Host malloc failed (vectors)");
     return 1;
yHostPtr[0] = 10.0; xIndHostPtr[0]=0; xValHostPtr[0]=100.0;
yHostPtr[1] = 20.0;
                          xIndHostPtr[1]=1; xValHostPtr[1]=200.0;
\begin{array}{lll} {\tt yHostPtr}\left[2\right] &= 30.0; \\ {\tt yHostPtr}\left[3\right] &= 40.0; & {\tt xIndHostPtr}\left[2\right] {\tt =} 3; & {\tt xValHostPtr}\left[2\right] {\tt =} 400.0; \end{array}
yHostPtr[4] = 50.0;
yHostPtr[5] = 60.0;
\mathtt{yHostPtr}\left[6\right]\ =\ 70.0;
yHostPtr[7] = 80.0;
//print the vectors
for (int j=0; j<2; j++){
     for (int i=0; i< n; i++){
           printf("yHostPtr[%d,%d]=%f n",i,j,yHostPtr[i+n*j]);
for (int i=0; i<nnz_vector; i++){
    printf("xIndHostPtr[%d]=%d ",i,xIndHostPtr[i]);
    printf("xValHostPtr[%d]=%f\n",i,xValHostPtr[i]);
/* allocate GPU memory and copy the matrix and vectors into it */
\verb| cudaStat1| = \verb| cudaMalloc| ((\verb| void**|) & \verb| cooRowIndex|, \verb| nnz*| & \verb| sizeof| (\verb| cooRowIndex| [0]) );
cudaStat2 = cudaMalloc((void**)&cooColIndex,nnz*sizeof(cooColIndex[0]));
\verb"cudaStat3" = \verb"cudaMalloc" ((\verb"void"**) \& \verb"cooVal", & \verb"nnz*sizeof" (\verb"cooVal" [0]"));
cudaStat4 = cudaMalloc((void**)&y,
                                                           2*n*sizeof(y[0]));
cudaStat5 = cudaMalloc((void**)&xInd,nnz_vector*sizeof(xInd[0]));
\mathtt{cudaStat6} = \mathtt{cudaMalloc}\left(\left(\underbrace{\mathtt{void}} **\right) \& \mathtt{xVal}, \mathtt{nnz\_vector} * \underbrace{\mathtt{sizeof}\left(\mathtt{xVal}\left[0\right]\right)}\right);
if ((cudaStat1 != cudaSuccess) |
     (cudaStat2 != cudaSuccess)
     (cudaStat3 != cudaSuccess)
     (cudaStat4 != cudaSuccess)
     (cudaStat5 != cudaSuccess)
     (cudaStat6 != cudaSuccess))
     CLEANUP ("Device malloc failed");
     return 1;
cudaStat1 = cudaMemcpy(cooRowIndex, cooRowIndexHostPtr,
                               (size_t)(nnz*sizeof(cooRowIndex[0])),
                               cudaMemcpyHostToDevice);
```

```
cudaStat2 = cudaMemcpy(cooColIndex, cooColIndexHostPtr;
                       (size_t)(nnz*sizeof(cooColIndex[0])),
                       cudaMemcpyHostToDevice);
cudaStat3 = cudaMemcpy(cooVal,
                                   cooValHostPtr
                       (size_t)(nnz*sizeof(cooVal[0])),
                       cudaMemcpyHostToDevice);
cudaStat4 = cudaMemcpy(y,
                                    yHostPtr
                       (size_t)(2*n*sizeof(y[0])),
                       cudaMemcpyHostToDevice);
cudaStat5 = cudaMemcpy(xInd,
                                   xIndHostPtr,
                       (size_t)(nnz_vector*sizeof(xInd[0])),
                       {\tt cudaMemcpyHostToDevice});\\
cudaStat6 = cudaMemcpy(xVal,
                                   xValHostPtr,
                       (size_t)(nnz_vector*sizeof(xVal[0])),
                       cudaMemcpyHostToDevice);
if ((cudaStat1 != cudaSuccess)
    (cudaStat2 != cudaSuccess)
    (cudaStat3 != cudaSuccess)
    (cudaStat4 != cudaSuccess)
    (cudaStat5 != cudaSuccess)
    (cudaStat6 != cudaSuccess)) {
    CLEANUP ("Memcpy from Host to Device failed");
    return 1;
/* initialize cusparse library */
status= cusparseCreate(&handle);
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP("CUSPARSE Library initialization failed");
    return 1:
/* create and setup matrix descriptor */
status= cusparseCreateMatDescr(&descr);
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP ("Matrix descriptor initialization failed");
   return 1;
cusparseSetMatType(descr, CUSPARSE_MATRIX_TYPE_GENERAL);
cusparseSetMatIndexBase(descr, CUSPARSE_INDEX_BASE_ZERO);
/* exercise conversion routines (convert matrix from COO 2 CSR format) */
cudaStat1 = cudaMalloc((void**)\&csrRowPtr,(n+1)*sizeof(csrRowPtr[0]));
if (cudaStat1 != cudaSuccess) {
    CLEANUP("Device malloc failed (csrRowPtr)");
    return 1:
status= cusparseXcoo2csr(handle,cooRowIndex,nnz,n,
                         csrRowPtr , CUSPARSE_INDEX_BASE_ZERO );
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP ("Conversion from COO to CSR format failed");
    return 1;
// csrRowPtr = [0 \ 3 \ 4 \ 7 \ 9]
/* exercise Level 1 routines (scatter vector elements) */
status = cusparseDsctr(handle, nnz_vector, xVal, xInd,
                      &y[n], CUSPARSE_INDEX_BASE_ZERO);
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP("Scatter from sparse to dense vector failed");
    return 1;
```

```
//y = [10 \ 20 \ 30 \ 40 \ | \ 100 \ 200 \ 70 \ 400]
/* exercise Level 2 routines (csrmv) */
\verb|status| = \verb|cusparseDcsrmv| (\verb|handle|, CUSPARSE_OPERATION_NON_TRANSPOSE|, |n|, |n|, |n|z|, \\
                           \& \mathtt{dtwo} \;,\;\; \mathtt{descr} \;,\;\; \mathtt{cooVal} \;,\;\; \mathtt{csrRowPtr} \;,\;\; \mathtt{cooColIndex} \;,
                           &y[0], &dthree, &y[n]);
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP("Matrix-vector multiplication failed");
     return 1;
//y = [10 \ 20 \ 30 \ 40 \ | \ 680 \ 760 \ 1230 \ 2240]
\verb"cudaMemcpy" (y HostPtr", y, (size_t)(2*n*sizeof(y[0])), cudaMemcpyDeviceToHost);
printf("Intermediate results:\n");
for (int j=0; j<2; j++){
for (int i=0; i<n; i++){
          printf("yHostPtr[%d,%d]=%f\n",i,j,yHostPtr[i+n*j]);
*/
/* exercise Level 3 routines (csrmm) */
cudaStat1 = cudaMalloc((void**)&z, 2*(n+1)*sizeof(z[0]));
if (cudaStat1 != cudaSuccess) {
     CLEANUP ("Device malloc failed (z)");
     return 1;
cudaStat1 = cudaMemset((void *)z,0, 2*(n+1)*sizeof(z[0]));
if (cudaStat1 != cudaSuccess) {
     CLEANUP ("Memset on Device failed");
     return 1;
status= cusparseDcsrmm(handle, CUSPARSE_OPERATION_NON_TRANSPOSE, n, 2, n,
                            \verb"nnz"\,,\ \& \verb"dfive"\,,\ \verb"descr"\,,\ \verb"cooVal"\,,\ \verb"csrRowPtr"\,,\ \verb"cooColIndex"\,,
                            y, n, \&dzero, z, n+1);
if (status != CUSPARSE_STATUS_SUCCESS) {
    CLEANUP ("Matrix-matrix multiplication failed");
     return 1;
/* print final results (z) */
{\tt cudaStat1} \, = \, {\tt cudaMemcpy} \, (\, {\tt zHostPtr} \, , \, \, {\tt z} \, , \, \,
                            (size_t)(2*(n+1)*sizeof(z[0])),
                            {\tt cudaMemcpyDeviceToHost})\;;
if (cudaStat1 != cudaSuccess) {
     CLEANUP ("Memcpy from Device to Host failed");
//\mathrm{z} = [950\ 400\ 2550\ 2600\ 0\ |\ 49300\ 15200\ 132300\ 131200\ 0]
printf("Final results:\n");
for (int j=0; j<2; j++){}
    for (int i=0; i< n+1; i++){}
         printf("z[\%d,\%d]=\%f \ n", i, j, zHostPtr[i+(n+1)*j]);
/* destroy matrix descriptor */
status = cusparseDestroyMatDescr(descr);
descr = 0;
if (status != CUSPARSE_STATUS_SUCCESS) {
```

```
CLEANUP("Matrix descriptor destruction failed");
     return 1;
/* destroy handle */
status = cusparseDestroy(handle);
handle = 0;
if (status != CUSPARSE_STATUS_SUCCESS) {
     CLEANUP ("CUSPARSE Library release of resources failed");
     return 1;
/* check the results */
/* Notice that CLEANUP() contains a call to cusparseDestroy(handle) */
if ((zHostPtr[0] != 950.0)
(zHostPtr[1] != 400.0)
(zHostPtr[2] != 2550.0)
(zHostPtr[3] != 2600.0)
     (zHostPtr[7] != 132300.0)
     (zHostPtr[8] != 131200.0)
     (zHostPtr[9] != 0.0)
(yHostPtr[0] != 10.0)
(yHostPtr[1] != 20.0)
     (yHostPtr[2] != 30.0)
     (yHostPtr[3] != 40.0)
     (yHostPtr[4] != 680.0)
(yHostPtr[5] != 760.0)
(yHostPtr[6] != 1230.0)
(yHostPtr[7] != 2240.0)){
     CLEANUP ("example test FAILED");
     return 1;
else{
     CLEANUP ("example test PASSED");
     return 0;
}
```

Chapter 12

Appendix C: CUSPARSE Fortran Bindings

The CUSPARSE library is implemented using the C-based CUDA toolchain, and it thus provides a C-style API that makes interfacing to applications written in C or C++ trivial. There are also many applications implemented in Fortran that would benefit from using CUSPARSE, and therefore a CUSPARSE Fortran interface has been developed.

Unfortunately, Fortran-to-C calling conventions are not standardized and differ by platform and toolchain. In particular, differences may exist in the following areas:

- ▶ Symbol names (capitalization, name decoration)
- ► Argument passing (by value or reference)
- ▶ Passing of pointer arguments (size of the pointer)

To provide maximum flexibility in addressing those differences, the CUSPARSE Fortran interface is provided in the form of wrapper functions, which are written in C and are located in the file cusparse_fortran.c. This file also contains a few additional wrapper functions (for cudaMalloc(), cudaMemset(), and so on) that can be used to allocate memory on the GPU.

The CUSPARSE Fortran wrapper code is provided as an example only and needs to be compiled into an application for it to call the CUSPARSE API functions. Providing this source code allows users to make any changes necessary for a particular platform and toolchain.

The CUSPARSE Fortran wrapper code has been used to demonstrate interoperability with the compilers g95 0.91 (on 32-bit and 64-bit Linux) and g95 0.92 (on 32-bit and 64-bit Mac OS X). In order to use other compilers, users have to make any changes to the wrapper code that may be required.

The direct wrappers, intended for production code, substitute device pointers for vector and matrix arguments in all CUSPARSE functions. To use these interfaces, existing applications need to be modified slightly to allocate and deallocate data structures in GPU memory space (using CUDA_MALLOC() and CUDA_FREE()) and to copy data between GPU and CPU memory spaces (using the CUDA_MEMCPY routines). The sample wrappers provided in cusparse_fortran.c map device pointers to the OS-dependent type size_t, which is 32 bits wide on 32-bit platforms and 64 bits wide on a 64-bit platforms.

One approach to dealing with index arithmetic on device pointers in Fortran code is to use C-style macros and to use the C preprocessor to expand them. On Linux and Mac OS X, preprocessing can be done by using the option '-cpp' with g95 or gfortran. The function GET_SHIFTED_ADDRESS(), provided with the CUSPARSE Fortran wrappers, can also be used, as shown in example B.

Example B shows the the C++ of example A implemented in Fortran 77 on the host. This example should be compiled with ARCH_64 defined as 1 on a 64-bit OS system and as undefined on a 32-bit OS system. For example, on g95 or gfortran, it can be done directly on the command line using the option '-cpp -DARCH_64=1'.

Example B, Fortran Application

```
#define ARCH_64 0
     #define ARCH_64 1
      program cusparse_fortran_example
      implicit none
      integer cuda_malloc
      external cuda_free
      integer cuda_memcpy_c2fort_int
      integer cuda_memcpy_c2fort_real
      integer cuda_memcpy_fort2c_int
      integer cuda_memcpy_fort2c_real
      integer cuda_memset
      integer cusparse_create
      external cusparse_destroy
      integer cusparse_get_version
      integer cusparse_create_mat_descr
      external cusparse_destroy_mat_descr
      integer cusparse_set_mat_type
      integer cusparse_get_mat_type
      integer cusparse_get_mat_fill_mode
      integer cusparse_get_mat_diag_type
      integer cusparse_set_mat_index_base
      integer cusparse_get_mat_index_base
      integer cusparse_xcoo2csr
      integer cusparse_dsctr
      integer cusparse_dcsrmv
      integer cusparse_dcsrmm
      external get_shifted_address
#if ARCH_64
      integer *8 handle
      integer *8 descrA
      integer *8 cooRowIndex
      integer *8 cooColIndex
      integer *8 cooVal
      integer *8 xInd
      integer *8 xVal
      integer *8 y
      integer *8 z
      integer *8 csrRowPtr
      integer *8 ynp1
#else
      integer *4 handle
```

```
integer *4 descrA
      integer *4 cooRowIndex
      integer *4 cooColIndex
      integer *4 cooVal
      integer*4 xInd
      integer *4 xVal
      integer *4 y
      integer *4 z
      integer*4 csrRowPtr
      integer *4 ynp1
#endif
      integer status
      integer cudaStat1, cudaStat2, cudaStat3
      {\tt integer} \ {\tt cudaStat4} \ , {\tt cudaStat5} \ , {\tt cudaStat6}
      {\tt integer} \ {\tt n} \, , \ {\tt nnz} \, , \ {\tt nnz\_vector}
      parameter (n=4, nnz=9, nnz_vector=3)
      integer cooRowIndexHostPtr(nnz)
      integer cooColIndexHostPtr(nnz)
      real *8 cooValHostPtr(nnz)
      integer xIndHostPtr(nnz_vector)
      real *8 xValHostPtr(nnz_vector)
      real *8 yHostPtr(2*n)
      real*8 zHostPtr(2*(n+1))
      integer i, j
      integer version, mtype, fmode, dtype, ibase
      real *8 dzero, dtwo, dthree, dfive
      real *8 epsilon
      write(*,*) "testing fortran example"
      predefined constants (need to be careful with them)
      dzero = 0.0
      \mathtt{dtwo} \ = \ 2.0
      \mathtt{dthree} = 3.0
      {\tt dfive}\,=\,5.0
      create the following sparse test matrix in {\tt COO} format
С
С
      (notice one-based indexing)
      1.0 2.0 3.0
С
          4.0
С
С
      5.0
              6.0 7.0
      8.0 9.0
С
      cooRowIndexHostPtr(1)=1
      cooColIndexHostPtr(1)=1
      cooValHostPtr(1)
                            =1.0
      {\tt cooRowIndexHostPtr\,(2)}{=}1
      cooColIndexHostPtr(2)=3
      cooValHostPtr(2)
      cooRowIndexHostPtr(3)=1
      cooColIndexHostPtr(3)=4
      cooValHostPtr(3) = 3.0
      cooRowIndexHostPtr(4)=2
      {\tt cooColIndexHostPtr}(4) = 2
      cooValHostPtr(4)
                            =4.0
      cooRowIndexHostPtr(5)=3
      {\tt cooColIndexHostPtr}(5){=}1
      cooValHostPtr(5)
                            =5.0
      cooRowIndexHostPtr(6)=3
      cooColIndexHostPtr(6)=3
      cooValHostPtr(6) = 6.0
      cooRowIndexHostPtr(7)=3
      cooColIndexHostPtr(7)=4
      cooValHostPtr(7) = 7.0
```

```
cooRowIndexHostPtr(8)=4
        cooColIndexHostPtr(8)=2
        cooValHostPtr(8) = 8.0
        cooRowIndexHostPtr(9)=4
       cooColIndexHostPtr(9)=4
       cooValHostPtr(9)
С
        print the matrix
        write(*,*) "Input data:"
       do i=1,nnz
           write(*,*) "cooRowIndexHostPtr[",i,"]=",cooRowIndexHostPtr(i)
write(*,*) "cooColIndexHostPtr[",i,"]=",cooColIndexHostPtr(i)
write(*,*) "cooValHostPtr[", i,"]=",cooValHostPtr(i)
       create a sparse and dense vector
С
       xVal = [100.0 200.0 400.0]
                                           (sparse)
       xInd = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix}
C
       y = [10.0 \ 20.0 \ 30.0 \ 40.0 \ | \ 50.0 \ 60.0 \ 70.0 \ 80.0] (dense)
       (notice one-based indexing)
       yHostPtr(1) = 10.0
        yHostPtr(2) = 20.0
       yHostPtr(3) = 30.0
       \mathtt{yHostPtr}(4) = 40.0
       \mathtt{yHostPtr}(5) = 50.0
        yHostPtr(6) = 60.0
        \mathtt{yHostPtr}(7) = 70.0
        yHostPtr(8) = 80.0
        xIndHostPtr(1)=1
        \mathtt{xValHostPtr}\,(1)\!=\!100.0
        xIndHostPtr(2)=2
        \mathtt{xValHostPtr}\,(2)\!=\!200.0
       xIndHostPtr(3)=4
       xValHostPtr(3) = 400.0
       print the vectors
С
       do j=1,2
           do i=1.n
                write (*,*) "yHostPtr[",i,",",j,"]=",yHostPtr(i+n*(j-1))
       enddo
        do i=1,nnz\_vector
           write(*,*) "xIndHostPtr[",i,"]=",xIndHostPtr(i)
write(*,*) "xValHostPtr[",i,"]=",xValHostPtr(i)
       allocate GPU memory and copy the matrix and vectors into it
       cudaSuccess=0
C.
       {\tt cudaMemcpyHostToDevice}{=}1
       \verb"cudaStat1" = \verb"cuda_malloc" (\verb"cooRowIndex", \verb"nnz*4")
        cudaStat2 = cuda_malloc(cooColIndex,nnz*4)
        \verb"cudaStat3" = \verb"cuda_malloc" (\verb"cooVal", mnz*8)
        {\tt cudaStat4} = {\tt cuda\_malloc(y,}
                                                        2*n*8)
        cudaStat5 = cuda_malloc(xInd,nnz_vector*4)
        cudaStat6 = cuda_malloc(xVal,nnz_vector*8)
       egin{array}{lll} 	ext{if} & 	ext{((cudaStat1 } /= 0) & .OR. \ 	ext{S} & 	ext{(cudaStat2 } /= 0) & .OR. \end{array}
             (cudaStat3 /= 0) .OR.
            (cudaStat4 /= 0) .OR.
            (\texttt{cudaStat5} \ /= \ 0) .OR.
      $
            (cudaStat6 /= 0)) then
            write(*,*) "Device malloc failed"
write(*,*) "cudaStat1=",cudaStat1
            write(*,*) "cudaStat2=",cudaStat2
```

```
write(*,*) "cudaStat3=",cudaStat3
          write(*,*) "cudaStat4=",cudaStat4
write(*,*) "cudaStat5=",cudaStat5
write(*,*) "cudaStat6=",cudaStat6
         stop
      endif
      cudaStat1 = cuda_memcpy_fort2c_int(cooRowIndex,cooRowIndexHostPtr,
                                              nnz*4,1)
      cudaStat2 = cuda_memcpy_fort2c_int(cooColIndex,cooColIndexHostPtr,
                                              nnz*4.1)
      cudaStat3 = cuda_memcpy_fort2c_real(cooVal,
                                                            cooValHostPtr ,
                                               nnz*8,1)
                                                        yHostPtr,
      cudaStat4 = cuda_memcpy_fort2c_real(y,
                                               2*n*8,1)
     $
      cudaStat5 = cuda_memcpy_fort2c_int(xInd,
                                                           xIndHostPtr.
                                              nnz_vector*4,1)
      nnz_vector*8,1)
      if ((cudaStat1 /= 0) .OR.
           (\mathtt{cudaStat2} \ /= \ 0) .OR.
     $
           (cudaStat3 /= 0) .OR.
           (cudaStat4 /= 0) .OR.
     $
          (cudaStat5 /= 0) .OR.
          (cudaStat6 /= 0)) then
     $
         write(*,*) "Memcpy from Host to Device failed"
write(*,*) "cudaStat1=",cudaStat1
write(*,*) "cudaStat2=",cudaStat2
          write (*,*) "cudaStat3=", cudaStat3
         write(*,*) "cudaStat4=",cudaStat4
write(*,*) "cudaStat5=",cudaStat5
write(*,*) "cudaStat6=",cudaStat6
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
         stop
      endif
      initialize cusparse library
      CUSPARSE_STATUS_SUCCESS=0
      status = cusparse_create(handle)
      if (status /= 0) then
          write (*,*) "CUSPARSE Library initialization failed"
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
         stop
      endif
      get version
С
      {\tt CUSPARSE\_STATUS\_SUCCESS}{=}0
      status = cusparse_get_version(handle, version)
      if (status \neq 0) then
          write(*,*) "CUSPARSE Library initialization failed"
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
```

```
call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy(handle)
         stop
      endif
      write(*,*) "CUSPARSE Library version", version
С
      create and setup the matrix descriptor
      {\tt CUSPARSE\_STATUS\_SUCCESS}{=}0
      CUSPARSE_MATRIX_TYPE_GENERAL=0
С
      {\tt CUSPARSE\_INDEX\_BASE\_ONE}{=}1
      status= cusparse_create_mat_descr(descrA)
      if (status /= 0) then
         write (*,*) "Creating matrix descriptor failed"
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy(handle)
         stop
      endif
      status = cusparse_set_mat_type(descrA, 0)
      status = cusparse_set_mat_index_base(descrA,1)
      print the matrix descriptor
      {\tt mtype} \; = \; {\tt cusparse\_get\_mat\_type} \, (\, {\tt descrA} \, )
      fmode = cusparse_get_mat_fill_mode(descrA)
      dtype = cusparse_get_mat_diag_type(descrA)
      ibase = cusparse_get_mat_index_base(descrA)
      write (*,*) "matrix descriptor
      write (*,*) "t=",mtype, "m=",fmode, "d=",dtype, "b=",ibase
      exercise conversion routines (convert matrix from COO 2 CSR format)
С
      \verb"cudaSuccess=0"
С
      {\tt CUSPARSE\_STATUS\_SUCCESS}{=}0
С
С
      CUSPARSE_INDEX_BASE_ONE=1
      cudaStat1 = cuda_malloc(csrRowPtr,(n+1)*4)
      if (cudaStat1 /= 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
         write(*,*) "Device malloc failed (csrRowPtr)"
         stop
      endif
      status= cusparse_xcoo2csr(handle,cooRowIndex,nnz,n,
                                  csrRowPtr, 1)
      if (status \neq 0) then
         call cuda_free(cooRowIndex)
         call cuda_free(cooColIndex)
         call cuda_free(cooVal)
         call cuda_free(xInd)
         call cuda_free(xVal)
         call cuda_free(y)
         call cuda_free(csrRowPtr)
         call cusparse_destroy_mat_descr(descrA)
         call cusparse_destroy(handle)
```

```
write (*,*) "Conversion from COO to CSR format failed"
          stop
      endif
С
      csrRowPtr = [0 \ 3 \ 4 \ 7 \ 9]
С
      exercise Level 1 routines (scatter vector elements)
      {\tt CUSPARSE\_STATUS\_SUCCESS}{=}0
С
      {\tt CUSPARSE\_INDEX\_BASE\_ONE}{=}1
С
      call get_shifted_address(y,n*8,ynp1)
      \begin{array}{lll} status = & cusparse\_dsctr\left( \texttt{handle} \;,\; \texttt{nnz\_vector} \;,\; \texttt{xVal} \;,\; \texttt{xInd} \;, \end{array}
                                 ynp1, 1)
      if (status /= 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(csrRowPtr)
          call cusparse_destroy_mat_descr(descrA)
          call cusparse_destroy(handle)
          write(*,*) "Scatter from sparse to dense vector failed"
          stop
      endif
С
      y = [10 \ 20 \ 30 \ 40 \ | \ 100 \ 200 \ 70 \ 400]
      exercise Level 2 routines (csrmv)
С
      {\tt CUSPARSE\_STATUS\_SUCCESS}{=}0
      {\tt CUSPARSE\_OPERATION\_NON\_TRANSPOSE}{=}0
С
      status= cusparse_dcsrmv(handle, 0, n, n, nnz, dtwo,
                                descrA , cooVal , csrRowPtr , cooColIndex ,
                                 y, dthree, ynp1)
      if (status \neq 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(csrRowPtr)
          {\tt call cusparse\_destroy\_mat\_descr(descrA)}
          call cusparse_destroy(handle)
          write(*,*) "Matrix-vector multiplication failed"
          stop
      endif
      print intermediate results (y)
С
      y = [10 \ 20 \ 30 \ 40 \ | \ 680 \ 760 \ 1230 \ 2240]
      cudaSuccess=0
С
       cudaMemcpyDeviceToHost=2
       cudaStat1 = cuda_memcpy_c2fort_real(yHostPtr, y, 2*n*8, 2)
       if (cudaStat1 \neq 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(csrRowPtr)
          call cusparse_destroy_mat_descr(descrA)
          call cusparse_destroy(handle)
          write(*,*) "Memcpy from Device to Host failed"
```

```
stop
       endif
       write(*,*) "Intermediate results:"
      do j=1,2
          do i=1,n
              write(*,*) "yHostPtr[",i,",",j,"]=",yHostPtr(i+n*(j-1))
          enddo
      enddo
      exercise Level 3 routines (csrmm)
С
      {\tt cudaSuccess}{=}0
С
      CUSPARSE_STATUS_SUCCESS=0
С
      {\tt CUSPARSE\_OPERATION\_NON\_TRANSPOSE}{=}0
С
       cudaStat1 = cuda_malloc(z, 2*(n+1)*8)
      if (cudaStat1 /= 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(csrRowPtr)
          call cusparse_destroy_mat_descr(descrA)
          {\tt call} \;\; {\tt cusparse\_destroy} \, (\, {\tt handle} \, )
          write(*,*) "Device malloc failed (z)"
          stop
      endif
       cudaStat1 = cuda\_memset(z, 0, 2*(n+1)*8)
      if (cudaStat1 /= 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(z)
          call cuda_free(csrRowPtr)
          call cusparse_destroy_mat_descr(descrA)
          {\tt call} \;\; {\tt cusparse\_destroy} \, (\, {\tt handle} \, )
          write (*,*) "Memset on Device failed"
          stop
      endif
      status = cusparse\_dcsrmm(handle, 0, n, 2, n, nnz, dfive,
                                 {\tt descrA}\;,\;\;{\tt cooVal}\;,\;\;{\tt csrRowPtr}\;,\;\;{\tt cooColIndex}\;,
                                 y, n, dzero, z, n+1)
      if (status \neq 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(z)
          call cuda_free(csrRowPtr)
          call cusparse_destroy_mat_descr(descrA)
          call cusparse_destroy(handle)
          write(*,*) "Matrix-matrix multiplication failed"
          stop
      endif
С
       print final results (z)
       \verb"cudaSuccess"=0
```

```
\verb"cudaMemcpyDeviceToHost"=2"
       cudaStat1 = cuda_memcpy_c2fort_real(zHostPtr, z, 2*(n+1)*8, 2)
       if (cudaStat1 /= 0) then
          call cuda_free(cooRowIndex)
          call cuda_free(cooColIndex)
          call cuda_free(cooVal)
          call cuda_free(xInd)
          call cuda_free(xVal)
          call cuda_free(y)
          call cuda_free(z)
          call cuda_free(csrRowPtr)
          {\tt call cusparse\_destroy\_mat\_descr(descrA)}
          call cusparse_destroy(handle)
          write(*,*) "Memcpy from Device to Host failed"
          stop
       endif
c
      z = [950 \ 400 \ 2550 \ 2600 \ 0 \ | \ 49300 \ 15200 \ 132300 \ 131200 \ 0]
       write(*,*) "Final results:"
      do j=1,2
          do i=1,n+1
              write (*,*) "z [",i,",",j,"]=",zHostPtr(i+(n+1)*(j-1))
      enddo
      check the results
       \mathtt{epsilon} \, = \, 0.00000000000001
      if ((DABS(zHostPtr(1) - 950.0)) .GT. epsilon) .OR.
           (DABS(zHostPtr(2) - 400.0) .GT. epsilon) .OR.
           \begin{array}{cccc} (\texttt{DABS}(\texttt{zHostPtr}(3) - 2550.0) & .\texttt{GT. epsilon}) \\ (\texttt{DABS}(\texttt{zHostPtr}(4) - 2600.0) & .\texttt{GT. epsilon}) \end{array}
                                                                .OR.
                                                                .OR.
                                              .GT. epsilon)
           (DABS(zHostPtr(5) - 0.0)
     $
                                                                . OR .
           (DABS(zHostPtr(6) - 49300.0) .GT. epsilon) .OR.
     $
           (DABS(zHostPtr(7) - 15200.0) .GT. epsilon) .OR.
     $
     $
          (\mathtt{DABS}(\mathtt{zHostPtr}(8) - 132300.0).\mathtt{GT}.\mathtt{epsilon})
                                                                . OR .
           $
                                                                . OR. .
     $
                                                                . OR .
                                                                . OR .
           (DABS(yHostPtr(2) - 20.0)
                                                               OR.
     $
                                           . \, \mathtt{GT} \, . \, \, \, \mathtt{epsilon} \, )
           (DABS(yHostPtr(3) - 30.0)
                                           .GT. epsilon)
                                                                . OR .
     $
           (DABS(yHostPtr(4) - 40.0)
     $
                                                                . OR .
           (DABS(yHostPtr(5) - 680.0) .GT. epsilon)
     $
                                                                OR.
          (\mathtt{DABS}(\mathtt{yHostPtr}(6) - 760.0) .GT. epsilon) .OR.
     $
           (\mathtt{DABS}(\mathtt{yHostPtr}(7) - 1230.0) .GT. epsilon)
     $
                                                                .OR.
           (DABS(yHostPtr(8) - 2240.0) .GT. epsilon)) then write(*,*) "fortran example test FAILED"
        else
            write(*,*) "fortran example test PASSED"
        endif
        deallocate GPU memory and exit
        call cuda_free(cooRowIndex)
        call cuda_free(cooColIndex)
        call cuda_free(cooVal)
        call cuda_free(xInd)
        call cuda_free(xVal)
        call cuda_free(y)
        call cuda_free(z)
        call cuda_free(csrRowPtr)
        call cusparse_destroy_mat_descr(descrA)
        call cusparse_destroy(handle)
        stop
```

end

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