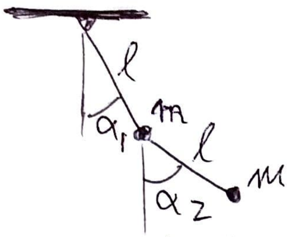
ESIQ4 (I)



USE SMALL ANGLES, IE

$\sin(\alpha_1) = \alpha_1$

$\sin(\alpha_2) = \alpha_2$

$|\alpha_1|, |\alpha_2| \leq 1$

$$T = \frac{1}{2} m (l\dot{\alpha}_1)^2 + \frac{1}{2} m (l\dot{\alpha}_1 + l\dot{\alpha}_2)^2$$

$$= \frac{1}{2} m l^2 (\dot{\alpha}_1^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2 + 2\dot{\alpha}_1\dot{\alpha}_2)$$

$$= \frac{1}{2} m l^2 (2\dot{\alpha}_1^2 + \dot{\alpha}_2^2 + 2\dot{\alpha}_1\dot{\alpha}_2)$$

$$V = -mgl(\cos\alpha_1 + (\cos\alpha_1 + \cos\alpha_2))$$

$$= -mgl\left(\left(1 - \frac{\alpha_1^2}{2}\right)\cdot 2 + 1 - \frac{\alpha_2^2}{2}\right) = -mgl\left(2 - \alpha_1^2 - \frac{\alpha_2^2}{2}\right)$$

REDEFINE V FOR CONVENIENCE:

$$V = mgl\left(\alpha_1^2 + \frac{\alpha_2^2}{2}\right)$$

$$T = KE = \frac{1}{2} \underline{\alpha}^T \underline{\underline{M}} \underline{\alpha} \Rightarrow \underline{\underline{M}} = ml^2 \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$V = PE = \frac{1}{2} \underline{\alpha}^T \underline{\underline{K}} \underline{\alpha} \Rightarrow \underline{\underline{K}} = mgl \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\underline{\underline{M}} \ddot{\underline{\alpha}} + \underline{\underline{K}} \underline{\alpha} = 0 \quad \text{WHERE } \underline{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} e^{i\omega t}$$

$$(-\omega^2 \underline{\underline{M}} + \underline{\underline{K}}) \underline{\alpha} = 0$$

$$\det(\underline{\underline{K}} - \omega^2 \underline{\underline{M}}) = 0$$

TO MAKE AGEBRA SIMPLER, TEMPORARILY SET $m = l = g = 1$

$$|\underline{\underline{K}} - \omega^2 \underline{\underline{M}}| = \begin{vmatrix} 2 - 2\omega^2 & -\omega^2 \\ -\omega^2 & 1 - \omega^2 \end{vmatrix} = 2(1-\omega^2)^2 - \omega^4 = 0$$

$$= 2(1 - 2\omega^2 + \omega^4) - \omega^4 = 0$$

$$= \omega^4 - 4\omega^2 + 2 = 0$$

$$\omega_{1,2}^2 = \frac{4 \pm \sqrt{16 - 4\cdot 1 \cdot 2}}{2} = 2 \pm \frac{\sqrt{8}}{2} = 2 \pm \sqrt{2}$$

ESIQ4(IV)

IF $\omega^2 = 2 + \sqrt{2}$

$$(-\omega^2 \underline{\underline{M}} + \underline{\underline{K}})\underline{\alpha} = 0$$

$$\begin{pmatrix} 2 - 2(2+\sqrt{2}) & -(2+\sqrt{2}) \\ -(2+\sqrt{2}) & 1-(2+\sqrt{2}) \end{pmatrix}\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix} -2-2\sqrt{2} & -(2+\sqrt{2}) \\ -(2+\sqrt{2}) & -1-\sqrt{2} \end{pmatrix}\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$(-2-2\sqrt{2})\alpha_1 - (2+\sqrt{2})\alpha_2 = 0$$

$$(-2-2\sqrt{2})\alpha_1 = (2+\sqrt{2})\alpha_2$$

$$\frac{\alpha_2}{\alpha_1} = \frac{-2-2\sqrt{2}}{2+\sqrt{2}} = -\sqrt{2}$$

↳ AFTER SOME ALGEBRA

IF $\omega^2 = 2-\sqrt{2}$

$$(-\omega^2 \underline{\underline{M}} + \underline{\underline{K}})\underline{\alpha} = 0$$

$$\begin{pmatrix} 2 - 2(2-\sqrt{2}) & -(2-\sqrt{2}) \\ -(2-\sqrt{2}) & 1-(2-\sqrt{2}) \end{pmatrix}\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\begin{pmatrix} -2+2\sqrt{2} & -(2-\sqrt{2}) \\ -(2-\sqrt{2}) & -1+\sqrt{2} \end{pmatrix}\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$(-2+2\sqrt{2})\alpha_1 - (2-\sqrt{2})\alpha_2 = 0$$

$$\frac{\alpha_2}{\alpha_1} = \frac{-2+2\sqrt{2}}{+(2-\sqrt{2})} = \sqrt{2}$$

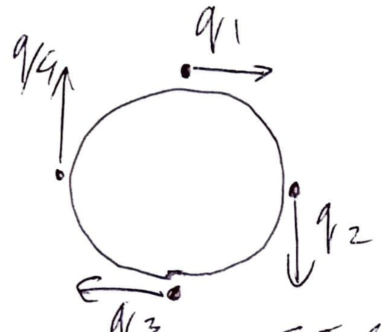
TWO NORMAL MODES & THEIR FREQUENCIES:

$$\underline{\alpha}^{(1)} = \begin{pmatrix}1\\-\sqrt{2}\end{pmatrix} \qquad \underline{\alpha}^{(2)} = \begin{pmatrix}1\\\sqrt{2}\end{pmatrix}$$

$$\omega^{(1)} = \sqrt{2+\sqrt{2}}\sqrt{\frac{g}{\ell}} \qquad \omega^{(2)} = \sqrt{2-\sqrt{2}}\sqrt{\frac{g}{\ell}}$$

ANSWER

$\boxed{\text{ESIQ5}}$ [LETS TRY DOING IT BY INTUITION, NOT LAGRANGIAN]



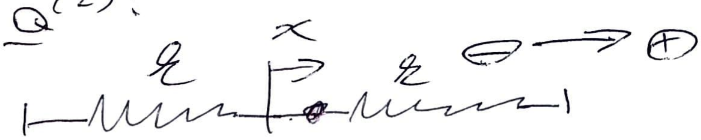

we need:
- rotation
- sth like this: 
- sth like this: 
- sth orthogonal to previous.

SO we have:

$$\underline{Q}^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad \underline{Q}^{(2)} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \qquad \underline{Q}^{(3)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \qquad \underline{Q}^{(4)} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

$\omega_1^2 = 0$ (ROTATION)

INSPECT $\underline{Q}^{(2)}$:




$$-2kx = m\ddot{x}$$

$$\ddot{x} + \frac{2k}{m}x = 0$$

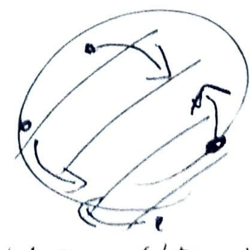
$$\omega^2 = \frac{2k}{m} \Rightarrow \omega_2 = \sqrt{\frac{2k}{m}}$$

& BY SYMMETRY: $\omega_3 = \sqrt{\frac{2k}{m}}$

$\omega_n^2 \propto$ MAX KE STORED = MAX PE STORED $\propto \sum$(DISPLACEMENT OF EACH MASS)$^2 \propto \underline{Q}^{(n)} \cdot \underline{Q}^{(n)} \Rightarrow \omega_n^2 \propto \underline{Q}^{(n)} \cdot \underline{Q}^{(n)}$

$$\omega_4^2 = \frac{\underline{Q}^{(4)} \cdot \underline{Q}^{(4)}}{\underline{Q}^{(2)} \cdot \underline{Q}^{(2)}} \omega_2^2 = \frac{4}{2} \frac{2k}{m} \Rightarrow 2\sqrt{\frac{k}{m}} = \omega_4$$

The $4^{TH}$ mode behaves like this:



[BAD DRAWING BUT I HOPE IT SHOWS THE POINT]

ESI Q6

$$T_{ij}\ddot{q}_j + V_{ij}q_j = 0 \quad T_{ij}=T_{ji};\ V_{ij}=V_{ji} \Rightarrow (T_{ij}\ddot{q}_i + V_{ij}q_i)\dot{q}_j + + (T_{ij}\ddot{q}_j + V_{ij}q_j)\dot{q}_i = 0$$

REGROUP/MULTIPLY BY $\frac{1}{2}$:

$$\Rightarrow \frac{1}{2}T_{ij}(\ddot{q}_i\dot{q}_j + \dot{q}_i\ddot{q}_j) + \frac{1}{2}V_{ij}(q_i\dot{q}_j + \dot{q}_i q_j) = 0$$

REEXPRESS:

$$\frac{\partial}{\partial T}\left(\frac{1}{2}T_{ij}\dot{q}_i\dot{q}_j + \frac{1}{2}V_{ij}q_i q_j\right) = 0$$

CONCLUDE:

$$\frac{1}{2}T_{ij}\dot{q}_i\dot{q}_j + \frac{1}{2}V_{ij}q_i q_j = C$$

ie

$$T + V = C$$ AS REQUIRED.

$C = T + V$ $\left(\frac{d}{dt}\right.$

$$0 = \frac{d}{dt}(T+V) = \frac{1}{2}T_{ij}(\ddot{q}_i\dot{q}_j + \dot{q}_i\ddot{q}_j) + + \frac{1}{2}V_{ij}(\dot{q}_i q_j + q_i\dot{q}_j) = 0$$

$$\Downarrow$$

$\cancel{T_{ij}\,q_i + \frac{1}{2}V_{ij}}$

REARRANGE:

$$\frac{1}{2}\dot{q}_j(T_{ij}\ddot{q}_i + V_{ij}q_i) + \frac{1}{2}\dot{q}_i(T_{ij}\ddot{q}_j + V_{ij}q_j) = 0$$

$\dot{q}_i$ & $\dot{q}_j$ ARE INDEPENDENT

$$\Rightarrow T_{ij}\ddot{q}_i + V_{ij}q_i = 0 \ \&\ T_{ij}\ddot{q}_j + V_{ij}q_j = 0$$

SO: YES, IT CAN.

ESIQ7 | I

CLOSURE: SATISFIED, SEE GROUP TABLE

IDENTITY: $f_1$

INVERSE: SATISFIED, SEE GROUP TABLE

ASSOCIATIVITY:

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
|---|---|---|---|---|---|---|
| $f_1$ | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
| $f_2$ | $f_2$ | $f_3$ | $f_1$ | $f_6$ | $f_4$ | $f_5$ |
| $f_3$ | $f_3$ | $f_1$ | $f_2$ | $f_5$ | $f_6$ | $f_4$ |
| $f_4$ | $f_4$ | $f_5$ | $f_6$ | $f_1$ | $f_2$ | $f_3$ |
| $f_5$ | $f_5$ | $f_6$ | $f_4$ | $f_3$ | $f_1$ | $f_2$ |
| $f_6$ | $f_6$ | $f_4$ | $f_5$ | $f_2$ | $f_3$ | $f_1$ |

SUBGROUPS: $\{f_1; f_4\}$; $\{f_1; f_5\}$; $\{f_1; f_6\}$;
$\{f_1; f_2; f_3\}$

$N \triangleleft G \Leftrightarrow \forall n \in N, \forall g \in G: gng^{-1} \in N$

(FROM WIKIPEDIA)

USING GROUP TABLE

$f_5 f_4 f_5^{-1} = f_5 f_4 f_5 = f_3 f_5 = f_6 \notin \{f_1, f_4\}$

$f_6 f_5 f_6^{-1} = f_6 f_5 f_6 = f_6 f_2 = f_4 \notin \{f_1, f_5\}$

$f_4 f_6 f_4^{-1} = f_4 f_6 f_4 = f_4 f_2 = f_5 \notin \{f_1, f_6\}$

THESE ARE NOT NORMAL SUBGROUPS.

ESIQ7 II

$$f_i f_1 f_i^{-1} = f_i f_i^{-1} = f_1$$

$$f_1 f_2 f_1^{-1} = f_1 f_2 f_1 = f_2$$

$$f_2 f_2 f_2^{-1} = f_2 f_1 = f_2$$

$$f_3 f_2 f_3^{-1} = f_3 f_2 f_2 = f_3 f_3 = f_2$$

$$f_4 f_2 f_4^{-1} = f_4 f_2 f_4 = f_4 f_6 = f_3$$

$$f_5 f_2 f_5^{-1} = f_5 f_2 f_5 = f_5 f_4 = f_3$$

$$f_6 f_2 f_6^{-1} = f_6 f_2 f_8 = f_6 f_5 = f_3$$

SAME PROCEDURE FOR $f_3$ TO SHOW $\{f_1, f_2, f_3\}$ IS NORMAL.

ANY NON-TEDIOUS WAY TO DO THIS?

$\boxed{\text{ESIQ8}}$ $C_4$: Z

| | | | |
|---|---|---|---|
| $I$ | $R$ | $R^2$ | $R^3$ |
| $R$ | $R^2$ | $R^3$ | $I$ |
| $R^2$ | $R^3$ | $I$ | $R$ |
| $R^3$ | $I$ | $R$ | $R^2$ |

$K_4$:

| | | | |
|---|---|---|---|
| $I$ | $x$ | $y$ | $z$ |
| $x$ | $I$ | $z$ | $y$ |
| $y$ | $z$ | $I$ | $x$ |
| $z$ | $y$ | $x$ | $I$ |

They are both abelian because both group tables are symmetric to main diagonal.

Isomorphic if:

$$g * h = j \iff f(g) \circ f(h) = f(j) \ \forall g, h, j \in G$$

$R^3 * R = I$ WHEREAS $f(R^3) \circ f(R) = z \circ x = y$

$f(I) = I \neq y \Rightarrow$ THEY ARE NOT ISOMORPHIC