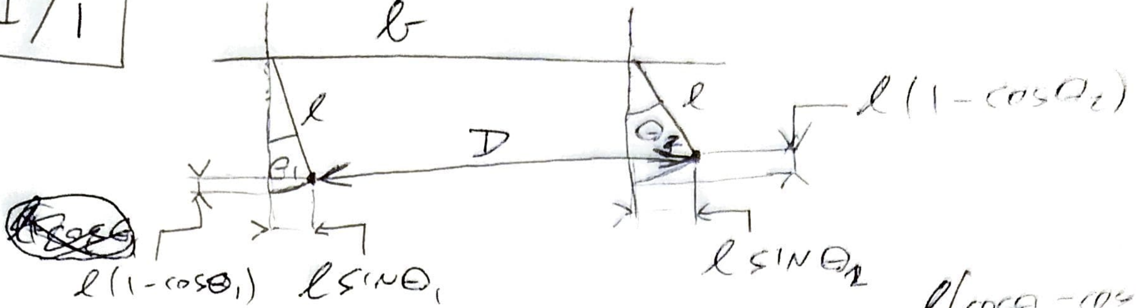
ESI/1



$$D^2 = \left(b + l\sin\theta_2 - l\sin\theta_1\right)^2 + \Big(\overbrace{l(1-\cos\theta_2) - l(1-\cos\theta_1)}^{l[\cos\theta_1 - \cos\theta_2]}\Big)^2$$

TO THIRD ORDER

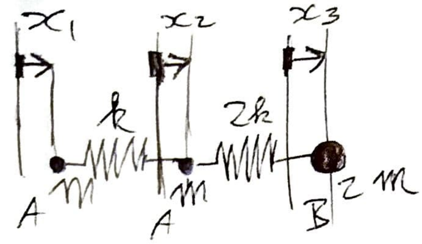
$$\approx \left(b + l\left[\left(\theta_2 - \frac{\theta_2^3}{3!}\right) - \left(\theta_1 - \frac{\theta_1^3}{3!}\right)\right]\right)^2 + \left[l\left(\left(1-\frac{\theta_1^2}{2}\right) - \left(1 - \frac{\theta_2^2}{2}\right)\right)\right]^2$$

TO FIRST ORDER IN θ

$$\approx \left(b + l(\theta_2 - \theta_1)\right)^2 \Rightarrow D = b + l(\theta_2 - \theta_1)$$

Extension $= D - b = \underline{l(\theta_2 - \theta_1)}$ AS REQUIRED

ESI/2 (I)



EQS OF MOTION, MATRIX FORM:

$$m\begin{pmatrix}1&0&0\\0&1&0\\0&0&2\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{x}_3\end{pmatrix} = k\begin{pmatrix}(x_2-x_1)\\(x_1-x_2)+2(x_3-x_2)\\2(x_2-x_3)\end{pmatrix}$$

LET'S USE THE FORM:

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}e^{i\omega t}$$

$$-m\omega^2\begin{pmatrix}1&0&0\\0&1&0\\0&0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\cancel{e^{i\omega t}} = k\begin{pmatrix}-1&1&0\\1&-3&2\\0&2&-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\cancel{e^{i\omega t}}$$

SET $m=1$, $k=1$ TO MAKE ALGEBRA SIMPLER.

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}\omega^2-1&1&0\\1&\omega^2-3&2\\0&2&2\omega^2-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\Rightarrow (\omega^2-1)\left[(\omega^2-3)(2\omega^2-2)-4\right] - \underbrace{(2\omega^2-2)}_{2(\omega^2-1)} = 0$$

$$\omega^2-1=0$$
$$\underline{\omega^2=1\left(\tfrac{k}{m}\right)}$$

$$(\omega^2-3)(2\omega^2-2)-4-2=0$$
$$2\omega^4-8\omega^2=0 \rightarrow \underline{\omega^2=0\left(\tfrac{k}{m}\right)}$$
$$\omega^2-4=0$$
$$\underline{\omega^2=4\left(\tfrac{k}{m}\right)}$$

NORMAL FREQUENCIES:

$$\boxed{\begin{aligned}\omega_0&=0\sqrt{\tfrac{k}{m}}\\ \omega_1&=1\sqrt{\tfrac{k}{m}}\\ \omega_2&=2\sqrt{\tfrac{k}{m}}\end{aligned}}$$

FOR $\omega=\omega_0$ CASE:

$$\begin{pmatrix}-1&1&0\\1&-3&2\\0&2&-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}$$

FOR $\omega=\omega_1$ CASE:

$$\begin{pmatrix}0&1&0\\1&-2&2\\0&2&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}-2\\0\\1\end{pmatrix}$$

FOR $\omega=\omega_2$ CASE:

$$\begin{pmatrix}3&1&0\\1&1&2\\0&2&6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\-3\\1\end{pmatrix}$$

ESI/2(II)

NORMALISED STATE VECTORS:

$$Q^{(1)}=\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\qquad Q^{(2)}=\frac{1}{\sqrt{5}}\begin{pmatrix}-2\\0\\1\end{pmatrix}\qquad Q^{(3)}=\frac{1}{\sqrt{11}}\begin{pmatrix}1\\-3\\1\end{pmatrix}$$

ORTHOGONALITY REL.: $\boxed{Q^{(n)T}\,T\,Q^{(m)}=\delta_{nm}}$

$$\frac{1}{\sqrt{3}}(1\ 1\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}\frac{1}{\sqrt{3}}=\frac{1}{3}(1\ 1\ 1)\begin{pmatrix}1\\1\\2\end{pmatrix}=\frac{4}{3}\neq 0$$

(NOT 1 BECAUSE RESCALING ISSUES. MAIN POINT: NOT ZERO)

$$\frac{1}{\sqrt{3}}(1\ 1\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\frac{1}{\sqrt{5}}\begin{pmatrix}-2\\0\\1\end{pmatrix}=\frac{1}{\sqrt{15}}(1\ 1\ 1)\begin{pmatrix}-2\\0\\2\end{pmatrix}=0$$

$$\frac{1}{\sqrt{3}}(1\ 1\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\frac{1}{\sqrt{11}}\begin{pmatrix}1\\-3\\1\end{pmatrix}=\frac{1}{\sqrt{33}}(1\ 1\ 1)\begin{pmatrix}1\\-3\\2\end{pmatrix}=0$$

$$\frac{1}{\sqrt{5}}(-2\ 0\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\begin{pmatrix}-2\\0\\1\end{pmatrix}\frac{1}{\sqrt{5}}=\frac{1}{\sqrt{25}}(-2\ 0\ 1)\begin{pmatrix}-2\\0\\2\end{pmatrix}=\cancel{6}\ \frac{6}{5}\neq 0$$

$$\frac{1}{\sqrt{5}}(-2\ 0\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\frac{1}{\sqrt{11}}\begin{pmatrix}1\\-3\\1\end{pmatrix}=\frac{1}{\sqrt{55}}(-2\ 0\ 1)\begin{pmatrix}1\\-3\\2\end{pmatrix}=0$$

$$\frac{1}{\sqrt{11}}(1\ -3\ 1)\begin{pmatrix}1&&0\\&1&\\0&&2\end{pmatrix}\frac{1}{\sqrt{11}}\begin{pmatrix}1\\-3\\1\end{pmatrix}=\frac{1}{\sqrt{121}}(1\ -3\ 1)\begin{pmatrix}1\\-3\\2\end{pmatrix}=\frac{12}{\sqrt{121}}\neq 0$$

"At t=0 all atoms ~~[illegible]~~" part

POSITIONS OF ATOMS AT t, AS A FUNCTION OF NORMAL-MODE COMPOSITION

$$=A\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot t+B\cdot\begin{pmatrix}-2\\0\\1\end{pmatrix}\sin(t)+C\begin{pmatrix}1\\-3\\1\end{pmatrix}\sin(2t)$$

$$\text{VELOCITY}\Big|_{t=0}=A\begin{pmatrix}1\\1\\1\end{pmatrix}+B\begin{pmatrix}-2\\0\\1\end{pmatrix}\cos(t)+C\begin{pmatrix}1\\-3\\1\end{pmatrix}2\cos(2t)\Bigg|_{t=0}=\begin{pmatrix}0\\0\\u_0\end{pmatrix}$$

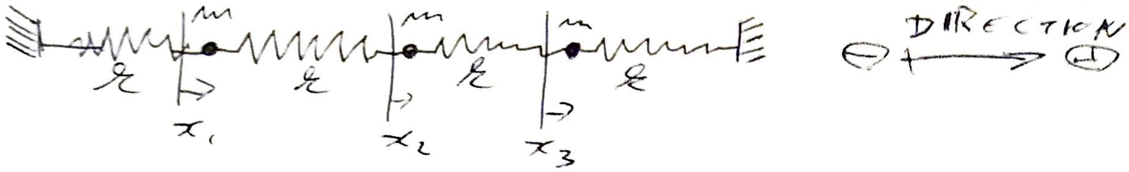
$$\begin{pmatrix}A-2B+2C\\A\quad-6C\\A+B+2C\end{pmatrix}=\begin{pmatrix}0\\0\\u_0\end{pmatrix}\Rightarrow\begin{array}{l}A=6C\\8C-2B=0\\B=4C\\6C+4C+2C=u_0\\C=\frac{u_0}{12}\\B=\frac{1}{3}u_0\\A=\frac{1}{2}u_0\end{array}$$

ESI/2 (III)

$$\text{POSITIONS}(t) = \frac{1}{2} u_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} t + \frac{1}{3} u_0 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \sin(t) + \frac{1}{12} u_0 \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \sin(2t)$$

DIMENSIONALLY THIS IS WRONG.

ESI/3 II



$$m\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{pmatrix} = k\begin{pmatrix}(x_2-x_1)-x_1\\(x_3-x_2)-(x_2-x_1)\\-x_3-(x_3-x_2)\end{pmatrix}$$

SEARCH FOR $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ IN THE FORM: $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}e^{i\omega t}$

SET $m=1$, $k=1$ TO MAKE ALGEBRA SIMPLER.

$$-\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\omega^2\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}-2&1&0\\1&-2&1\\0&1&-2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

$$\begin{pmatrix}-\omega^2+2&-1&0\\-1&-\omega^2+2&-1\\0&-1&-\omega^2+2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=0$$

$$(-\omega^2+2)\left[(-\omega^2+2)(-\omega^2+2)-1\right]-1(-\omega^2+2)=0$$

$$-\omega^2+2=0$$
$$\underline{\omega^2=2}$$

$$(-\omega^2+2)(-\omega^2+2)-1-1=0$$

$$\omega^4-4\omega^2+4-2=0$$

$$\omega^4-4\omega^2+2=0$$

$$\omega^2=\frac{4\pm\sqrt{16-4.1.2}}{2}=\underline{2\pm\sqrt{2}}$$

ESI/3(#)

CASE I: $\omega^2 = 2$

$$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = Q^{(1)}$$

CASE II: $\omega^2 = 2 + \sqrt{2}$

$$\begin{pmatrix} \sqrt{2} & -1 & 0 \\ -1 & -\sqrt{2} & -1 \\ 0 & -1 & -\sqrt{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix} = Q^{(2)}$$

CASE III: $\omega^2 = 2 - \sqrt{2}$

$$\begin{pmatrix} \sqrt{2} & -1 & 0 \\ -1 & \sqrt{2} & -1 \\ 0 & -1 & \sqrt{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} = Q^{(3)}$$

WHAT IS "PURE" NORMAL MODE?