1.

1 mol H_2
0.5 mol O_2
2 mol N_2
$P_0 = 10^5 Pa$
$T = 300K$

$\xrightarrow{V_0}$ IGNITION $\longrightarrow$

1 mol H_2O
2 mol N_2

$\downarrow$

$2.8 \cdot 10^5 J$
EXTRACTED

$C_V = 26.4 \frac{J}{mol K}$ (N_2)

$C_V = 39.1 \frac{J}{mol K}$ (H_2O)

(a)

$$P_{FINAL} = P_0 \cdot \frac{1+2}{1+0.5+2} = P_0 \cdot \frac{6}{7} = \frac{6}{7} \cdot 10^5 Pa \quad \checkmark$$

(b) FIRST LAW:

$dU = dQ + dW$

$dW = 0$ BECAUSE
$dW = -P \, dV \, \& \, dV = 0.$

(FROM: $PV = nRT$

$P = \frac{nRT}{V}$,

IF R, T, V CONSTANT $P \propto n$)

HEAT EXTRACTED IS FROM U of H_2O & N_2.

$$26.4 \cdot 2 \cdot (T_{MAX} - 300) + 39.1 \cdot 1 \cdot (T_{MAX} - 300) = 2.8 \cdot 10^5$$

(IE $\sum C_V \Delta T = $ HEAT EXTRACTED)

$$T_{MAX} = 3346.79 K \approx \underline{3350K} \quad \checkmark$$

(c) FOLLOWING HINT:

$$\underbrace{(C_V + R)}_{\substack{N_2 \\ C_P}} \cdot 2(T_{MAX} - 300) + \left(\underset{H_2O}{C_V} + R\right)(T_{MAX} - 300) = 2.8 \cdot 10^5$$

$$T_{MAX} = 2696.9 \approx \underline{2700K} \quad \checkmark$$

It is lower because if we do it in constant pressure, gas is doing pV work (ie it expands) so not all the E released from the chemical reaction goes into the increase of $U(T)$, but some of it is used to do work on surroundings. The heat which we could extract from this process is therefore also less.

$\checkmark$

(a) $TV^{\gamma-1} = \text{CONSTANT}$ $\qquad T_i V_i^{\gamma-1} = T_\ell V_\ell^{\gamma-1}$

$\gamma = \dfrac{C_p}{C_v} = \dfrac{\frac{5}{2}R + R}{\frac{5}{2}R} = \dfrac{7}{5}$ $\qquad T_f = T_i \left(\dfrac{V_i}{V_\ell}\right)^{\gamma-1}$

$\left(\dfrac{V_i}{V_\ell}\right) = \left(\dfrac{\frac{nRT_i}{p_i}}{\frac{nRT_\ell}{p_\ell}}\right) = \dfrac{T_i\, p_\ell}{p_i\, T_\ell}$ $\longrightarrow$ $T_\ell = T_i \left(\dfrac{T_i\, p_\ell}{p_i\, T_\ell}\right)^{\gamma-1}$

$$T_f = \left(\dfrac{T_i^\gamma\, p_\ell^{\gamma-1}}{p_i^{\gamma-1}}\right)^{\frac{1}{\gamma}} = $$

$$= \left(\dfrac{273.15^{\frac{7}{5}}\,(10^6)^{\frac{7}{5}-1}}{(10^5)^{\frac{7}{5}-1}}\right)^{\frac{5}{7}} = 273.15 \cdot 2.6\,\cancel{1} = 527.37 \approx \underline{527\,K}$$

$\checkmark$

(b) isothermal: $\Delta T = 0 \Rightarrow dU = 0 \Rightarrow dQ = -dW$

$$Q = \int_i^f dW = \int_i^f p\,dV = \int_i^f \dfrac{RTn}{V}\,dV = RTn\,\ln\left(\dfrac{V_f}{V_i}\right)$$

$n = \dfrac{pV}{RT} = \dfrac{10^5 \cdot 1}{8.314 \cdot 273} = 44.06 \text{ mol}$

$\qquad V_f = \dfrac{nRT}{p_\ell} = \dfrac{V_i\, p_i}{p_\ell}$

$Q = RTn\,\ln\left(\dfrac{V_f}{V_i}\right) = 8.314 \cdot 273.15 \cdot 44.06 \cdot \ln\left(\dfrac{10^5 \cdot 1}{10^6}\right)$

$= -230.4\,kJ \Rightarrow \approx \underline{230\,kJ}$ MUST BE REMOVED

$\checkmark$

3(I) [ANSWER REVERSE-MATHED]

"THERMALLY ISOLATED CYLINDERS" $\Rightarrow dQ=0 \Rightarrow$ CHANGE IS ADIABATIC.

$$\Rightarrow T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i \left(\frac{V_f}{V_i}\right)^{-(\gamma-1)}$$

$$= T_0 \cdot 2^{-\left(\frac{5}{3}-1\right)} = 2^{-\frac{2}{3}} T_0$$

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{3}{2} + 1}{\frac{3}{2}} = \frac{5}{2} / \frac{3}{2} = \frac{5}{3}$$

By $dU = dQ + dW$ WHAT HAPPENS: A IS FULLY PUSHED IN.

$$\underset{\to 0}{} \int dW = \int p\, dV = pV \quad \text{SINCE } p \text{ IS CONSTANT.}$$

$$T_f = T_0 + \Delta T = T_0 + \frac{W}{C_p}$$

$$dU = C_p\, dT$$
$$dQ = 0$$
$$\longrightarrow dW = C_p\, dT$$
$$dT = \frac{dW}{C_p}$$
$$\Delta T = \frac{W}{C_p}$$

$$= T_0 + \frac{pV}{C_p} = T_0 + \frac{nRT_0}{C_p}$$

$$= T_0\left(1 + \frac{nR}{C_p}\right) = T_0\left(1 + \frac{nR}{n \cdot C_p}\right) \quad \text{PER MOLE}$$

$$= T_0\left(1 + \frac{R}{\left(\frac{3}{2}+1\right)R}\right) = T_0\left(1 + \frac{2}{5}\right) = T_0 \frac{7}{5}$$

C/

A   B

η mol GAS T_0 V

$$T_{B_{\text{INITIAL}}} = \lim_{\eta \to 0} \frac{pV}{R\eta} = 0$$

$\eta \to 0$

IN THE END, η_R GAS REMAINS IN A.

ENERGY FOR CONSTANT p COMPRESSION   ENERGY FOR CONSTANT V EXPANSION

$$C_p \eta_R \Delta T_A + C_V (\eta - \eta_R)\Delta T_B =$$

FOR THE GAS REMAINING IN A.   THIS MANY GAS TO B

$$= -p\Delta V \qquad \text{I}$$

THE WORK WE DO ON THE GAS.

EQ. OF STATE FOR FINAL STATE:

$$pV_{\text{REM}} = \eta_R R(T_0 + \Delta T_A) \qquad \text{II}$$

$$pV = (\eta - \eta_R) R\, \Delta T_B \qquad \text{III}$$

3(II)

$$-\Delta V = V - n_R R (T_0 + \Delta T_A)\frac{1}{n} \qquad V_{REM} = V + \Delta V = \frac{n_R R}{n}(T_0 + \Delta T_A)$$

REWRITE EQS:

I: $C_p n_R \Delta T_A + C_V (n - n_R)\Delta T_B = nV - n_R R(T_0 + \Delta T_A)$

II: $n \frac{n_R R}{n}(T_0 + \Delta T_A) = n_R R (T_0 + \Delta T)$ (NOT THAT HELPFUL)

III: $nV = (n - n_R) R \Delta T_B$

3 UNKNOWNS $(n_R, \Delta T_A, \Delta T_B)$; 2 "USEFUL" EQS (I & III)

LET: $n = 1$, $n = 1$, $V = 1$, $R = 1$, $T_0 = 1$

I BECOMES:

$$\frac{5}{2} n_R \Delta T_A + \frac{3}{2}(1 - n_R)\Delta T_B = 1 - n_R(1 + \Delta T_A)$$

III BECOMES:

$$1 = (1 - n_R)\Delta T_B$$

$$\downarrow$$

$$n_R = \frac{\Delta T_B - 1}{\Delta T_B}$$

SUB IN TO I:

$$\frac{5}{2}\frac{\Delta T_B - 1}{\Delta T_B}\Delta T_A + \frac{3}{2}\left(1 - \frac{\Delta T_B - 1}{\Delta T_B}\right)\Delta T_B = 1 - \frac{\Delta T_B - 1}{\Delta T_B}(1 + \Delta T_A)$$

IF ΔT_B [MISSING STEP HERE]

IF $\Delta T_A = 0$, AS SOLUTION SUGGESTS

$$\frac{3}{2}\left(1 - \frac{\Delta T_B - 1}{\Delta T_B}\right)\Delta T_B = 1 - \frac{\Delta T_B - 1}{\Delta T_B}$$

$$\Delta T_B = \frac{2}{3}$$

SO $T_B = T + \Delta T_B =$ ~~5/3~~

$$= 1 + \frac{2}{3} = \frac{5}{3},\ \text{WHICH}$$

AGREES WITH SOLUTION.
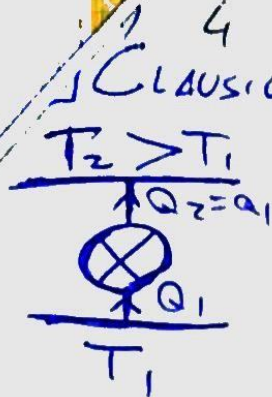
see also S.V.

3(IV)

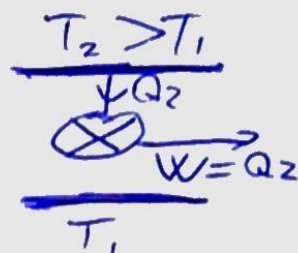This is just Joule expansion $\Rightarrow \Delta T = 0$, $T_A = T_B = T_0$ ✓

(e)

see ✓

4

↓CLAUSIUS: No process is possible whose only effect is to transfer heat from a colder body to a hotter body.

$T_2 > T_1$

$Q_2 = Q_1$

Q_1

T_1

KELVIN: No process is possible whose only effect is the complete conversion of heat into work.
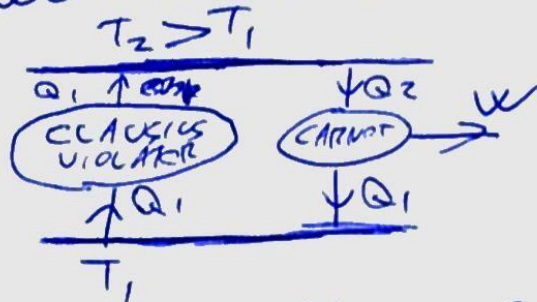
$T_2 > T_1$

Q_2

$W = Q_2$

T_1

CARNOT'S THM: No ~~an~~ engine operating between two reservoirs ~~can~~ can be more efficient than a Carnot engine.

SUPPOSE WE HAVE KELVIN-VIOLATOR. RUN ~~A~~ CARNOT ENGINE WITH IT BACKWARDS (IE HEAT PUMP). NET EFFECT: TRANSFER HEAT FROM T_1 TO T_2 ⟹ IT IS A CLAUSIUS VIOLATOR.
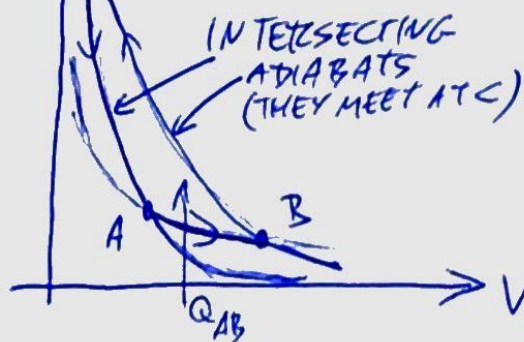
$T_2 > T_1$

$W = Q$ $W + Q_1$

KELVIN VIOLATOR — W → CARNOT

Q_1

T_1
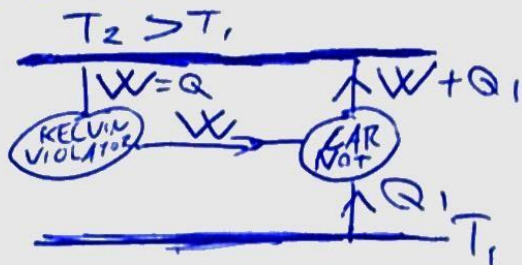
SUPPOSE WE HAVE CLAUSIUS-VIOLATOR. RUN IT IN ∥ WITH CARNOT:

$T_2 > T_1$

Q_1 Q_2 W NET EFFECT: KELVIN VIOLATOR

CLAUSIUS VIOLATOR   CARNOT →

Q_1 Q_1

T_1

✓ straight from the lecture notes.

So the two laws are the same.

(e) ADIABATS: $V^\partial = $ CONSTANT
ISO-THERMS: $p = \frac{MRT}{V} = \frac{\text{CONSTANT}}{V}$

INTERSECTING ADIABATS (THEY MEET AT C)

A   B

Q_{AB}

V

✓ correct, but there is a more fundamental argument!

CONSIDER ABC LOOP:

$\partial Q_{BC} = \partial Q_{CA} = 0$ (BECAUSE THEY ARE ADIABATS)

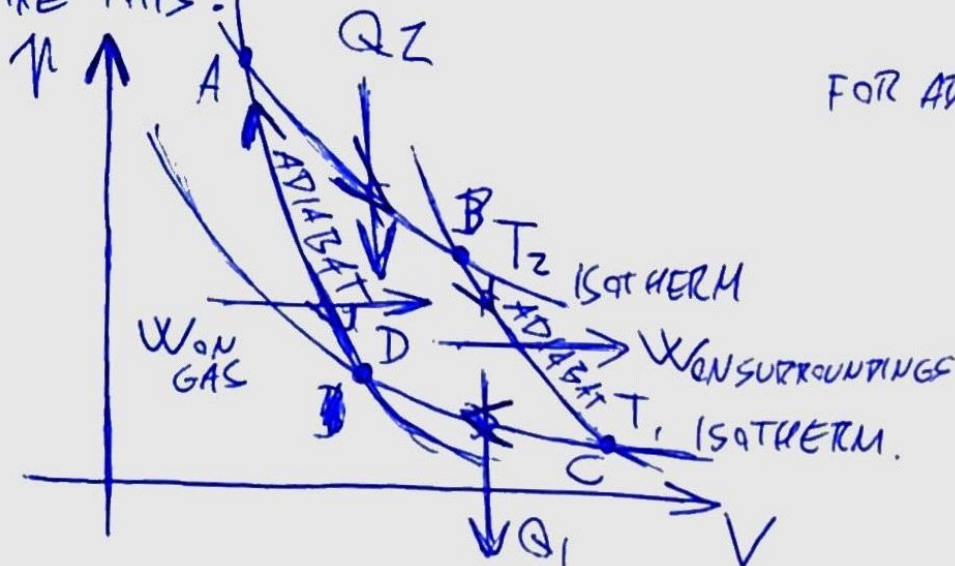$dU_{AB} = \partial Q_{AB} + \partial W_{AB} = 0$ (BECAUSE $U = U(t)$ & T IS CONSTANT ON AB)

$Q_{AB} = \int_A^B p\, dV = \int^? \frac{RT}{V} dV = RT \cdot \ln \frac{V_B}{V_A}$

$= -W$

THIS CYCLE IS A KELVIN-VIOLATOR ⟹ C MUST NOT EXIST (IE THEY DON'T INTERSECT)

5(I)

A CARNOT CYCLE IS A CYCLE
LIKE THIS:

FOR ADIABATS: $TV^{2} = $ CONSTANT

$$T_2 = \frac{TV}{nR}$$

$$T_1 = \frac{TV}{nR}$$
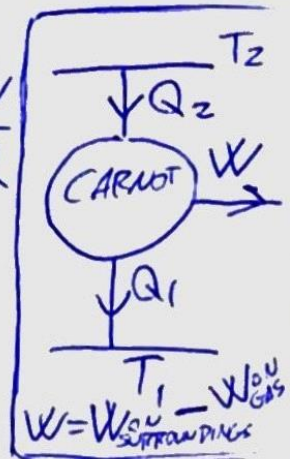
WORK IS DONE ON SURROUNDINGS THROUGH STEPS: $A \to B \to C$

$A \to B$: isothermal expansion.

"isothermal" $\Rightarrow dT = 0 \Rightarrow dU(T\ ONLY) = 0$

$$dU = 0 = dQ + dW_{ON\ GAS} = dQ - dW^{ON}_{SURROUNDINGS} = 0$$

$$dQ = dW_{SURROUNDINGS}$$

Q_2 ABSORBED FROM HIGH T RESERVOIR,
W_2 WORK IS DONE ON ENVIRONMENT.

$$(Q_2 = W_2)$$

$B \to C$: DISCONNECT FROM HIGH T RES. AT B,
COOL TO T AT C (IE MOVE TO LOWER T ISOTHERM)
ADIABATICALLY. $dQ = 0 \Rightarrow dU = dW_{ON\ GAS}$

$$\int_{B}^{C} TdV \quad \text{WORK IS DONE ON SURROUNDINGS.}$$

$C \to D$: ISOTHERMAL COMPRESSION.

$$dT = 0 \Rightarrow dU(T\ ONLY) = 0 = dQ + dW_{ON\ GAS} = 0$$

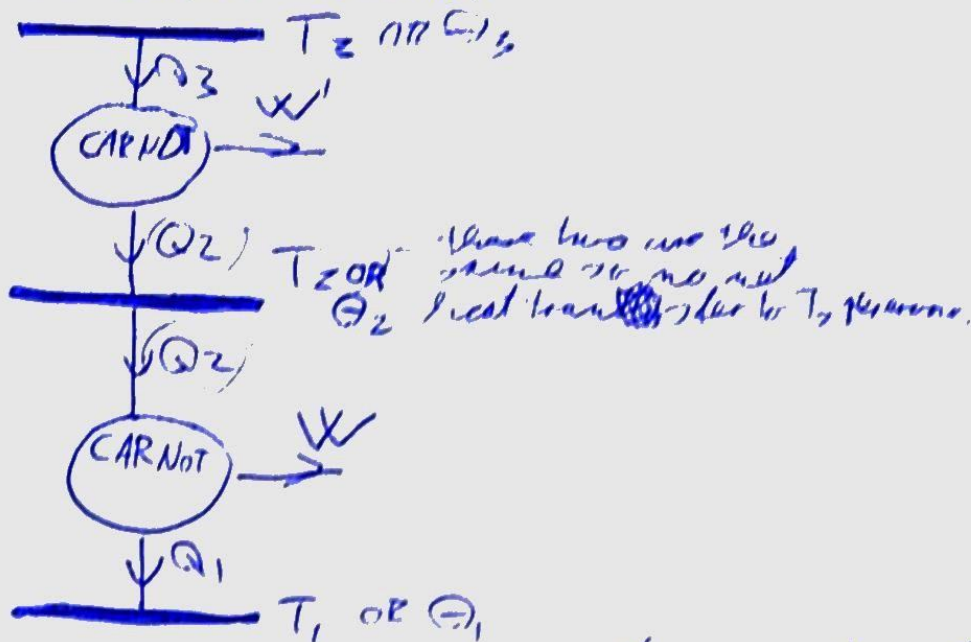GAS LOSES HEAT Q_1,
SURROUNDINGS DO WORK W_1

$$Q_1 = W_1.$$

$D \to A$: ADIABATIC COMPRESSION

$$dQ = 0 \Rightarrow dU = dW_{ON\ GAS}$$

SURROUNDINGS DO WORK ON GAS, IT WARMS UP
TO T_2 AGAIN.

Carnot cycle is more general.

-] CONSIDER TWO CARNOT ENGINES
IN SERIES.

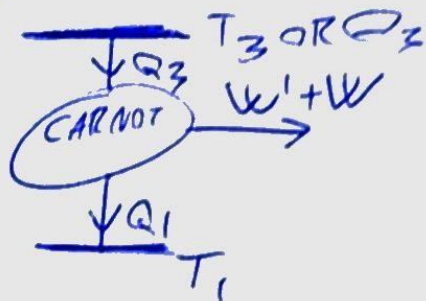T_2 OR Θ_3

$\downarrow Q_3$ W'

CARNOT →

$\downarrow (Q_2)$ T_2 OR   these two are the
Θ_2 same or no net
heat transfer to T_2 reservoir.

$\downarrow (Q_2)$

CARNOT → W

$\downarrow Q_1$

T_1 OR Θ_1

Carnot th.m: Carnot cycle is the most efficient between two
different T reservoirs.

⟹ FOR A GIVEN W' & W, Q_3 & Q_2 AND Q_2 & Q_1 IS
ONLY A FUNCTION of THE T OF THE RESERVOIRS.

⟹ $\dfrac{Q_2}{Q_1} = f(\Theta_1, \Theta_2)$ $\dfrac{Q_3}{Q_2} = f(\Theta_2, \Theta_3)$

The above setup is equivalent
to this:

T_3 OR Θ_3

$\downarrow Q_3$ W' + W

CARNOT →

$\downarrow Q_1$

T_1

$\dfrac{Q_3}{Q_1} = f(\Theta_1, \Theta_3)$

WHAT I DON'T LIKE
ABOUT MY WORK HERE
IS THAT I SAY THAT FOR
A GIVEN W' & W, ONLY A
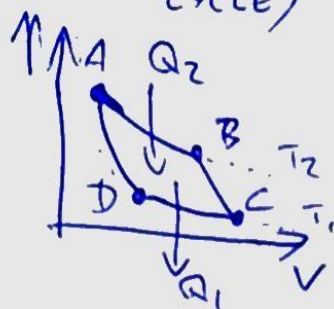FUNCTION, YET I USE
THE SAME FUNCTION f
FOR W, W', W + W'

$$\dfrac{Q_3}{Q_1} = \underbrace{f(\Theta_1, \Theta_3)}_{\text{NO } \Theta_2 \text{ DEPENDENCE}} = \dfrac{Q_3}{Q_1}\dfrac{Q_2}{Q_2} = \dfrac{Q_3}{Q_2}\dfrac{Q_2}{Q_1} = \underbrace{f(\Theta_2, \Theta_3) \cdot f(\Theta_1, \Theta_2)}_{\forall \Theta_1, \Theta_2, \Theta_3}$$

⟹ Θ_2 DEPENDENCE
MUST CANCEL ON LHS TOO.   HERE
LHS MUST HAVE THE FORM: $\dfrac{g(\Theta_3)}{g(\Theta_2)} \cdot \dfrac{g(\Theta_2)}{g(\Theta_1)}$

TAKE $g(\Theta_{\text{WHICHEVER}})$ AS DEFINITION
OF THERMODYNAMIC T SCALE:

⑪

1) 5(IV)

WE KNOW: $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \text{mm } \textcircled{1} \equiv T$

(FOR CARNOT CYCLE)

(UP TO SOME MULTIPLICATIVE FACTOR)

DERIVATION OF THIS:

$$Q_2 = \int_{V_A}^{V_B} \underbrace{\frac{RT_2}{V}}_{P} \, dV = RT_2 \ln \frac{V_B}{V_A}$$
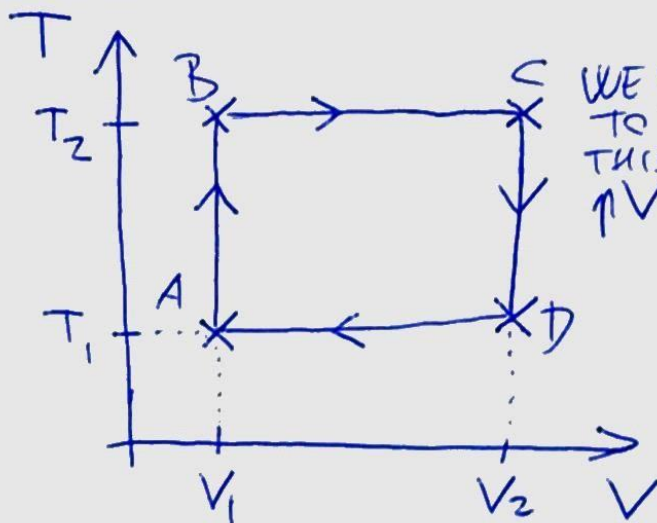
SIMILARLY:

$$Q_1 = RT_1 \ln \frac{V_C}{V_D}$$

BC ADIABAT: $T_2 V_B^{\gamma-1} = T_1 V_C^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_C}{V_B}\right)^{\gamma-1}$ $\searrow \frac{V_C}{V_B} = \frac{V_D}{V_A}$

$$T_2 V_A^{\gamma-1} = T_1 V_D^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \nearrow$$

$$\Downarrow$$
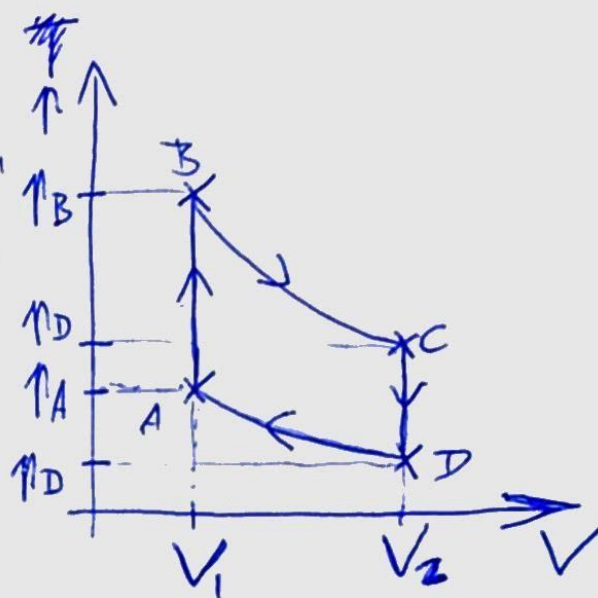
$$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

$$\frac{Q_2}{Q_1} = \frac{RT_2 \ln \text{some ratio}}{RT_1 \ln \text{the same ratio}} = \frac{T_2}{T_1}$$

STIRLING HEAT ENGINE

WE WANT TO TRANSFORM THIS TO A PV DIAGRAM.

$$P_A = \frac{RT_1}{V_1} \qquad P_B = \frac{RT_2}{V_1}$$

$$P_C = \frac{RT_2}{V_2} \qquad P_D = \frac{RT_1}{V_2}$$

$$P = \frac{RT}{V} \Rightarrow \text{ON THE } PV \text{ DIAGRAM,}$$
ISOTHERMS GO LIKE $\frac{1}{x}$.

1) $dU = dQ + dW$

$\underset{\text{#2}}{A \to B}$: $dV=0 \Rightarrow dW=0 \Rightarrow dU=dQ$

$$\underset{\substack{\text{INTO}\\\text{GAS}}}{Q} = \int dU = \int_{T_A}^{T_B} C_V \, dT = \frac{3R}{2}(T_2 - T_1)$$

$B \to C$: $dT=0 \Rightarrow dU=0$ (U IS ONLY FUNCTION OF T)

$dQ = -dW$

$$\underset{\text{BY GAS}}{W} = Q = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} \frac{RT_2}{V} \, dV = RT_2 \ln \frac{V_2}{V_1}$$

$C \to D$: $Q_{OUT} = \frac{3R}{2}(T_2 - T_1)$

$D \to A$: $\underset{\text{ON GAS}}{W} = RT_1 \ln \frac{V_2}{V_1}$ (THIS MUCH WORK PERFORMED ON GAS, THIS MUCH Q LEAVES GAS, SO IT COOLS)

$$\varepsilon = \frac{\text{HOW MUCH WORK I GET}}{\underset{\text{GAS}}{Q_{\text{INTO}}} + \underset{\text{ON GAS}}{W}} = \frac{RT_2 \ln \frac{V_2}{V_1}}{\frac{3}{2}R(T_2-T_1) + RT_1 \ln \frac{V_2}{V_1}} = \frac{T_2 \ln \frac{V_2}{V_1}}{\frac{3}{2}(T_2-T_1) + T_1 \ln \frac{V_2}{V_1}}$$

$$\frac{1}{\varepsilon} = \frac{\frac{3}{2}(T_2-T_1) + T_1 \ln \frac{V_2}{V_1}}{T_2 \ln \frac{V_2}{V_1}} = \frac{3}{2} \frac{1}{\ln \frac{V_2}{V_1}}(T_2-T_1) + \frac{T_1}{T_2}$$

WHICH IS WRONG.
GIVEN THAT I'M NOT FULLY UNDERSTANDING WHY I AM DOING WHAT I AM DOING, IT'S NOT THAT SURPRISING.