UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Escola Tècnica Superior d'Enginyeria
Industrial de Barcelona

Master's degree in **Automatic Control and Robotics**
course 2016/2017

Modelling, Identification and Simulation of Dynamic Systems

EXERCICES FOR SESSION 4: NON-LINEAR SYSTEMS

This list contains exercises that you will practice in problems session 4. One of them will have to delivered to the teacher at the end of the class.

Objectives

1. Get skilled in representing and manipulating non-linear systems.
2. Use simulink to create simulations of dynamic systems.
3. Convert system representations between transfer function, state space and block representations.

1. Represent the following systems in state space mode and calculate the equilibrium points.

(a) $\ddot{x} + \dot{x} + x + x^2 = 0$

(b) [2], example 7.2

$$0 = \ddot{y} + \dot{v}^2 + y$$
$$0 = \dot{y}^2 + \ddot{v} + vy$$

(c) [2], example 7.3

$$0 = \ddot{y} + \dddot{v} + \dot{v}^2 + y$$
$$0 = \dot{y}^2 + \ddot{v} + vy$$

2. Represent the following systems in state space mode.

(a) $\overset{(n)}{y} = g(t, y, \dot{y}, \ldots, \overset{(n-1)}{y}, u)$

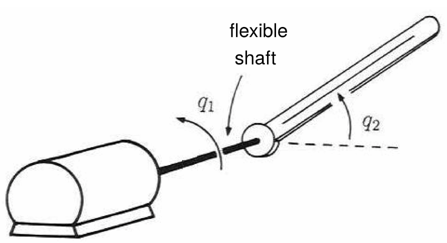
(b) $\overset{(n)}{y} = g_1(t, y, \dot{y}, \ldots, \overset{(n-1)}{y}, u) + g_2(t, y, \dot{y}, \ldots, \overset{(n-2)}{y})\dot{u}$

g_2 is differentiable for each one of its arguments.
(Note: you can try $x_n = \overset{(n-1)}{y} - g_2(t, y, \dot{y}, \ldots, \overset{(n-2)}{y})u$)

3. The following equations and figures describe the behavior of dynamic systems. They are examples of non-linear mechanical and electrical systems. Each one has to be represented using state space equations (you have to choose the state variables) and simulated using diagram block representation.

(a) *Flexible link manipulator*, [1]. q_1 and q_2 are the angular positions of the shaft and the manipulator end, J_1 and J_2 are the inertia moments, K is the torsion constant and m is the mass of the manipulator. The system's input is T, the applied motor torque.

$$J_1\ddot{q}_1 + K(q_2 - q_1) = T$$
$$J_2\ddot{q}_2 + K(q_2 - q_1) + mgd\cos(q_2) = 0$$



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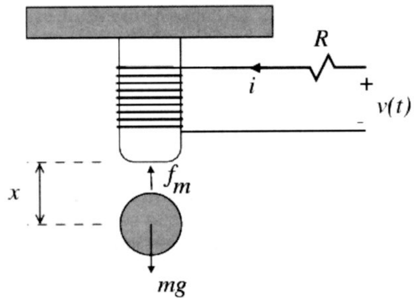
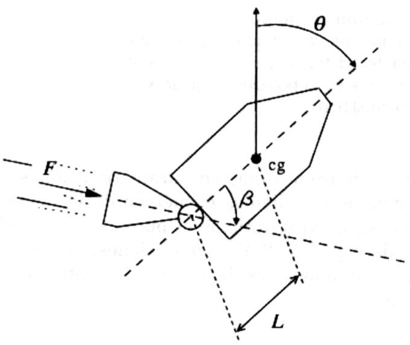
(b) *Magnetic levitation system.* This electrical assembly produces electromagnetic force through the voltage v applied at the input of the system. The force can counteract the gravity and levitate an iron made object. L and R are the inductance and resistance of the coil and i the intensity circulating through it. m is the mass of the levitating object and f_m is the electromagnet attraction force, which depends on a constant, c, the intensity and the distance, x, of the object.

$$L\frac{di}{dt} = -Ri + v(t)$$
$$m\frac{d^2x}{dt^2} = mg - f_m = mg - \frac{ci(t)^2}{x(t)}$$



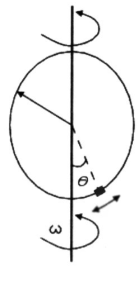
(c) *Satellite orientation system.* This satellite has a pivoting nozzle at its base to orientate the spacecraft. The angle of the nozzle in relation to the satellite is β and θ is the angular position of the satellite. u is the control action and L is the distance to the mass center, cg. The point where the reaction force to the gas ejection is applied is L.

$$J\frac{d^2\theta}{dt^2} = F\sin(\beta)L$$
$$\frac{d\beta}{dt} = Ru$$



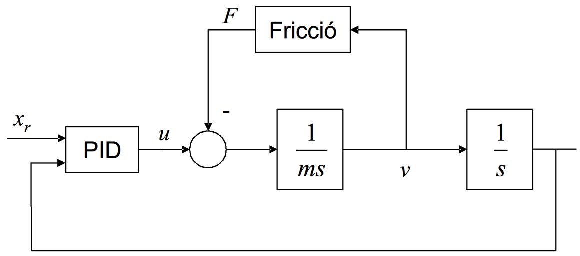
(d) *Spinning arc.* This system is a spinning arc with a frictionless ring sliding around it. The natural rest position of the ring is down but the centrifugal force takes the ring away from that position. The input of the system is the spinning velocity, ω, and the output is the angular position of the ring, θ. a is the radius of the arc.

$$a\frac{d^2\theta}{dt^2} = -g\sin(\theta) + a\omega^2\sin(\theta)\cos(\theta)$$

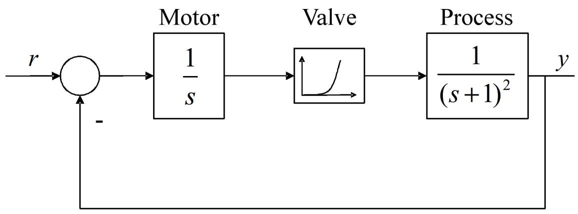


4. The following diagrams represent control diagrams with non-linear terms (blocks) made explicit. Each one has to be represented using state space equations (you have to choose the state variables) and simulated using diagram block representation (try different references or initial conditions).

(a) *Mechanical system with friction*, [3]. This diagram represents a simplified DC motor control system. The PID block can be substituted by its static gain, K_p, and the friction function can be substituted by its most simple definition: $F(x) = F_0\text{sign}(x)$.



(b) *Process with non-linear valve*, [3]. A second order process (chemical, for instance) depends on the aperture of a valve. The valve is actuated by a motor and the controller of the system is a proportional ($K_p = 1$) controller. The valve characteristic is given by the, not too realistic for $x < 0$, function $f(x) = x^2$. Try different references and check the values for which the output is stable and the ones for which it is unstable.



5. Linear regression models. The next exercises provide the difference equations that describe a nonlinear system which can be expressed as a linear regression model. Define the regressor vector, the parameter vector and the type of model you would use to identify the parameters.

(a) [2], example 5.1. *Solar-heated house*

$$x(t+1) - x(t) = d_2 I(t) - d_3 x(t) - d_0 x(t) u(t)$$
$$y(t+1) - y(t) = d_0 x(t) u(t) - d_1 y(t)$$

The state $x(t)$ is the temperature of the collector panel at time t, $I(t)$ is the heat supplied by the sun and $u(t)$ is the pump control input.

(b) [2], exercise 5E.1

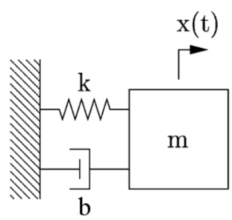
$$x(t) - a_1 x(t-1) + a_2 x(t-2) = b_1 u(t-1) + b_2 u(t-2) + c_1 x(t-1) u(t-1)$$
$$y(t) = x(t) + v(t)$$

(c) *Nonlinear mass-damper-spring system.* Consider the system of the course 2012/2013 exam. Assuming an Euler's discretization with sample time 1 second, the equations of the system become:

$$x_1(t) = x_1(t-1) + x_2(t)$$
$$x_2(t) = x_2(t-1) - \tfrac{b}{m} x_2(t)|x_2(t)| - \tfrac{k_0}{m} x_1(t) - \tfrac{k_1}{m} x_1^2(t)$$
$$y(t) = x_1(t)$$

6. *Course 2012/2013 exam. Nonlinear mass-damper-spring system.* Consider the system shown in the figure below. From Newton's second law the following equation was obtained:

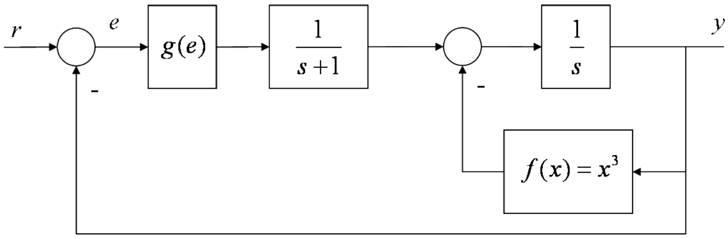
$$m\ddot{x} + b\dot{x}|\dot{x}| + k_0 x + k_1 x^3 = 0$$



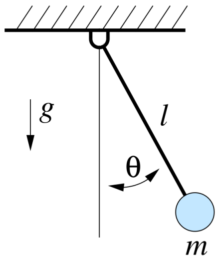
where m is the mass, x is the displacement, b is the nonlinear damper constant and k_0, k_1 are the nonlinear

spring terms.
Obtain the state space representation and calculate the equilibrium points of this dynamic system.

7. *Course 2013/2014 exam.* Consider the system shown in the figure below. Obtain the state space representation.



8. *Course 2014/2015 exam.* Consider the system shown in the figure below and the equation that describes its behaviour.



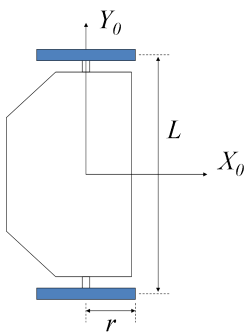
$$J\ddot{\theta} + mgl\sin(\theta) = -b\dot{\theta} + u$$

Where J is the moment of inertia of the pendulum about its axis of rotation, u denotes an input torque provided by a DC motor, b models a damping torque (typically produced by friction), m is the mass of the pendulum and l is the constant distance from the mass to its attachment point.

(a) Obtain the state space representation.

(b) Draw the block diagram of the system, considering θ as the output

9. *Course 2015/2016 exam.* Consider the robotic mobile platform depicted in the figure below and the equations that describe its behaviour. This is the differential drive robot model.



$$\dot{x} = \frac{r}{2}(\omega_l + \omega_r)\cos\theta$$
$$\dot{y} = \frac{r}{2}(\omega_l + \omega_r)\sin\theta$$
$$\dot{\theta} = \omega$$
$$\omega = \frac{r}{L}(\omega_r - \omega_l)$$

Most of the symbol's meanings can be deduced from the figure. ω_l and ω_r are the the rotation speeds of the left and right wheel, respectively, and represent the inputs to the system. θ is the robot's angle with respect to a non inertial frame of reference (X_0, Y_0).

(a) Draw the block diagram of the system.

(b) Consider x and y as the outputs of the system and an Euler discretization (with sampling time t_s and assuming $t-k$ represents $t-kt_s$). Select a model (**only one**) for identification, assuming you have enough input-output data, and justify your selection. Give details about the regressor vector and all the parameters the model might have, just as if you had to set up an automatic identification tool.

References

[1] H. K. Khalil. *Nonlinear Systems.* Prentice-Hall, Englewood Cliffs, NJ, 2nd edition, 1996.

[2] Lennart Ljung and Torkel Glad. *Modeling of Dynamic Systems.* PTR Prentice Hall, Englewood Cliffs, 1994.

[3] K. H. Johansson M. Johansson, B. Bernhardsson. Exercises in nonlinear control systems. Exercises from Nonlinear Control and Servo systems, LTH, Lund University, 12 2014. Available (2015) at http://www.control.lth.se/media/Education/EngineeringProgram/FRTN05/2014/compendium.pdf.