CALC OF VARIATIONS

- FUNCTIONAL FORM
  (WE USUALLY CONCERNED WITH
- FUNCTIONAL DERIVATIVE
  - VARIATION OF G (THE FUNCTIONAL) BY δy
  - INTEGRAL FORM
  - SIMPLIFY TO FIRST ORDER
    - TAYLOR EXP. USING EXACT-DERIVATIVE-LIKE FORM
  - INTEGRATE BY PARTS (NOTE THAT ONLY ONE TERM, LIKE IN SL PROBLEMS)
  - VARIATION OF G INVOLVING FUNCTIONAL DERIVATIVE
    - CONSIDER SUM OF ALL xS WHERE G HAS A CERTAIN RATE OF CHANGE WITH y & G IS CHANGED BY δy, SUM FOR ALL xS.
  - COMPARE PREVIOUS TWO STEPS, OBTAIN RESULT
- EULER-LAGRANGE EQUATION
  - HOLDS IF [CONDITION]
- FIRST INTEGRAL
  - NO EXPLICIT DEPENDENCE OF WHAT IN WHAT
  - EXPAND SOMETHING'S DERIVATIVE WHICH IS EQUAL TO ZERO
    (REMEMBER, FIRST INTEGRAL FORM... $= C$, THAT'S WHY WE ARE DOING THIS.
    - USE CHAIN RULE
  - APPLY EULER-LAGRANGE
  - REWRITE, OBTAIN RESULT
- NO y DEPENDENCE IN E-L
  - WRITE OUT "PRODUCT-RULED" VERSION OF E-L.
  - ZERO ZERO TERMS
  - REWRITE WHAT'S LEFT
  - OBTAIN RES ($\frac{d}{dx}[\square] = 0 \Rightarrow \square = C$)

  GRANDPA NOTES PAGE 4.
- NO y' DEPENDENCE IN E-L
  - "PRODUCT-RULED" VERSION OF E-L
  - ZERO ZERO TERMS
  - OBTAIN RESULT

  GRANDPA PAGES.

- EQUIVALENCE OF FUNCTIONAL EXTREMISATION & SL EIGENVALUE PROBLEM
  - CONSIDER RELATIVE PROBABILITIES OF CERTAIN OUTCOMES OF AN OPERATOR, INTEGRAL FORM ($F[y]=\ldots$)
    - INTEGRATE BY PARTS, ASSUME AWAY BTS
  - CONSIDER WEIGHTED NORM OF y
  - MAKE SMALL VARIATIONS IN F, OBTAIN FUNCTIONAL DERIVATIVE
    - TAYLOR ~~EXP.~~ EXPANSION USING EXACT-DERIVATIVE-LIKE FORM
    - INTEGRATE BY PARTS
    - COMPARE WITH A VARIATION-OF-F EXPRESSION, WHERE USE "INTEGRAL" OF SUM OF ALL xS WHERE F HAS A CERTAIN RATE OF CHANGE WITH y & RATE IS CHANGED BY δy; SUM FOR ALL xS.
  - FUNCTIONAL DERIVATIVE OF G
    - TAYLOR EXP. USING EXACT-DERIVATIVE-LIKE FORM
    - COMPARE WITH — " —
  - CONSIDER RATIO OF FUNCTIONAL DERIVATIVES
  - CHANGE IN THIS RATIO WITH VARYING NOMINATOR-DENOMINATOR, TO FIRST TERM
    - PUT DENOMINATOR INTO 1+... FORM
    - APPROXIMATE $\frac{1}{1+\text{SMALL}}$
    - SIMPLIFY
  - ~~DIVIDE~~ BY A SMALL CHANGE IN THE FUNCTION TO OBTAIN FUNCTIONAL DERIVATIVES (LAGRANGE'S SPINNING IN HIS GRAVE)
  - PLUG IN PREVIOUSLY OBTAINED EXPRESSION FOR FUNCTIONAL DERIVATIVES
  - CONDITION TO ZERO OUT THIS DERIVATIVE
- FERMAT'S PRINCIPLE
- HAMILTON'S PRINCIPLE
  - ACTION FUNCTIONAL
- LAGRANGE'S EQUATIONS
  - SAME DERIV. AS E-L EQS (PREV. PAGE)
  - WHAT IS THE CONSTANT IN FIRST INTEGRAL?