Modelling, Identification and Simulation of Dynamic Systems

EXERCISES FOR SESSION 3: SYSTEM IDENTIFICATION

Report the results in an informal little document (txt or pdf) and upload it at the end of the session. Upload also the Matlab/Simulink files used to obtain the results. Upload them as **separated files**.

1. Consider the system represented by the model

$$y(t) = 0.8y(t-1) + 0.7u(t-1) + e(t) + 0.5e(t-1)$$

with $u(t) = 10step(t)$, $y(0) = 0$, $Ee(t) = 0$, $Ee^2(t) = 0.4$.

(a) Use the model to simulate the generation of a set of experimental data. The duration of this experiment, denoted by N, has to be four times the duration of the transient. Keep this data set in a safe place because only this data set can be used in the following steps when necessary. (Report value of N)(Report plot)(Data set has to be uploaded)

(b) Compute a one-step predictor. (Report computation of predictor expression)

(c) Compute the one-step prediction of the $y(t)$ in the experiment using the predictor.

(d) Compute the square root of the power of the prediction error, $\sqrt{V_N} = \sqrt{\frac{1}{N}\sum_N \varepsilon^2(t)}$. Normalize it by the square root of the power of the prediction, $\sqrt{\frac{1}{N}\sum_N \hat{y}^2(t)}$ in percent. (Report$\sqrt{V_N}$ and the one normalized in percent and comment if this value is consistent with the model)

(e) Compute the prediction of $y(t)$ considering that the model has no error and that only $y(0)$ and $u(t)$ are known. Show that this is the same predictor that is obtained if the model is assumed to be an Output Error model. (Report computation of predictor expression with given assumptions and OE predictor)

(f) Compare the simulation data and the previous two predictions in one single plot (Report plot with theoretical interpretation of differences).

2. The following model gives the velocity $v(t)$ of a landing drone as a function of the propeller signal $m(t)$,

$$v(t) = v(t-1) + T\frac{K_a m(t-1) - Mg}{M}$$

Parameter K_a characterizes the propeller, T is the sampling time, M is the mass of the drone and g is the gravitational constant. A velocity sensor gives a signal

$$c(t) = K_s v(t) + e(t)$$

where parameter K_s characterizes the sensor and $e(t)$ is zero mean white noise. Consider the auxiliary signal $\Delta m(t)$ defined in such a way that

$$m(t) = \frac{Mg}{K_a} + \Delta m(t)$$

(a) Can the relation between $\Delta m(t)$ and $c(t)$ be embedded into an OE model structure? Compute the parameters of the OE structure as a function of the physical ones. (Report model)

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(b) Compute the one-step predictor for the OE model structure.(Report predictor)

(c) Can the relation between $\Delta m(t)$ and $c(t)$ be embedded into an ARMAX model structure? Compute the parameters of the ARMAX structure as a function of the physical ones.(Report model)

(d) Compute the one-step predictor for the ARMAX model structure. (Report predictor)

(e) Use the model to simulate an experiment so that a set of experimental data of duration $10s$ is obtained. The experiment has to be the following one: initial conditions: $v(0) = -2m/s$, inputs: $\Delta m(t)$ has to be such that during the experiment the drone tends to stop, i.e. velocity tends to 0 monotonically; the noise signal $e(t)$ is a zero mean uniformly distributed random signal that is in the range $[-0.2, 0.2]V$. The parameters of the model are $M = 1kg$, $K_a = 1N/V$, $K_s = 1V/m$ and $g = 9.8m/s^2$. The sampling time is chosen so that the significant dynamics of the evolution of the drone is captured. (Report T)(Report plot)(Data set has to be uploaded)

(f) Compute the one-step prediction of the $c(t)$ in the experiment using the OE predictor and compare it with the experimental one. Give a quantitative measure of the prediction error. (Report plot and measure)

(g) Compute the one-step prediction of the $c(t)$ in the experiment using the ARMAX predictor and compare it with the experimental one. Give a quantitative measure of the prediction error. (Report plot and measure)

3. Consider the system represented by

$$y(t) = 0.8y(t-1) + 0.7u(t-1) + e(t)$$

where $e(t)$ is gaussian white noise with zero mean and variance 0.4.

(a) Use the model to simulate the generation of a set of experimental data considering that the input $u(t)$ is gaussian white noise with zero mean and variance 0.4 (generated by a random signal generator). The duration of this experiment has to be chosen according to the dynamics of the system. It is denoted by N. (Report value of N and used criterion for choice)

(b) Use the data of the experiment to give an estimate $\hat{\theta}_N, \theta = (a, b)^T$ of the values of the parameters within the model structure

$$y(t) = -ay(t-1) + bu(t-1) + e(t)$$

by minimizing the power of the prediction error.

(c) Compute the square root of the power of the prediction error, $\sqrt{V_N} = \sqrt{\frac{1}{N}\sum_N \varepsilon^2(t)}$. Normalize it by the square root of the power of the prediction, $\sqrt{\frac{1}{N}\sum_N \hat{y}^2(t)}$ in percent. (Report$\sqrt{V_N}$ and the one normalized in percent and comment if this value is consistent with the model)

(d) Generate 99 experiments more and compute the distribution, the mean and the variance of the estimated parameters. (Report figure of distribution, mean and variance of estimated parameters)

(e) Choose a factor by which you would like to reduce the variance of the parameters. (Report chosen factor and reason for it)

(f) Repeat the previous study changing only the variance of $u(t)$ until the variance of the parameters reduces by the previously chosen factor. (Report new variance of $u(t)$, figure of parameter distribution, mean and variance)

(g) Repeat the previous study changing only the duration of the experiment until the variance of the parameters reduces by the previously chosen factor. (Report new duration, figure of parameter distribution, mean and variance)

(h) Repeat the previous study changing the variance of $u(t)$ and the duration of the experiment until the variance of the parameters reduces by the previously chosen factor. (Report new variance of $u(t)$, new duration, figure of parameter distribution, mean and variance)