Practice 4. Gradient Images and Singular points

The Harris & Stephens corner detection algorithm

Harris and Stephens improved upon Moravec's corner detector by considering the differential of the corner score with respect to direction directly, instead of using shifted patches. Without loss of generality, we will assume a grayscale 2-dimensional image is used. Let this image be given by I. Consider taking an image patch over the area (u,v) and shifting it by (x,y). The weighted *sum of squared differences* (SSD) between these two patches, denoted S, is given by:

$$S(x,y) = \sum_u \sum_v w(u,v)\ \left(I(u+x,v+y) - I(u,v)\right)^2$$

$I(u+x,v+y)$ can be approximated by a Taylor expansion. Let I_x and I_y be the partial derivatives of I, such that

$$I(u+x,v+y) \approx I(u,v) + I_x(u,v)x + I_y(u,v)y$$

This produces the approximation

$$S(x,y) \approx \sum_u \sum_v w(u,v)\ \left(I_x(u,v)x + I_y(u,v)y\right)^2,$$

which can be written in matrix form:

$$S(x,y) \approx \begin{pmatrix} x & y \end{pmatrix} A \begin{pmatrix} x \\ y \end{pmatrix},$$

where *A* is the structure tensor,

$$A = \sum_u \sum_v w(u,v) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$$

This matrix is a Harris matrix, and angle brackets denote averaging (i.e. summation over (u,v)). If a circular window (or circularly weighted window, such as a Gaussian) is used, then the response will be isotropic.

A corner (or in general an interest point) is characterized by a large variation of S in all directions of the vector $\begin{pmatrix} x & y \end{pmatrix}$. By analyzing the eigenvalues of A, this characterization can be expressed in the following way: A should have two "large" eigenvalues for an interest point. Based on the magnitudes of the eigenvalues, the following inferences can be made based on this argument:

1. If $\lambda_1 \approx 0$ and $\lambda_2 \approx 0$ then this pixel (x,y) has no features of interest.
2. If $\lambda_1 \approx 0$ and λ_2 has some large positive value, then an edge is found.
3. If λ_1 and λ_2 have large positive values, then a corner is found.

Harris and Stephens note that exact computation of the eigenvalues is computationally expensive, since it requires the computation of a square root, and instead suggest the following function M_c, where κ is a tunable sensitivity parameter:

$$M_c = \lambda_1 \lambda_2 - \kappa \left(\lambda_1 + \lambda_2\right)^2 = \det(A) - \kappa \operatorname{trace}^2(A)$$

Therefore, the algorithm does not have to actually compute the eigenvalue decomposition of the matrix A and instead, **it is sufficient to evaluate the determinant and trace of A to find corners, or rather interest points in general.**

The value of κ has to be determined empirically, and in the literature values in the range 0.04–0.15 have been reported as feasible.

Exercise:

Create a MATLAB code that calculates the M_c value for all the pixels in a gray input image I (x,y).

Steps:

1. Calculate the matrices I_x and I_y, the partial derivatives of I(x,y)
2. Compute the matrices $(I_x)^2$, $(I_y)^2$ and $(I_x. I_y)$.
3. Show these matrices as gray level images and combined in a RGB image (pseudocolor)
4. Average the results with a Gaussian filter and build the A matrix for every pixel of the original image. $A = \begin{bmatrix} \langle I_x^2 \rangle & \langle I_x I_y \rangle \\ \langle I_x I_y \rangle & \langle I_y^2 \rangle \end{bmatrix}$
5. For every pixel calculate the function M_c, from *det(A)* and *trace(A).*
6. Test for different values of *k.*
7. Present the results as an output gray image *Mc.*

To assess the results, try to identify the most singular N points and locate them on the original image. Use cross marks on the original image to indicate the singular point location.

Also compare the results with those from the MATLAB *corner(I)* and *cornermetric(I)* functions.

Write a little report explaining the developed work (including the code commented) and some discussion/conclusions on the obtained results. Include some real images showing your results, and synthetic test images as the next one: