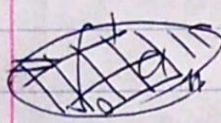


$$(a) \int_0^1 y_n y_m w dx = \delta_{nm}$$

$$(b) \int_0^1 w(x) f(x)^2 dx = \int_0^1 w(x) \left(\sum_{n=0}^{\infty} a_n y_n(x) \right)^2 dx$$

$$= \int_0^1 w(x) \sum_{n=0}^{\infty} a_n y_n(x) \sum_{m=0}^{\infty} a_m y_m(x) dx$$

$$= \int_0^1 w(x) \sum_{n=0}^{\infty} a_n a_n y_n y_n dx + \underbrace{\int_0^1 \sum_{n=0}^{\infty} \sum_{m=0}^{n-1} w a_n a_m y_n y_m dx}_{\text{0 USING ORTHOGONALITY}}$$



$$= \int_0^1 w(x) \sum_{n=0}^{\infty} a_n^2 y_n^2 dx$$

$$= \sum_{n=0}^{\infty} a_n^2 \underbrace{\int_0^1 w(x) y_n^2 dx}_{1, \text{ USING ORTHOGONALITY}} = \sum_{n=0}^{\infty} a_n^2$$

$$(c)(i) \int Y(x) = w(x) [\alpha Y(x) + f(x)]$$

$$f(x) = \sum_{n=0}^{\infty} a_n y_n(x)$$


$$\int Y = w [\alpha Y + \sum a_n y_n]$$

$$\int Y = \int \sum_{n=0}^{\infty} b_n y_n = \sum_{n=0}^{\infty} b_n \int y_n = \sum_{n=0}^{\infty} b_n \int_0^1 w y_n$$

COMBINE LAST TWO LINES, CANCEL w :

$$\sum_{n=0}^{\infty} b_n \int_0^1 w y_n = \alpha Y + \sum_{n=0}^{\infty} a_n y_n$$

$$= \sum_{n=0}^{\infty} \alpha b_n y_n + a_n y_n = \sum_{n=0}^{\infty} (\alpha b_n + a_n) y_n$$



$$\Rightarrow \sum_{n=0}^{\infty} (1_n - \alpha) b_n = \sum_{n=0}^{\infty} a_n$$

$$\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} \frac{a_n}{1_n - \alpha}$$

$$\Rightarrow Y = \sum_{n=0}^{\infty} b_n y_n = \sum_{n=0}^{\infty} \frac{a_n}{1_n - \alpha} y_n$$

(c)(i) $\lambda_0 = 1$ $\lambda_1 = 2$ $f = 2[y_0 + y_1] - \alpha y_1$

$a_0 = \int_0^1 f y_0 w dx = 2$ $a_1 = \int_0^1 f y_1 w dx = 2 - \alpha$

USING RESULT OF (c)(i):

$Y = \frac{a_0}{1-\alpha} y_0 + \frac{a_1}{2-\alpha} y_1$

• $\int_0^1 w(x) Y(x)^2 dx = 2$ ~~(ORTHOGONALITY OF EIG)~~

• ORTHOGONALITY OF EIGENFUNCTIONS

$\Rightarrow \int_0^1 w \left(\frac{a_0}{1-\alpha} \right)^2 y_0^2 + w \left(\frac{a_1}{2-\alpha} \right)^2 y_1^2 = 2$

(No $y_0 y_1$ TERMS BECAUSE THEY INTEGRATE TO 0)

$\left(\frac{a_0}{1-\alpha} \right)^2 + \left(\frac{a_1}{2-\alpha} \right)^2 = 2$

$\left(\frac{2}{1-\alpha} \right)^2 + \left(\frac{2-\alpha}{2-\alpha} \right)^2 = 2$

$\left(\frac{2}{1-\alpha} \right)^2 + 1 = 2$

$\Rightarrow \alpha = 3$

$Y = \frac{2}{1-3} y_0 + \frac{2-3}{2-3} y_1$

$= -y_0 + y_1$