2016 P2 Q1 (I)

(a) $\int_a^b y_1 \mathcal{L} y_2 \, dx = \int_a^b y_1 \cdot \left[-(p y_2')' + q y_2 \right] dx =$

$= \left[-(p y_2') y_1 \right]_a^b - \int_a^b -(p y_2') y_1' + q y_2 y_1 \, dx$

$\int_a^b y_2 \mathcal{L} y_1 \, dx = \int_a^b y_2 \cdot -(p y_1')' + q y_1 y_2 \, dx =$

$= \left[-(p y_1') y_2 \right]_a^b - \int_a^b -(p y_1') y_2' + q y_1 y_2 \, dx$

SELF-ADJOINT IF: $\langle y_1 | \mathcal{L} y_2 \rangle - \langle y_2 | \mathcal{L} y_1 \rangle = 0$

i.e. $\left[p (y_1' y_2 - y_1 y_2') \right]_a^b = 0$

$p\left(-\frac{B_1}{B_2} y_1 y_2 - - y_1 \frac{B_1}{B_2} y_2 \right) - p\left(-\frac{A_1}{A_2} y_1 y_2 - - \frac{A_1}{A_2} y_2 y_1 \right) =$

$= 0 \Longrightarrow \mathcal{L}$ IS SELF-ADJOINT.

I AM USING: $A_1 y_1(a) + A_2 y_2'(a) = 0$

$B_1 y_1(b) + B_2 y_2'(b) = 0$

WHICH IS NOT WHAT IS GIVEN.

CONDITION FOR SELF-ADJOINTNESS IS IN TERMS OF y_1 & y_2,

BC IS IN TERMS OF: y ITSELF.

$y = \sum c_i y_i$

THIS ISSUE IS A BIT CONFUSING.

(b) $\langle y | \mathcal{L} y \rangle = \langle y | \lambda y \rangle = \lambda \langle y^2 \rangle \Longrightarrow \lambda = \frac{\langle y | \mathcal{L} y \rangle}{\langle y^2 \rangle}$

$\lambda = \frac{\int_a^b y \cdot -(p y')' + q y^2 \, dx}{\int_a^b w y^2 \, dx} \underset{\int \text{BY PARTS}}{=} \frac{\int_a^b p y'^2 + q y^2 \, dx - [p y' y]_a^b}{\int_a^b w y^2 \, dx}$ AS REQUIRED

(c)

$\lambda = \frac{\int_0^l y'^2 + q y^2 \, dx - [y y']_0^l}{\int_0^l y^2 \, dx}$

BCS: $y(0) = 0$

$k y(l) + y'(l) = 0$

$[y y']_0^l = y(l) \cdot (-k) y(l) - 0 =$

$= -k \, y(l)^2$

$\lambda = \frac{\int_0^l y'^2 + q y^2 \, dx + k \, y(l)^2}{\int_0^l y^2 \, dx}$

NOMINATOR & DENOMINATOR STRICTLY POSITIVE $\Longrightarrow$ λs ARE STRICTLY POSITIVE.

16P2QI(II)

(d)

$a=0,\ b=l,\ p(x)=1,\ q(x)=0,\ w(x)=1,\ A_1=1,\ A_2=0,\ B_1=\varepsilon>0,\ B_2=1$

$$\mathcal{L}y = -y''$$

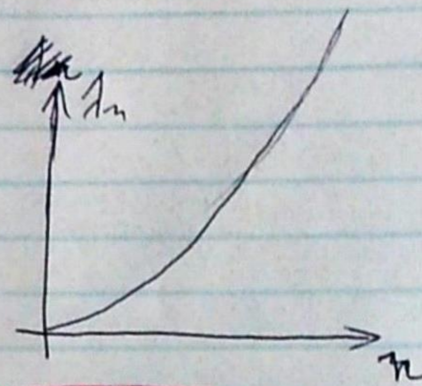
$$-y'' - \lambda y = 0$$

$$y'' + \lambda y = 0 \Rightarrow y = A\sin\left(\frac{n\pi x}{l}\right)$$

$$\lambda = \frac{\int_0^l A^2\left(\frac{n\pi}{l}\right)^2\cos^2\left(\frac{n\pi x}{l}\right)dx - \cancel{\left[A^2\sin\left(\frac{n\pi x}{l}\right)\left(\frac{n\pi}{l}\right)\cos\left(\frac{n\pi x}{l}\right)\right]_0^l}^{\to 0}}{\int_0^l A^2\sin^2\left(\frac{n\pi x}{l}\right)dx}$$

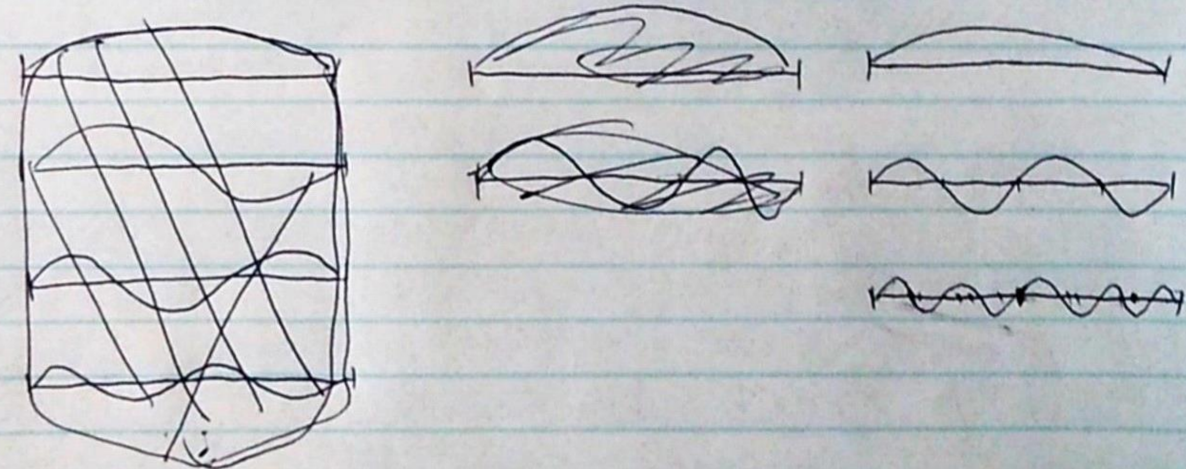
$$= \cancel{\frac{A^2 \frac{n^2\pi^2}{l^2} l}{2}} = \frac{A^2\frac{n^2\pi^2}{l^2}\frac{l}{2}}{A^2\frac{l}{2}} = \frac{n^2\pi^2}{l^2}$$

$\lambda_n \to \infty$ AS $n \to \infty$



~~(STH PROB. WENT WRONG)~~

EIGENVALUES HERE REPRESENT ANGULAR FREQUENCY-LIKE QUANTITY.



IT CAN BE AS HIGH AS WE WISH.

(e)

$\lambda_n \to \infty$ AS $n \to \infty$

(STH PROBABLY WENT WRONG)