UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Escola Tècnica Superior d'Enginyeria
Industrial de Barcelona

Master's degree in **Automatic Control and Robotics**
course 2015/2016

# Modelling, Identification and Simulation of Dynamical Systems

## State Estimation

# Controlability vs Observability

- Controlability
  - We can find $u(t)$ that drives $x(t_0)$ to $x(t_1)=0$ *in* $t_1>t_0$ finite.
- Observability
  - We can know $x(t_0)$ from $y_{[t0,t1]}$ and $u_{[t0,t1]}$ *in* $t_1>t_0$ finite.

# Criteria(i)

- Controlability

$$P = \left[ B \vdots AB \vdots A^2 B \vdots \ldots \vdots A^{n-1} B \right]$$

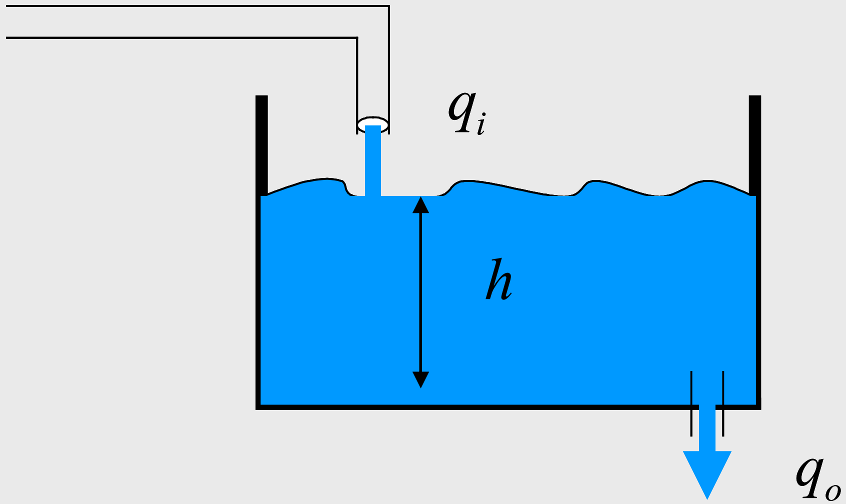
$$rank(P) = n$$

- Observability

$$Q = \left[ C^T \vdots A^T C^T \vdots A^{2T} C^T \vdots \ldots \vdots A^{n-1T} C^T \right]$$

$$rank(Q) = n$$

# Criteria(ii)




$$P = \left[ \frac{1}{C} \vdots \frac{1}{C} \frac{-1}{CR} \right]$$

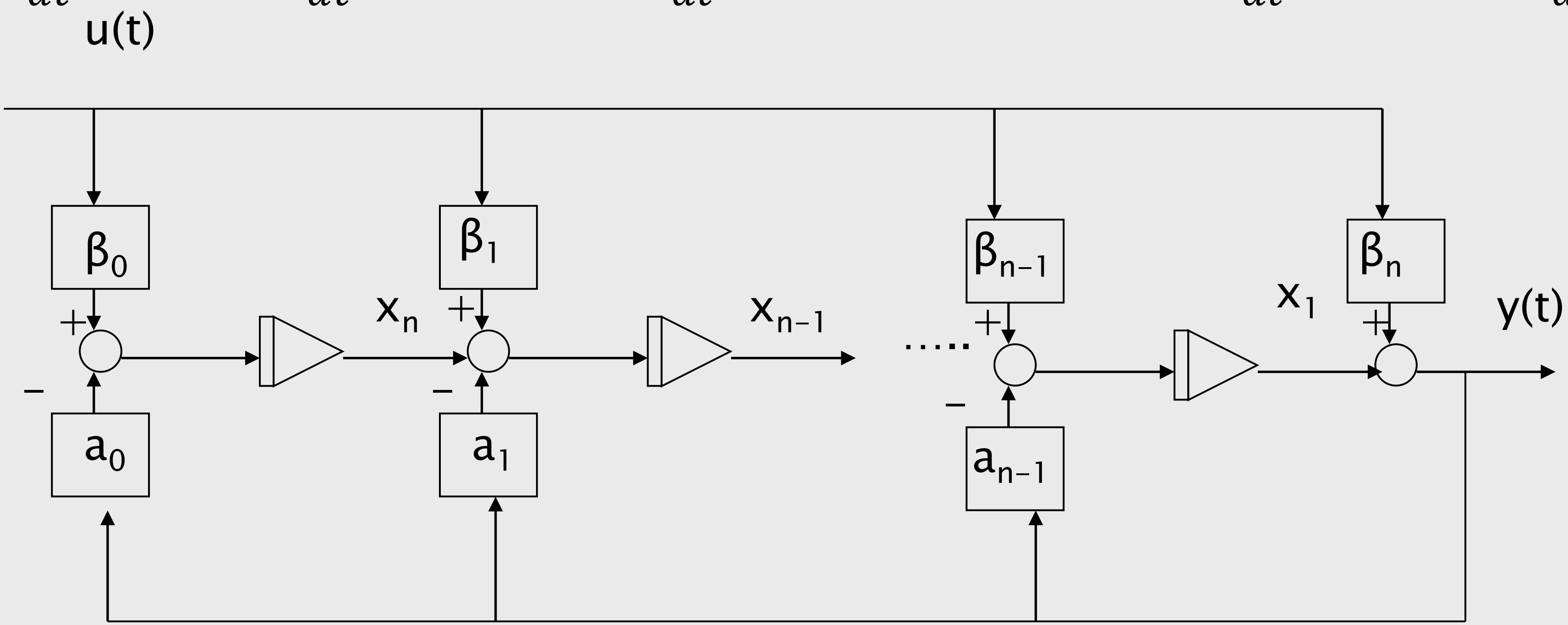
$$rank(P) = 1 = n$$

$$Q = \left[ 1 \vdots \frac{-1}{RC} 1 \right]$$

$$rank(Q) = 1 = n$$

# Zeros

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = \beta_0 u(t) + \beta_1\frac{du(t)}{dt} + \cdots + \beta_m\frac{d^m u(t)}{dt^m}$$



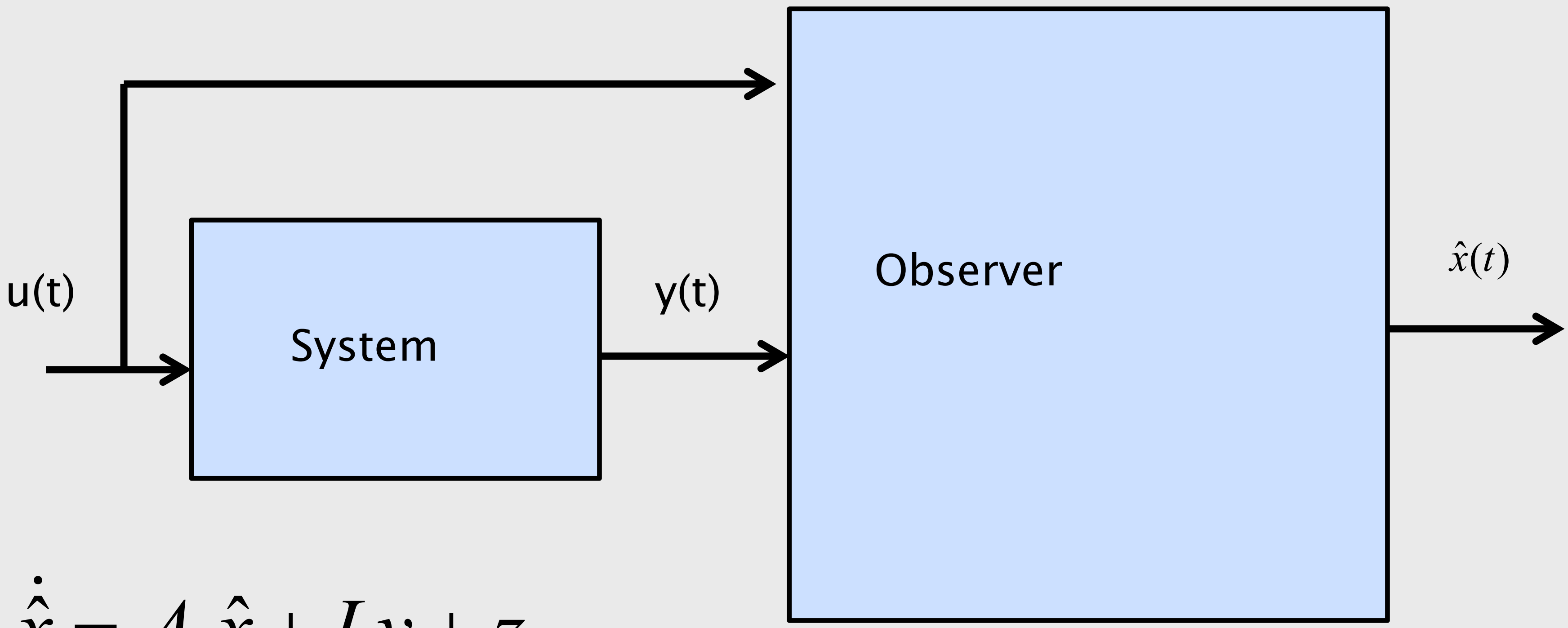

Obserbable canonical form

# Observable Canonical Form

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -a_{n-1} & 1 & \dots & 0 \\ -a_{n-2} & 0 & \dots & 0 \\ . & . & \dots & . \\ -a_0 & 0 & \dots & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \beta_{n-1} - a_{n-1}\beta_n \\ \beta_{n-2} - a_{n-2}\beta_n \\ . \\ \beta_0 - a_0\beta_n \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \mathbf{x}(t) + \beta_n u(t)$$

# State Observer

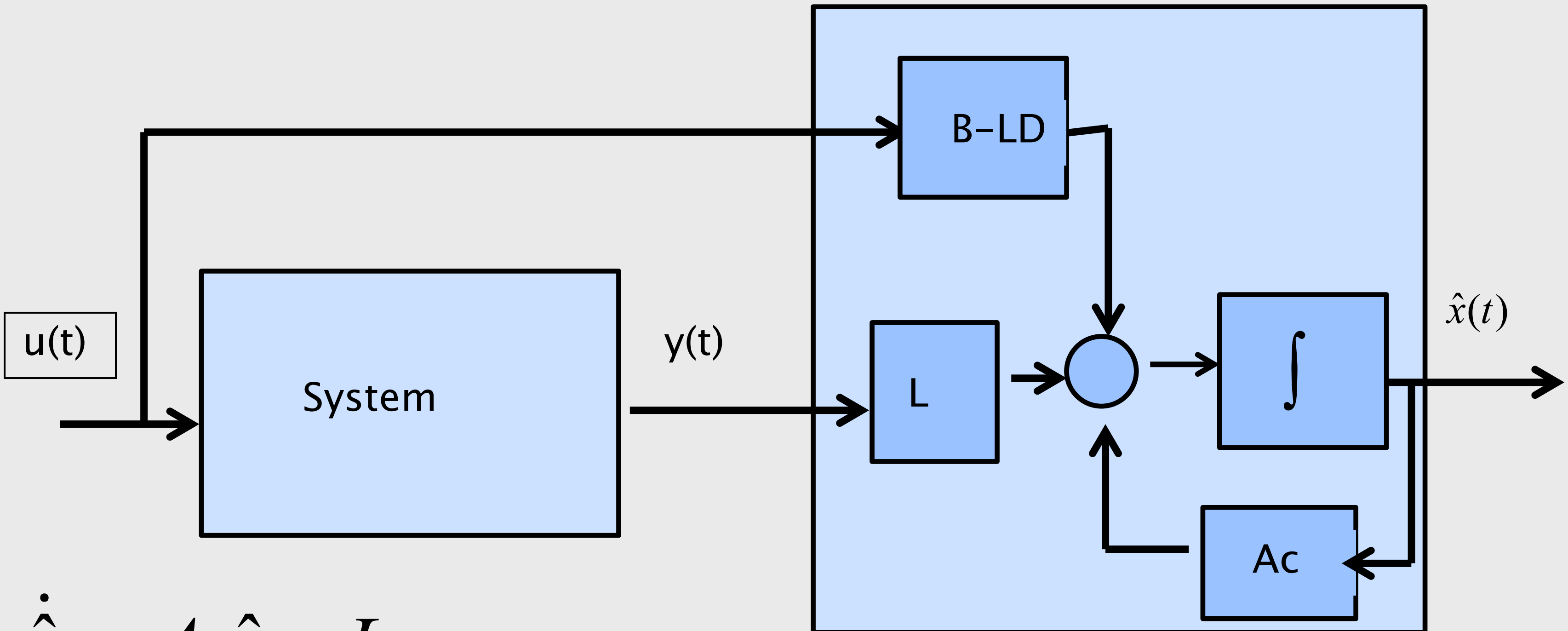



$$\dot{\hat{x}} = A_c\hat{x} + Ly + z$$

$$e = x - \hat{x}$$

$$\dot{e} = Ax - A_c\hat{x} - Ly + Bu - z$$

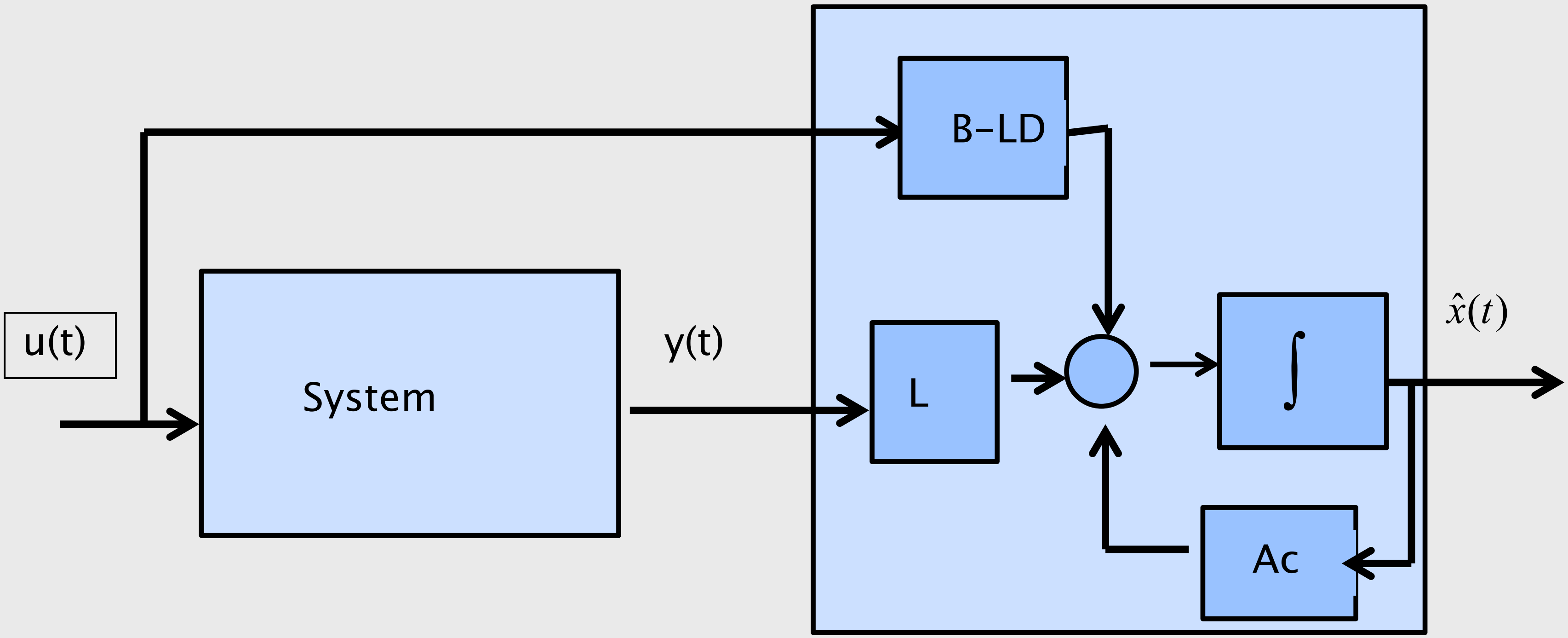
# State Observer




$$\dot{\hat{x}} = A_c \hat{x} + Ly + z$$

$$z = (B - LD)u$$

# State Observer

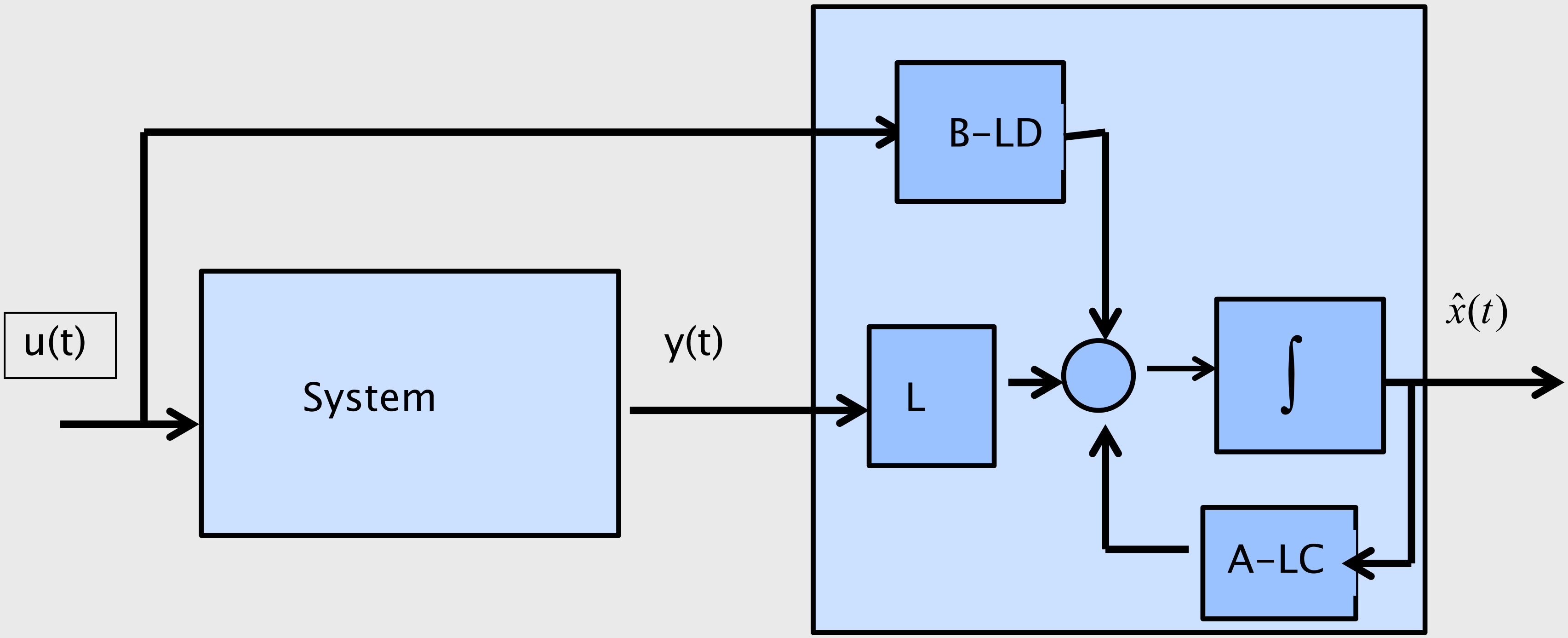



$$y = Cx + Du$$

$$z = (B - LD)u$$

$$\dot{e} = Ax - A_c\hat{x} - Ly + Bu - z = (A - LC)x - A_c\hat{x}$$

# State Observer

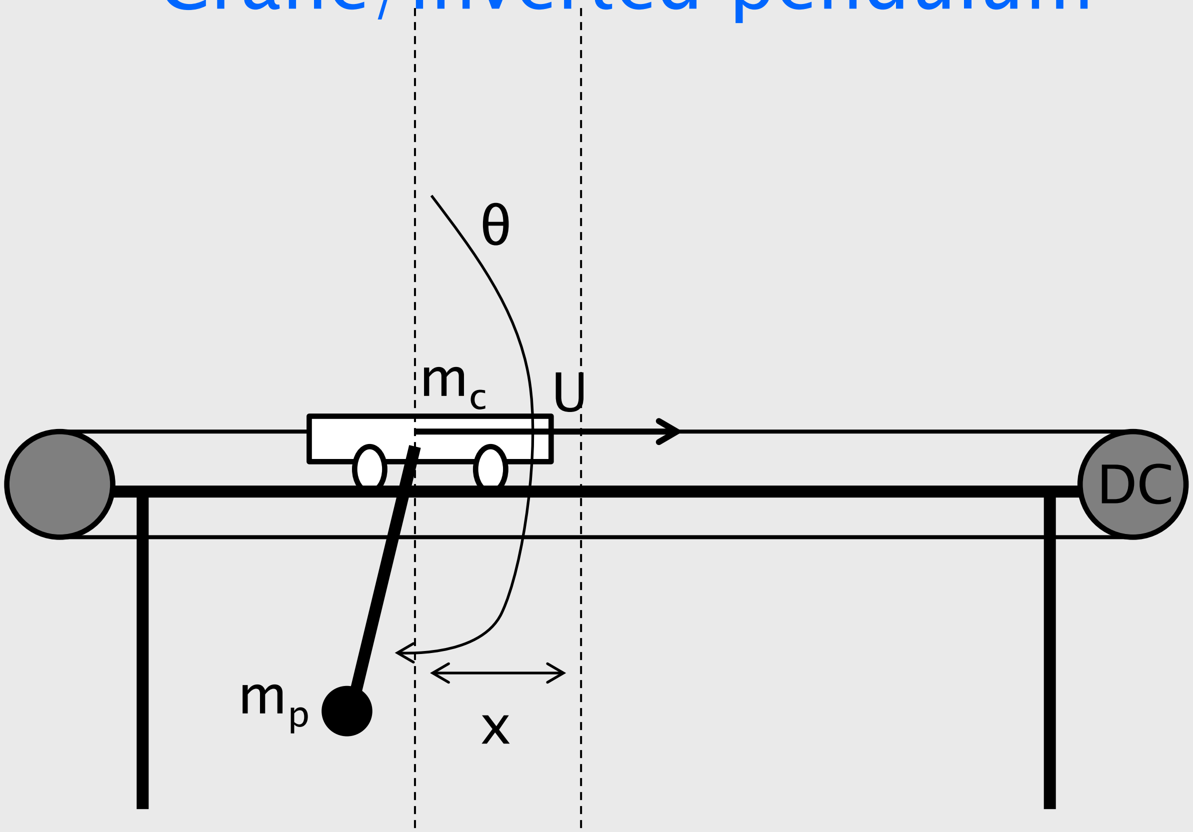



$$\dot{e} = \left(A - LC\right)x - A_c\hat{x}$$

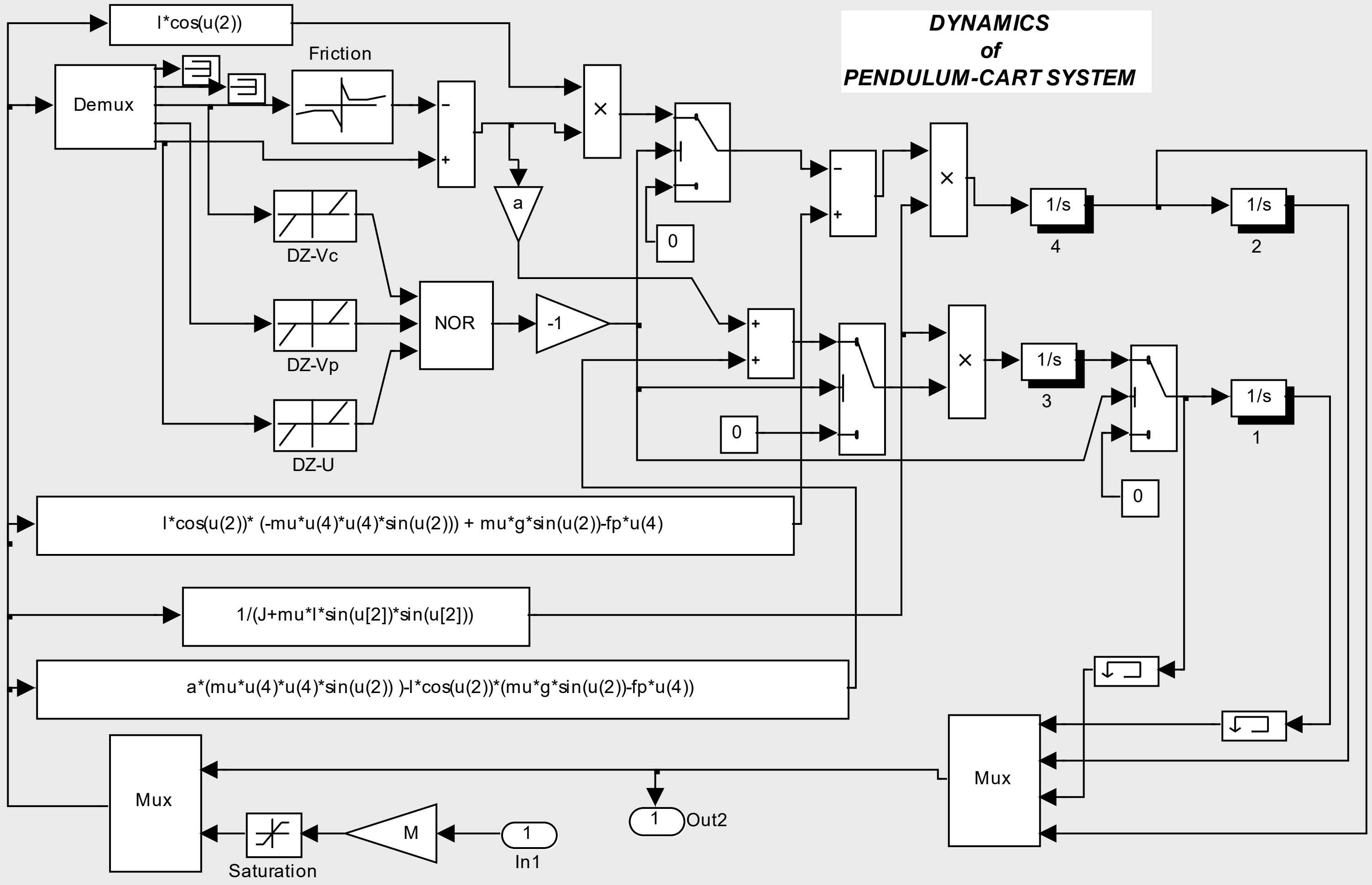
$$\dot{e} = (A - LC)e$$

$$A_c = A - LC$$

# Crane/inverted pendulum

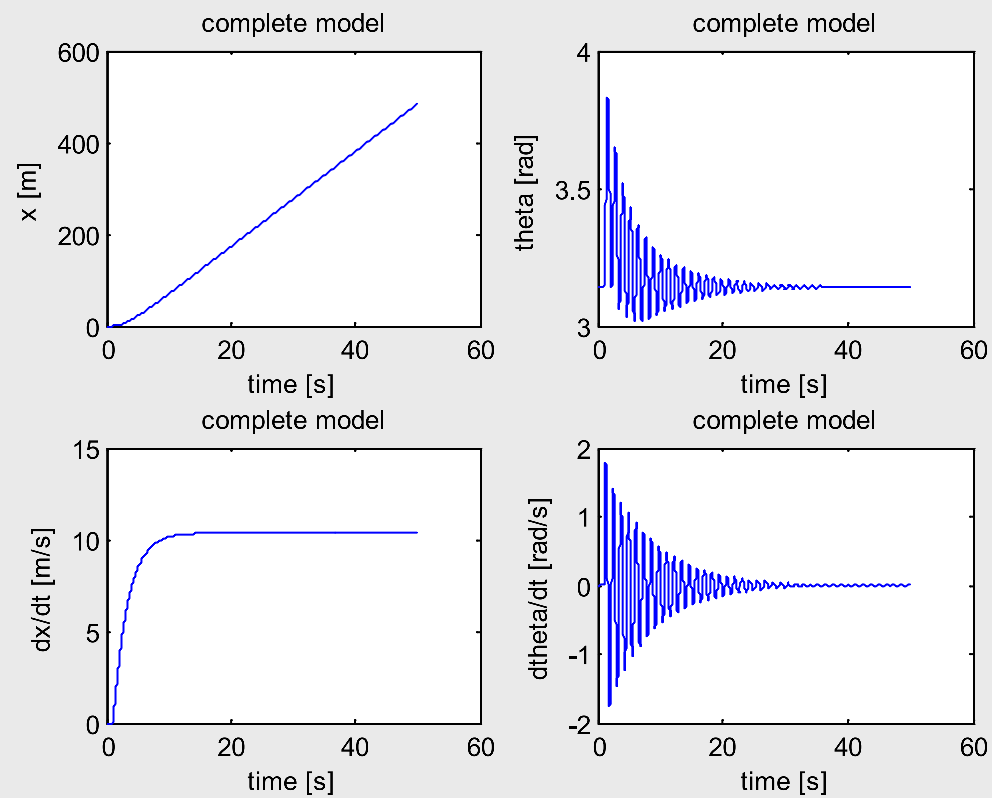


# Complete model

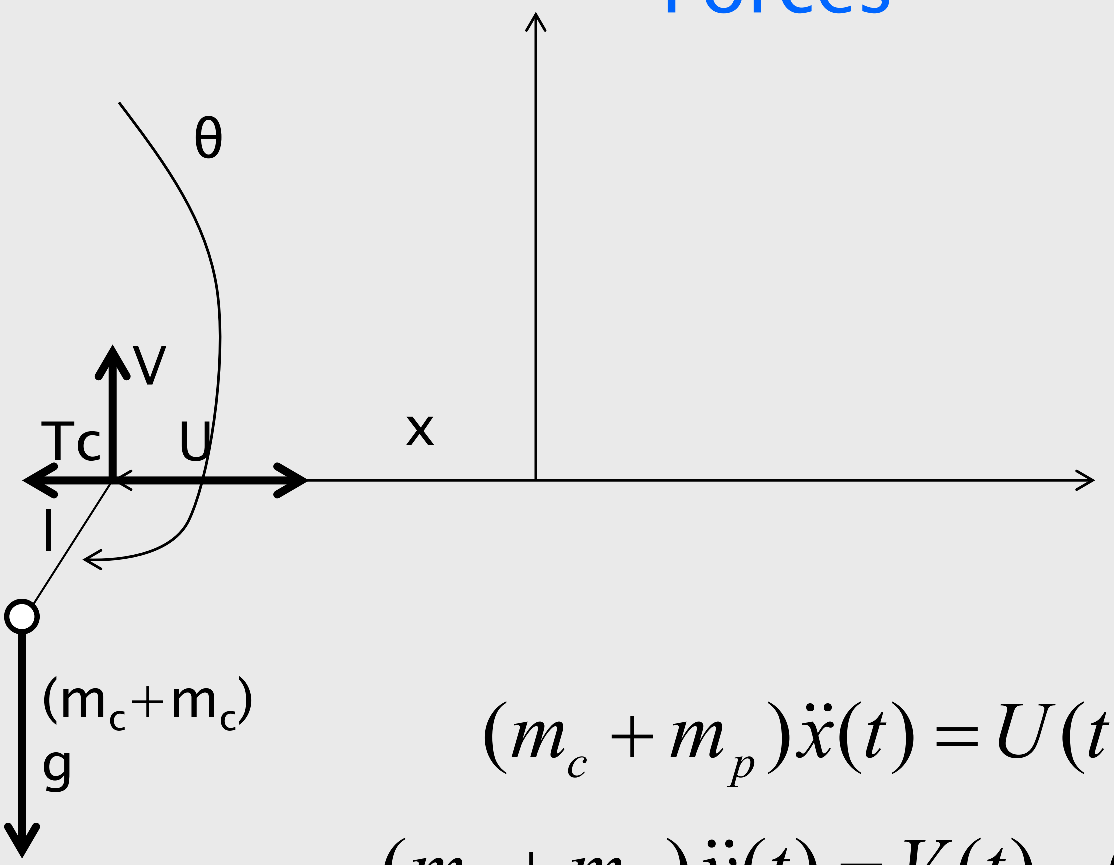


# Complete model simulation

$$U(s) = \frac{0.3}{s}$$



# Forces



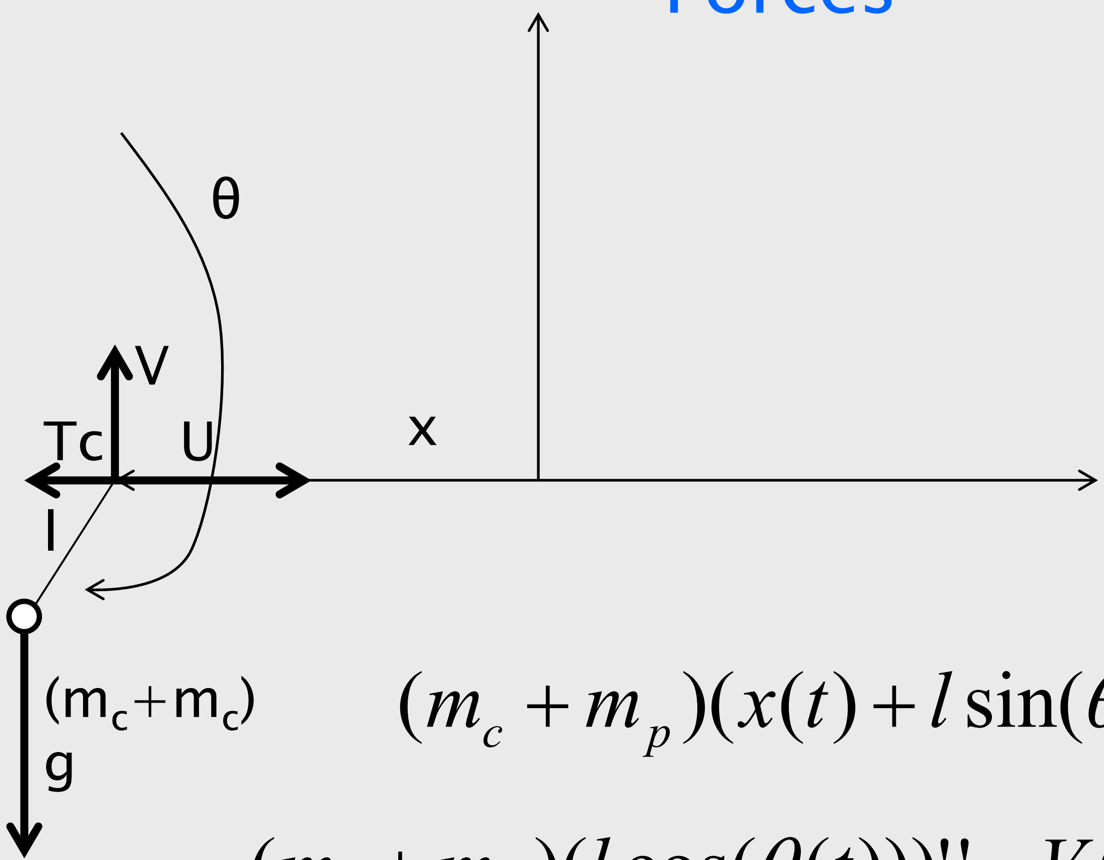

$$(m_c + m_p)\ddot{x}(t) = U(t) - T_c(t)$$

$$(m_c + m_p)\ddot{y}(t) = V(t) - (m_c + m_p)g$$

$$J\ddot{\theta}(t) = (U(t) - Tc(t))l\cos(\theta(t)) + V(t)l\sin(\theta(t) - D_p(t))$$

# Order?

## Forces




$$T_c = f_c \dot{x}$$

$$D_p = f_p \dot{\theta}$$

$$(m_c + m_p)(x(t) + l\sin(\theta(t)))'' = U(t) - T_c(t)$$

$$(m_c + m_p)(l\cos(\theta(t)))'' = V(t) - (m_c + m_p)g$$

$$J\ddot{\theta}(t) = (-U(t) + Tc(t))l\cos(\theta(t)) + V(t)l\sin(\theta(t) - D_p(t)$$

## State Space model

$$x_1 = x; x_2 = \theta; x_3 = \dot{x}; x_4 = \dot{\theta}$$

Non Linear

$$\dot{x}_1 = x_3$$

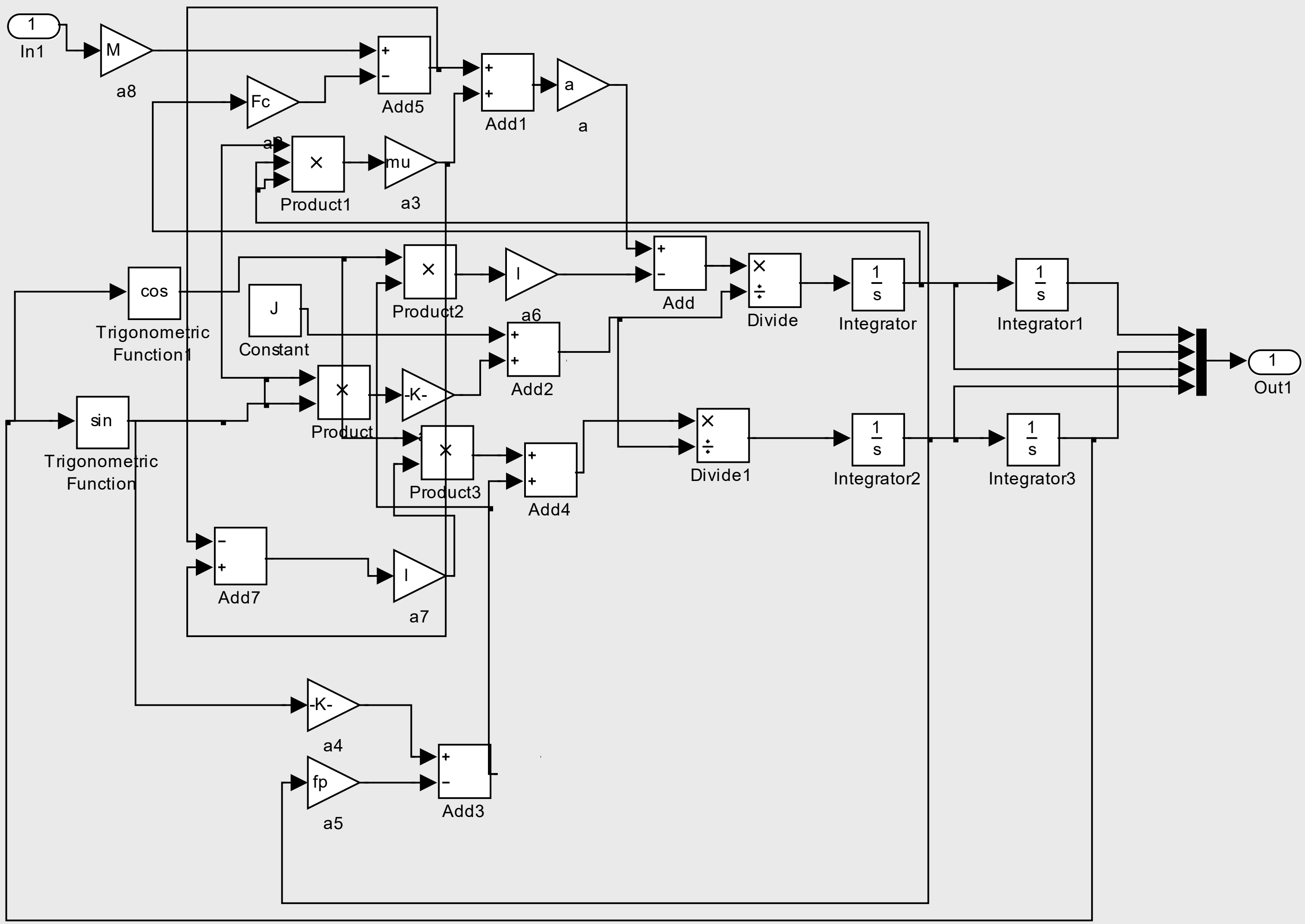
$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{a(U - F_c x_3 + \mu x_4^2 \sin x_2) - l \cos x_2 (\mu g \sin x_2 - f_p x_4)}{J + \mu l \sin^2 x_2}$$

$$\dot{x}_3 = \frac{l \cos x_2 (-U + F_c x_3 + \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu l \sin^2 x_2}$$

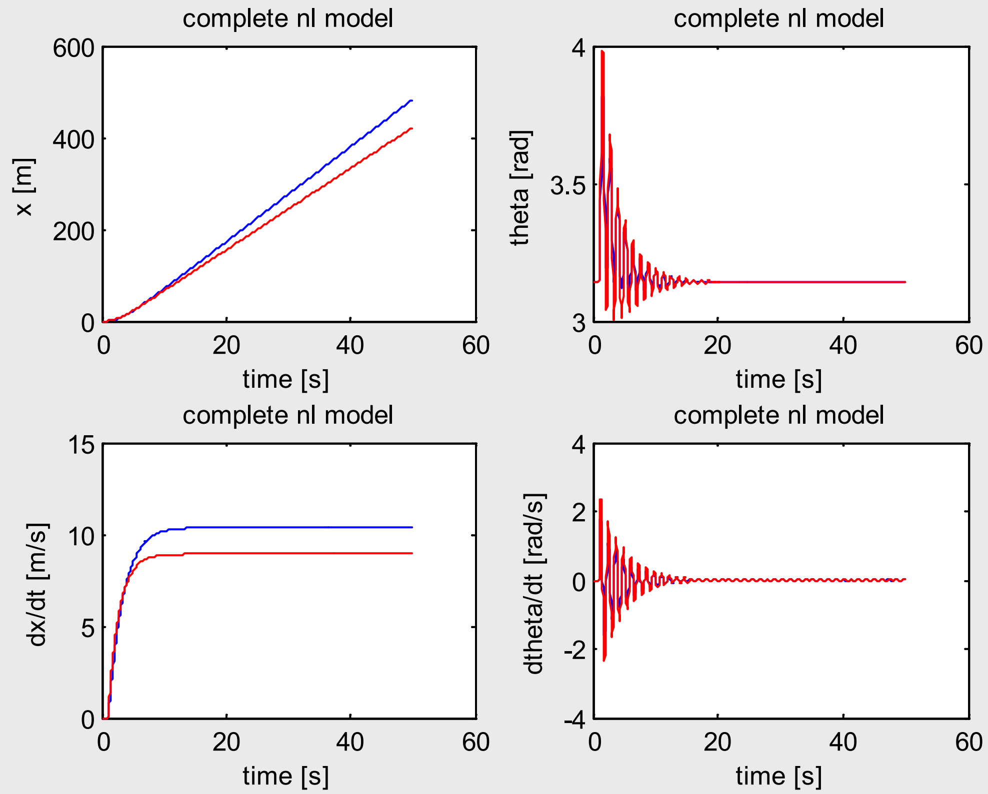
$$a = l^2 + \frac{J}{m_c + m_p}; \mu = (m_c + m_p) l$$

# Non-linear model



# Non-linear model simulation

$$U(s) = \frac{0.3}{s}$$



# Linearisation

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{a}{J}U - \frac{aF_c}{J}x_3 - \frac{l\mu g}{J}x_2 - \frac{lfp}{J}x_4$$

$$\dot{x}_4 = \frac{l}{J}U - \frac{lF_c}{J}x_3 - \frac{\mu g}{J}x_2 - \frac{fp}{J}x_4$$

# Linearisation

$$\dot{x}(t)=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{l\mu g}{J} & -\dfrac{aF_c}{J} & -\dfrac{lfp}{J}\\ 0 & -\dfrac{\mu g}{J} & -\dfrac{lF_c}{J} & -\dfrac{fp}{J}\end{bmatrix}x(t)+\begin{bmatrix}0\\ 0\\ \dfrac{a}{J}\\ \dfrac{l}{J}\end{bmatrix}u(t)$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}$$

# Linearisation

A =

$$A = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0 & -0.4480 & -0.5891 & -0.0085 \\ 0 & -26.6835 & -1.6790 & -0.5067 \end{bmatrix}$$

B =

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.7855 \\ 2.2387 \end{bmatrix}$$

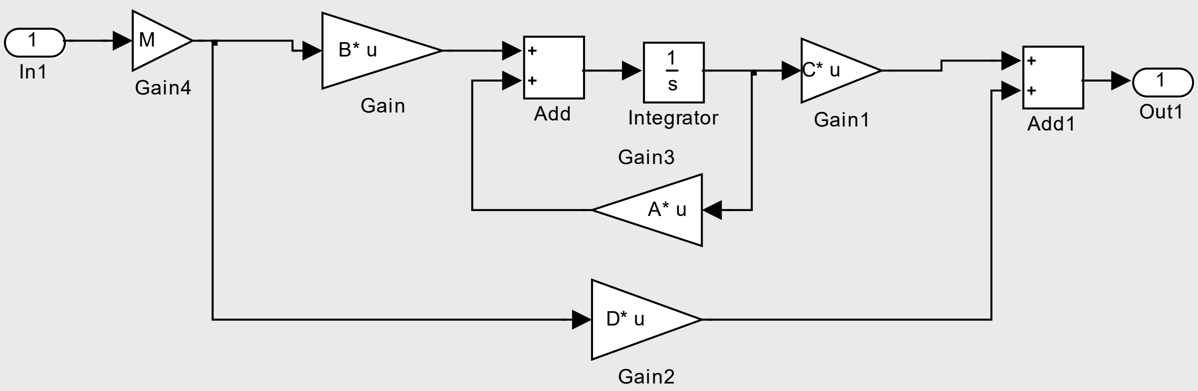
C =

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

D =

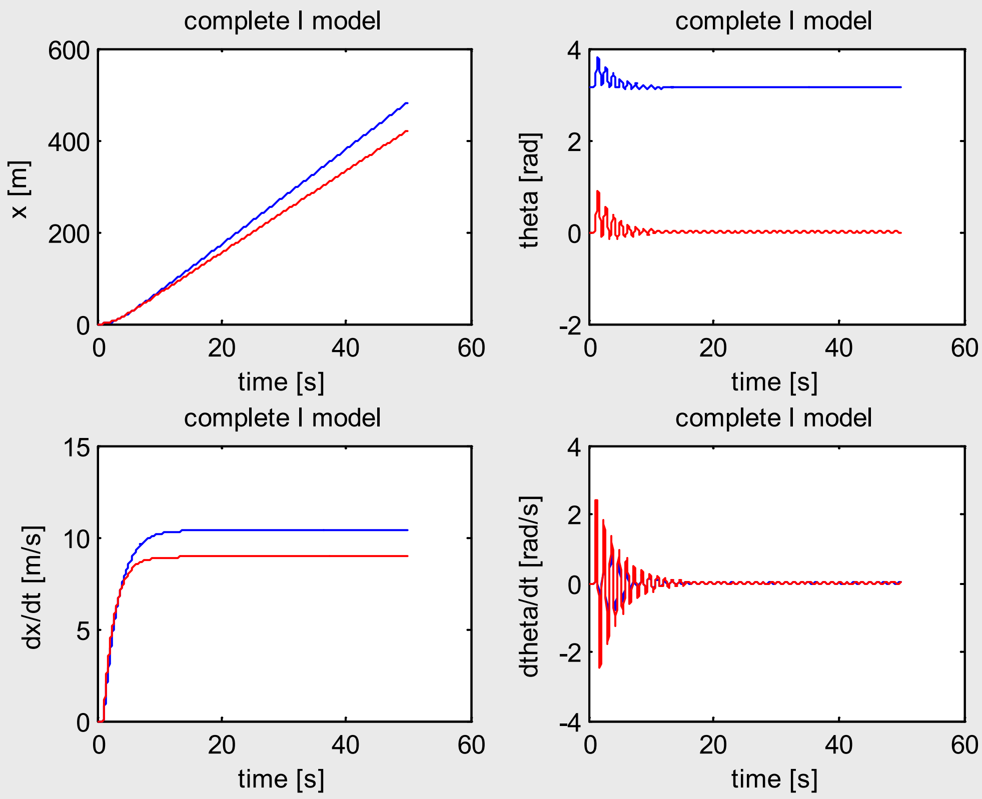
$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Linear model



# Linear model simulation

$$U(s) = \frac{0.3}{s}$$



# Transfer function

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

$$s\mathbf{x}(s) - \mathbf{x}(t=0) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{u}(s)$$

$$\mathbf{y}(s) = \mathbf{C}\mathbf{x}(s) + \mathbf{D}\mathbf{u}(s)$$

$$\mathbf{x}(s) = \left[s\mathbf{I}_n - \mathbf{A}\right]^{-1}\mathbf{B}\mathbf{u}(s)$$

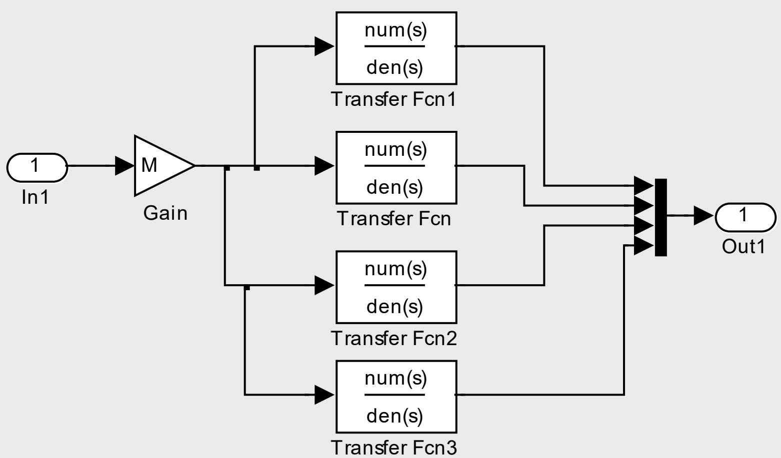
$$\mathbf{y}(s) = \left\{\mathbf{C}\left[s\mathbf{I}_n - \mathbf{A}\right]^{-1}\mathbf{B} + \mathbf{D}\right\}\mathbf{u}(s)$$

$$\mathbf{G}(s) = \mathbf{C}\left[s\mathbf{I}_n - \mathbf{A}\right]^{-1}\mathbf{B} + \mathbf{D}$$

```
[num,den]=ss2tf(A,B,C,D)

num =

         0    0.0000    0.7855    0.3789
19.9557
         0   -0.0000    2.2387   -0.0000         0


den =

    1.0000    1.0958   26.9677   14.9668
0
```

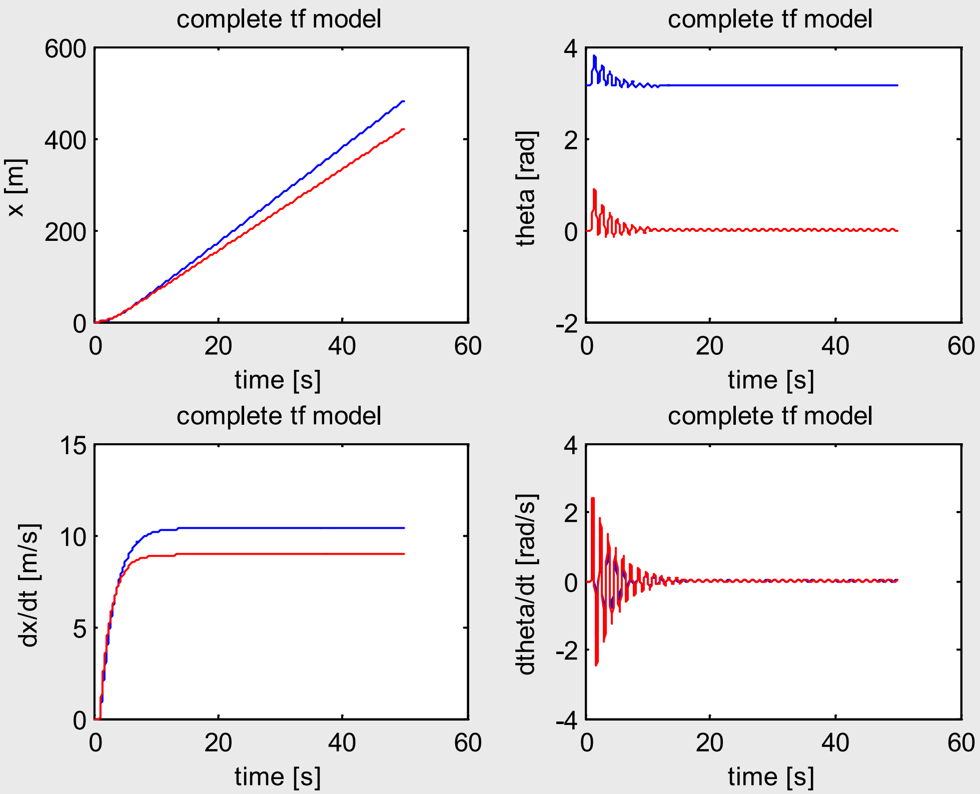
# Matrix transfer function

$$y(s)=\begin{bmatrix}\dfrac{num_{11}(s)}{den_{11}(s)}\\ \dfrac{num_{21}(s)}{den_{21}(s)}\end{bmatrix}u(s)=\begin{bmatrix}\dfrac{0.8s^2+0.4s+20}{s^4+s^3+27s^2+15s}\\ \dfrac{2.2s^2}{s^4+s^3+27s^2+15s}\end{bmatrix}u(s)$$



# Transfer function simulation

$$U(s) = \frac{0.3}{s}$$



# Poles

eig(A)

0
−0.2673 + 5.1572i
−0.2673 − 5.1572i
−0.5612

roots(den)

0
−0.2673 + 5.1572i
−0.2673 − 5.1572i
−0.5612

# Canonical form

```
[A,B,C,D]=tf2ss(num,den)

A =

   -1.0958  -26.9677  -14.9668         0
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0


B =

     1
     0
     0
     0

C =

    0.0000    0.7855    0.3789   19.9557
   -0.0000    2.2387   -0.0000         0


D =

     0
     0
```

# Discretisation

$$\mathbf{x}(k+1) = \mathbf{A_1}\mathbf{x}(k) + \mathbf{B_1}\mathbf{u}(k)$$

$$\mathbf{A}_1 = e^{\mathbf{A}T}$$

$$\mathbf{B_1} = \int e^{\mathbf{A}} d\tau \mathbf{B} = \left[\mathbf{A_1} - \mathbf{I}\right]\mathbf{A}^{-1}\mathbf{B}$$

```
A1=expm(A*0.1)
B1=(A1-eye(4))*inv(A)*B
```

```
A1 =

    1.0000   -0.0021    0.0971   -0.0001
         0    0.8718   -0.0079    0.0932
         0   -0.0405    0.9430   -0.0029
         0   -2.4843   -0.1519    0.8246
```

```
B1 =

   NaN
   NaN
   NaN
   NaN
```

# Discretisation

$$\mathbf{x}(k+1) = \mathbf{A_1}\mathbf{x}(k) + \mathbf{B_1}\mathbf{u}(k)$$

$$\mathbf{A}_1 = e^{\mathbf{A}T}$$

$$\mathbf{B_1} = \int e^{\mathbf{A}} d\tau \mathbf{B} = [\mathbf{A_1} - \mathbf{I}]\mathbf{A}^{-1}\mathbf{B}$$

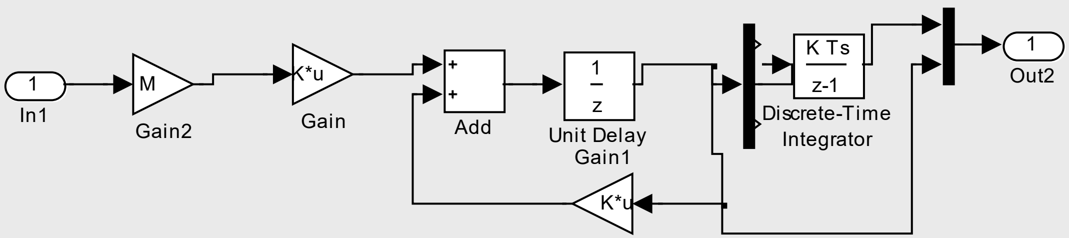
```
A1=expm(A(2:4,2:4)*0.1)
B1=(A1-eye(3))*inv(A(2:4,2:4))*B(2:4)
```

```
A1 =

    0.8718   -0.0079    0.0932
   -0.0405    0.9430   -0.0029
   -2.4843   -0.1519    0.8246
```

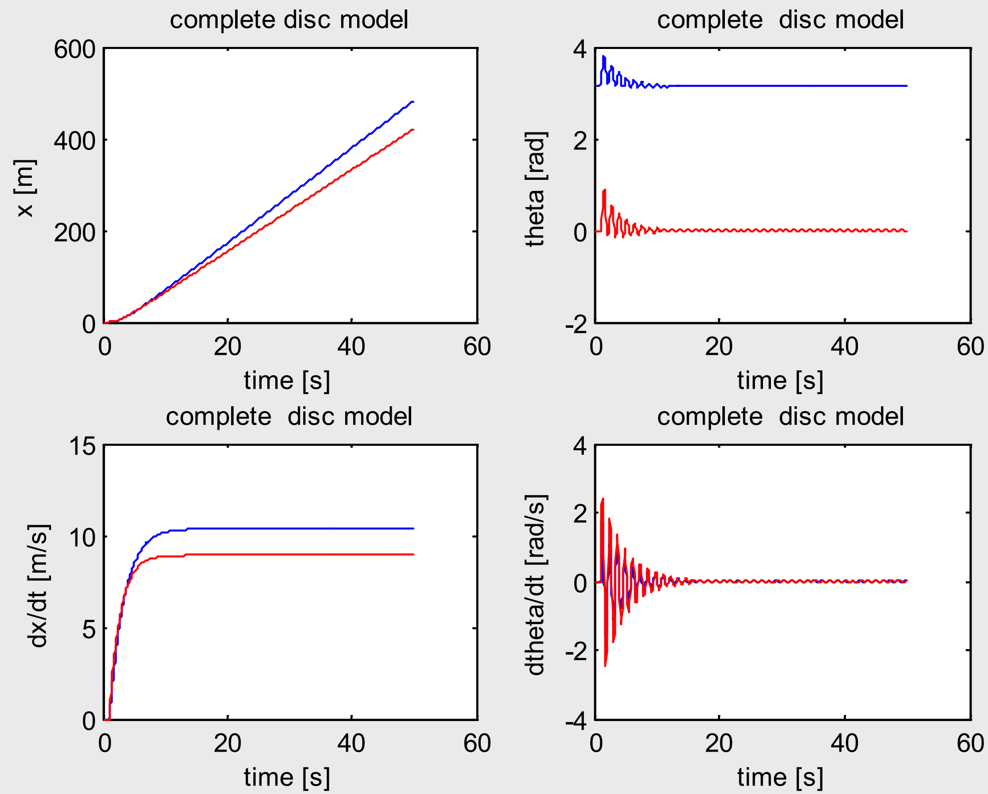
```
B1 =

    0.0106
    0.0760
    0.2025
```

```
eig(A1)

ans =

   0.8470 + 0.4802i
   0.8470 - 0.4802i
   0.9454
```

# Discrete model



# Discrete model simulation



# Controlability

- Controlability

P=[B,A*B,(A*A)*B,(A*A*A)*B]

$$P = \left[ B \vdots AB \vdots A^2 B \vdots \ldots \vdots A^{n-1} B \right]$$

rank(P)

$$rank(P) = n$$

```
P =

         0    0.7855   -0.4818   -0.6983
         0    2.2387   -2.4531  -57.6847
    0.7855   -0.4818   -0.6983    2.0011
    2.2387   -2.4531  -57.6847   95.8562

>> rank(P)

ans =

     4
```

# Observability

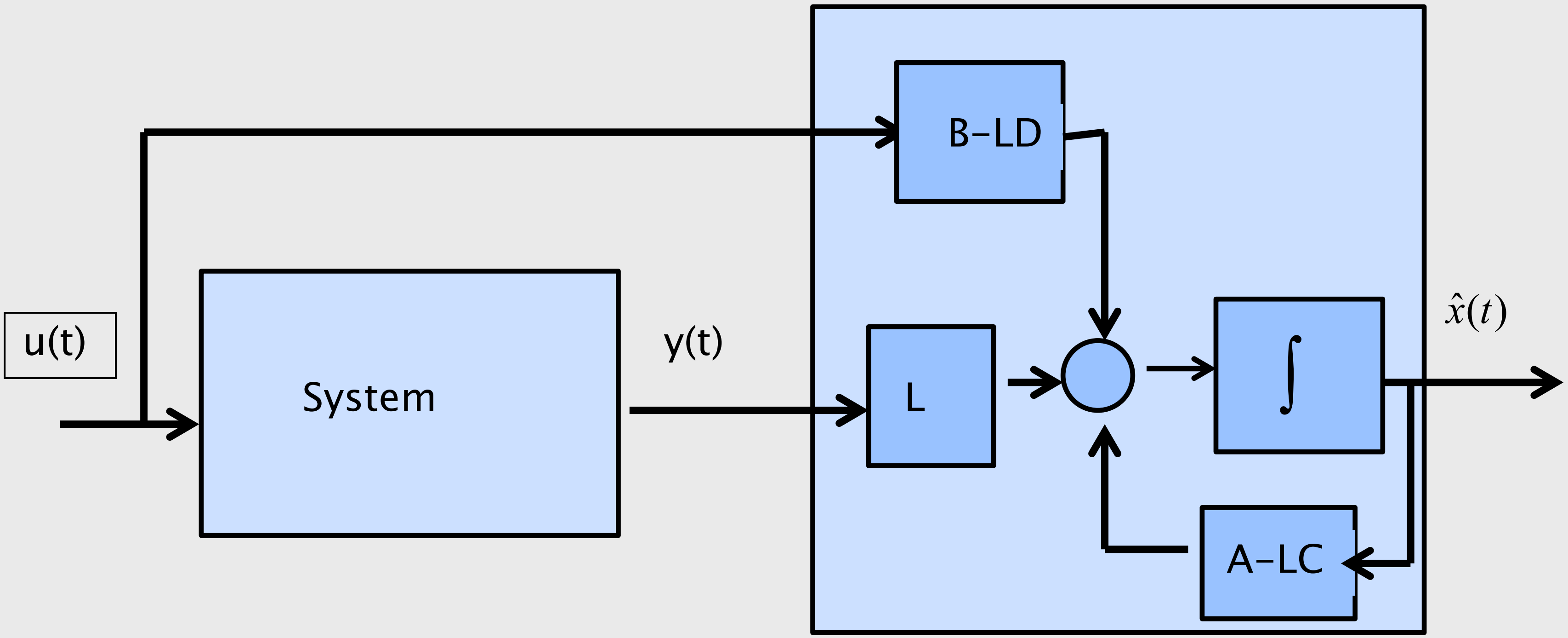
- Observability

$$Q = \left[ C^T \vdots A^T C^T \vdots A^{2T} C^T \vdots \ldots \vdots A^{n-1T} C^T \right]$$

$$rank(Q) = n$$

```
>> Q=[C',A'*C',(A*A)'*C',(A*A*A)'*C']

Q =

    1.0000         0         0         0
         0         0   -0.4480    0.4907
         0    1.0000   -0.5891    0.3613
         0         0   -0.0085   -0.4387

>> rank(Q)

ans =

     4
```

# State Observer



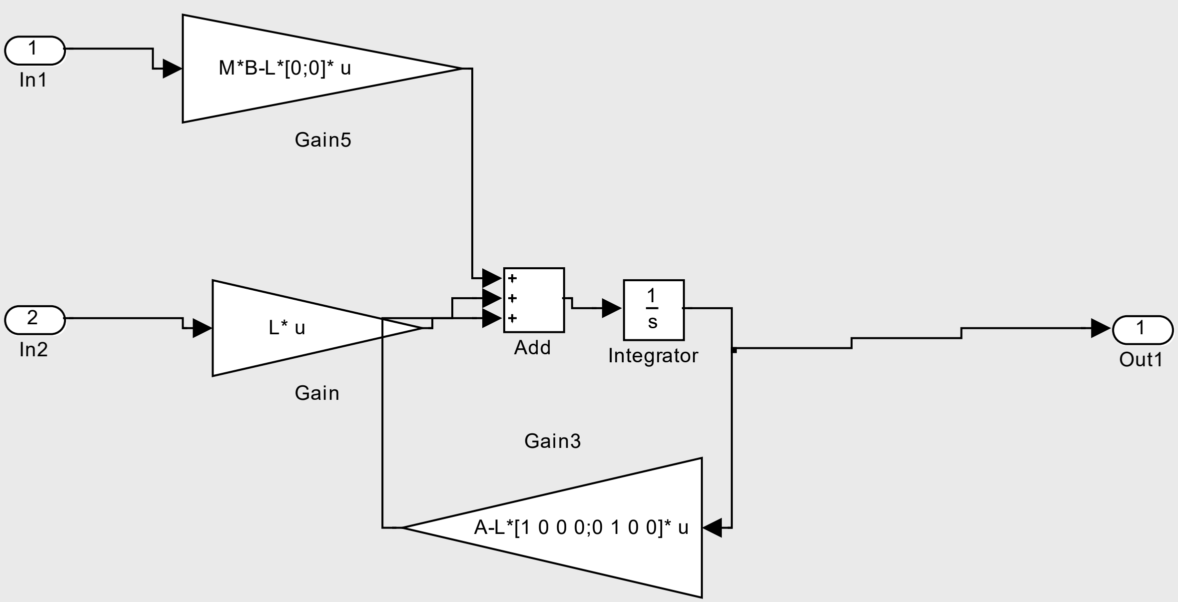

$$\dot{e} = (A - LC)e$$

$$A_c = A - LC$$

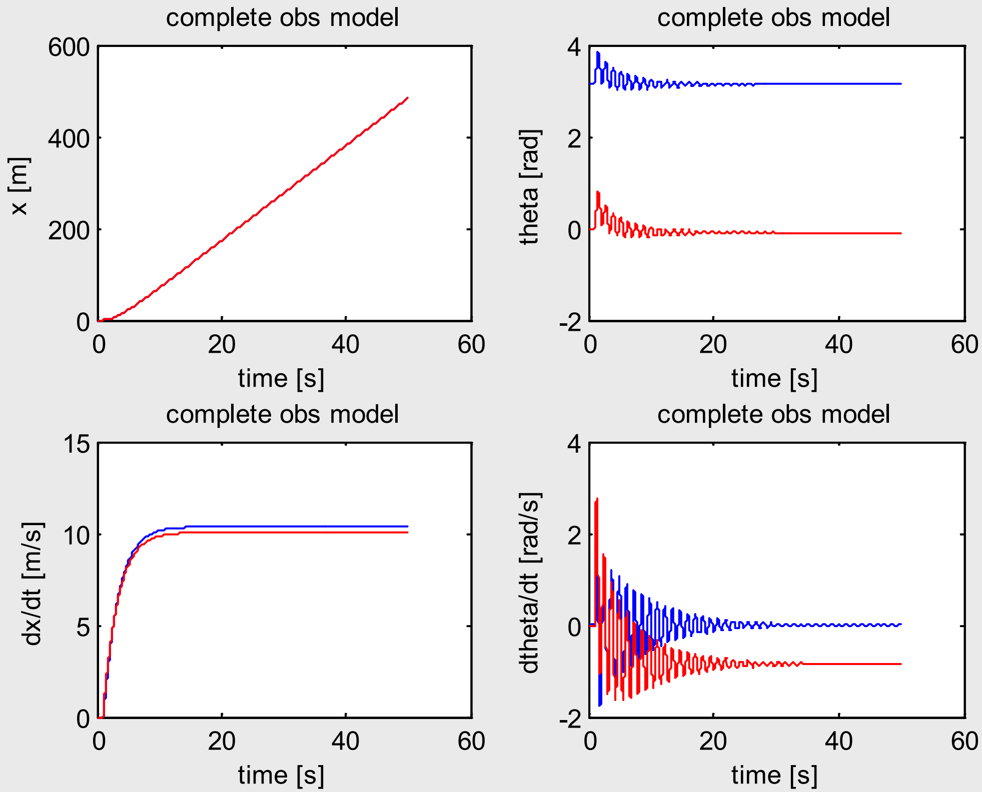
```
L=place(A',[1 0 0 0;0 1 0 0]',[-5,-5.1,-5.2,-5.3]);
L=L';
```

# State Observer

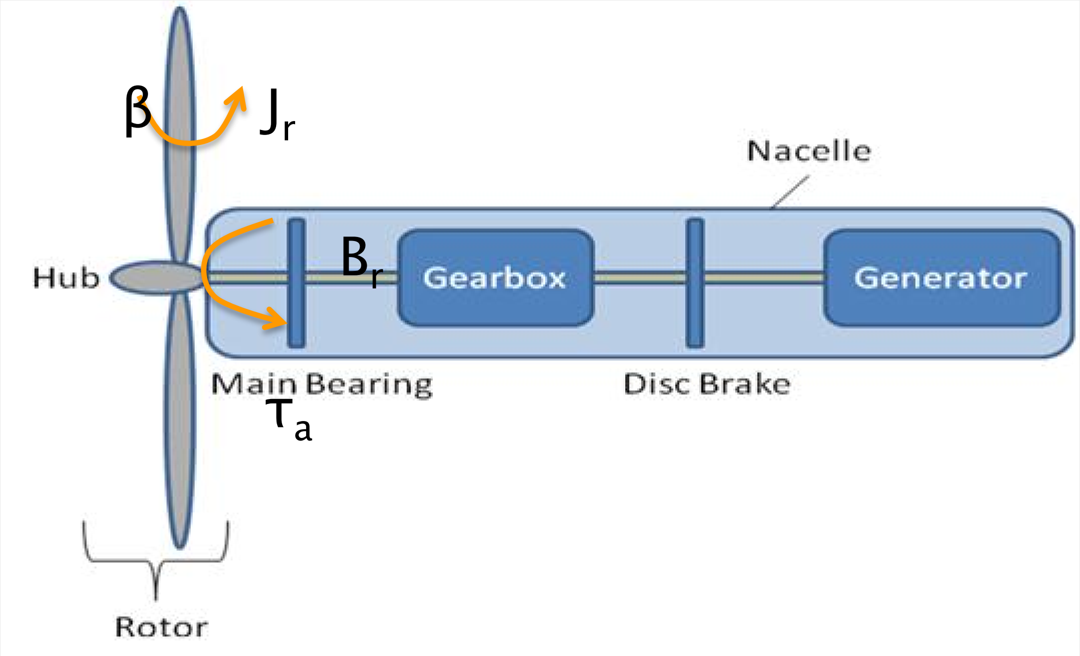
L=place(A',[1 0 0 0;0 1 0 0]',[-5,-5.1,-5.2,-5.3]);
L=L';



# State Observation



# Wind Turbine



# Rotation mechanism

$$J_r \dot{w}_r = \tau_a - B_r w_r - N \tau_{hs}$$

- $J_r$ inertia
- $W_r$ angular speed
- $B_r$ friction coefficient
- $\tau_{hs}$ required torque
- $N=w_g/w_r$ reduction
- $W_g$ high angular speed

# High speed

$$J_g \dot{w}_g = \tau_{hs} - B_g w_g - \tau_{em}$$

- $J_g$ inertia
- $W_g$ angular speed
- $B_g$ friction coefficient
- $\tau_{hs}$ required torque
- $\tau_{em}$ required torque electrical part
- $W_g$ high angular speed

# Global mechanical system

$$J_t \dot{w}_r = \tau_a - B_t w_r - \tau_g$$

- $J_t = J_r + N^2 J_g$
- $B_t = B_r + N^2 B_g$
- $\tau_g = \tau_{em}$

# Aerodynamics

$$\tau_a = \frac{1}{2}\rho\pi R^3 v^2 C_q(\lambda, \beta)$$

- R rotor radius
- v wind speed
- $C_q$ produced torque
- λ tip speed ratio
- β pitch angle

# Electric generator

$$P_g = K^2 \left( \frac{R_L}{R_L^2 + X_g^2} \right) I_f^2 N^2 w_r^2$$

- $P_g$ produced power
- $R_L$ resistive charge
- $X_g$ generator reactance
- $I_f$ intensity

# Electric generator

$$\tau_g = \frac{P_g}{\eta_g \eta_m w_g}$$

- $\eta_g$ electrical eficiency
- $\eta_m$ mechanical eficiency

## Global system

$$J_r \dot{w}_r = \tau_a - B_r w_r - N\tau_{hs}$$

$$\tau_a = \frac{1}{2}\rho\pi R^3 v^2 C_q(\lambda, \beta)$$

$$P_g = K^2 \left(\frac{R_L}{R_L^2 + X_g^2}\right) I_f^2 N^2 w_r^2$$

$$\tau_g = \frac{P_g}{\eta_g \eta_m w_g}$$

$$\beta(s) = \frac{K_\beta}{T_\beta s^2 + s + K_\beta}\beta_{ref}(s)$$

# Linearised model

$$\begin{bmatrix} W_r(s) \\ P_g(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-86{,}41}{15s + 0{,}37} & \dfrac{-10{,}89}{15s + 0{,}37}\dfrac{0{,}15}{2s^2 + s + 0{,}15} \\ \dfrac{29324{,}85s + 327{,}75}{15s + 0{,}37} & \dfrac{-51{,}07}{15s + 0{,}37}\dfrac{0{,}15}{2s^2 + s + 0{,}15} \end{bmatrix} \begin{bmatrix} I_f(s) \\ \beta_{ref}(s) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{15s + 0{,}37} \\ \dfrac{4{,}7}{15s + 0{,}37} \end{bmatrix} V(s)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0{,}025 & 0 \\ 0 & -0{,}025 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} I_f \\ \beta_{ref} \end{bmatrix}$$

$$\begin{bmatrix} W_r \\ P_g \end{bmatrix} = \begin{bmatrix} -0{,}72 & -0{,}03 \\ -3{,}3 & -1{,}7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1955 & 0 \end{bmatrix} \begin{bmatrix} I_f \\ \beta_{ref} \end{bmatrix}$$