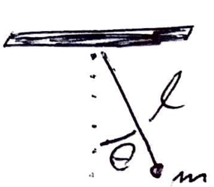
I SMALL OSCILLATIONS

① – EQUILIBRIUM IS BORING
– DEPARTURE FROM EQUILIBRIUM, START OFF SMALL
– PLAN: SIMPLE EXAMPLES, GENERAL THEORY OF SMALL OSC), CONCLUDE WITH EXAMPLES (WITH SYMMETRY)

③

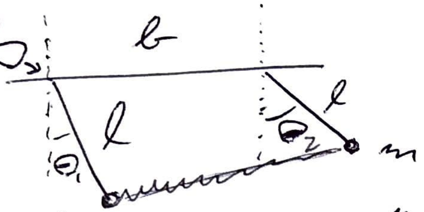


$m l\ddot{\theta} = - mg\sin\theta$

④ $\theta = C\sin(\omega t) + D\cos(\omega t)$

$\theta = A\sin(\omega(t - t_0))$

⑤



- SMALL ANGLES ⇒ VERTICAL DISPLACEMENTS SMALL, HORIZONTAL DISPLACEMENTS $\approx l\theta_1, l\theta_2$
- ASSIGN COORDINATES: ORIGIN O; POSITION VECTORS $\underline{r}_1$ & $\underline{r}_2$ FOR THE TWO MASSES.

⑥ $m l\ddot{\theta}_1 = - mg\theta_1 + kl(\theta_2 - \theta_1)$

$m l\ddot{\theta}_2 = - mg\theta_2 + kl(\theta_1 - \theta_2)$

⑦ in phase sol: $ml\ddot{\theta}_1 = -mg\theta_1 \Rightarrow \theta_1 = \theta_2 = A\sin(\omega(t - t_0))$ with $\omega^2 = \frac{g}{l}$

⑧ $180°$ out of phase sol: $\theta_1 = -\theta_2$

$$ml\ddot{\theta}_1 = -(mg + 2kl)\theta_1$$

SOLUTION: $\theta_1 = -\theta_2 = B\sin(\Omega(t - t_1))$ WITH $\Omega^2 = \frac{g}{l} + \frac{2k}{m}$

These are harmonic sols.: periodic sols described by a single angular frequency

⑨ Normal modes with corresponding normal frequencies.

GEN SOL CAN BE WRITTEN AS LIN. COMB OF 2 LIN. INDEP. SOL:

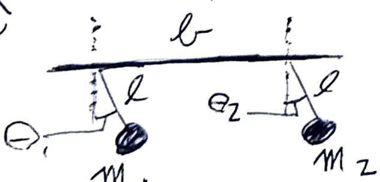
$$\theta_1 = A\sin(\omega(t - t_0)) + B\sin(\Omega(t - t_1))$$

$$\theta_2 = A\sin(\omega(t - t_0)) - B\sin(\Omega(t - t_1))$$

when $\theta_1 = \theta_2$, that is $B = 0$, & that's what the above two shows

when $\theta_1 = -\theta_2$, i.e. $A = 0$, $\theta_1 = -\theta_2$ is reflected on the above two eqs.

⑩



⑪ $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$ SAME FOR θ_2 $(1 \to 2)$

$T = \frac{1}{2} m_1 l^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l^2 \dot{\theta}_2^2$

⑫ $PE = \frac{1}{2} k l^2 (\theta_2 - \theta_1)^2$

(13)
$$\begin{pmatrix} m_1 l & 0 \\ 0 & m_2 l \end{pmatrix}\begin{pmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{pmatrix} = \begin{pmatrix} -m_1 g - kl & kl \\ kl & -m_2 g - kl \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

IE $\underline{\underline{T}}\,\ddot{\underline{q}} = -\underline{\underline{V}}\,\underline{q}$

$q = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ $T = \begin{pmatrix} m_1 l^2 & 0 \\ 0 & m_2 l^2 \end{pmatrix}$

$V = \begin{pmatrix} m_1 g l + kl^2 & -kl^2 \\ -kl^2 & m_2 g l + kl^2 \end{pmatrix}$

(NOTE EXTRA l FACTOR)

(14)
$$L = \frac{1}{2}\dot{\underline{q}}^T \underline{\underline{T}}\,\dot{\underline{q}} - \frac{1}{2}\underline{q}^T \underline{\underline{V}}\,\underline{q}$$

$$= \frac{1}{2} T_{ij}\dot\theta_i\dot\theta_j - \frac{1}{2}V_{ij}\theta_i\theta_j$$

BY CONSTRUCTION THIS IS NORMAL MODE: IE SINGLE FREQ

$$\underline{q} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{i\omega t}$$

(15) det of det & group by powers of m

(17)
$$m_1\ddot\theta_1 + m_2\ddot\theta_2 = -\frac{g}{l}(m_1\theta_1 + m_2\theta_2)$$

$$\ddot\theta_1 - \ddot\theta_2 = -\left[\frac{g}{l} + k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right](\theta_1 - \theta_2)$$