Final Assignment

Identification of a quadrotor system

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ETSEIB

Question 1

Using both the systemIdentification MATLAB toolbox and the role/position data from the quadrotor system one can create three sets of data depicted in the figure 1 below:

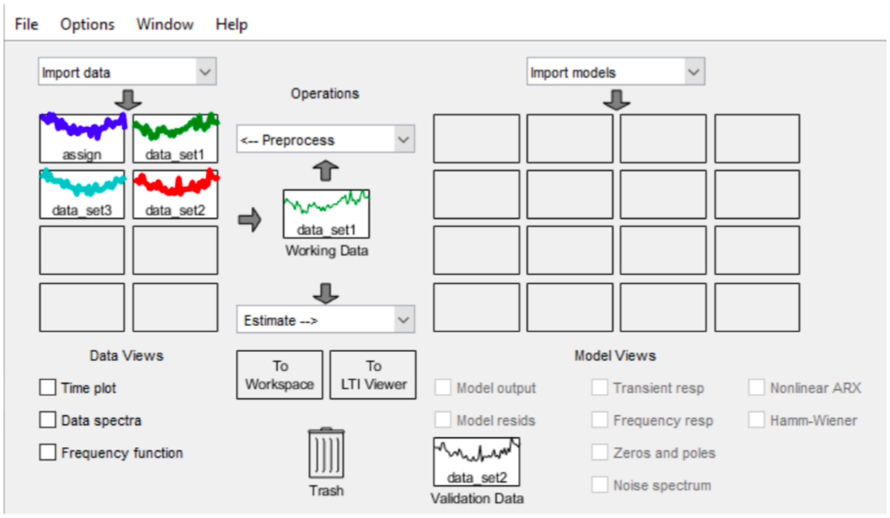

Figure 1: Identification, validation, final model comparison sets of data

Remark 1. *Let us note that the first set of data was used as "working data" and that the data to predict (reference data) is here considered as being the second set of data.*

Have in mind that the considered input is the ref-pitch called "mr" in the previous work MATLAB code, and the output is v_y.

Question 2

To estimate the delay between the input and the output, one should generate a step-response plot using a "Quick-Start" process to have various models estimation. Then, selecting the transient response check box, it displays the left plot of Figure 2 and if we make a zoom (right picture), one can evaluate the delay.

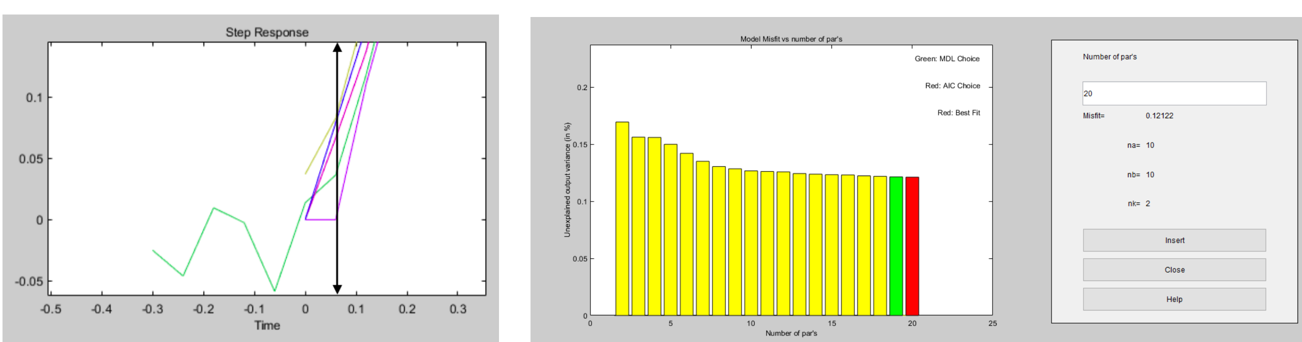

Figure 2: Delay's estimation, step input & linear ARX model structure

The previous step-response (left picture) plot shows a time delay of about 1 sample (0,06s) before the system responds to the input. However one can see the presence of noise which is distorting the response, it follows the use of the linear ARX model structure to get a more relevant approximation of that delay. From the right picture of Figure 2, the delay would be about 2 samples (0,12 seconds). Let us therefore choose a delay of 2 samples.

Question 3

Using as an estimation the linear ARX model structure let us specify the following parameters :

- Orders=[1:10 1:10 1:10] % Order tuning involves the comparison of various order models.

It gives us the right graph of Figure 2 and Figure 3:

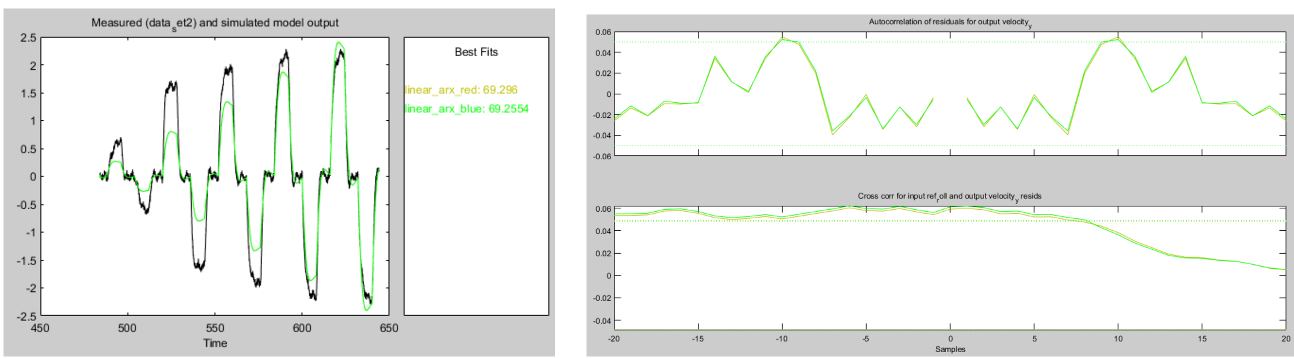

Figure 3: Performance (left picture) and residual (right picture) analysis

According to the MATLAB tutorial, the best fit minimizes the sum of the squares of the difference between the validation data output and the model output. According to Figure 2 the red rectangle (corresponding to n_a= 10 n_b=10 and n_k=2) indicates the overall best fit as well as the "AIC" choice[1]. Finally from Figure 3, one should see the comparison between the red and the blue model from Figure 2. From the residual[2] and performance analysis, the red model (order 2 for n_k) is more accurate than the blue model (order 1 for n_k). It therefore results in the choice of (the red model) an order 10 for n_a and n_b and a second order for n_k.

Question 4

From what is above, follows the use of the model of Hammerstein-Wiener. Analogously to the previous question, one can use the performance and residual analysis to design the model order, it results in the following tuning parameters[3]:

- n_a= 10 n_b=2 and n_k=2

[1] the "AIC" criteria should be applied to select the more precise model order when one uses the same data set for estimation and validation.

[2] Residuals represent the portion of the validation data not explained by the model, they need to be close to 0

[3] it will be left to the reader to carry out the analysis which is strictly similar to that of question 3

However, using the MATLAB Identification toolbox one should use the non-linearity option to improve its model, indeed, knowing that the system includes saturation let us use, as described below, the saturation mode for the input and therefore improving the model's fit.

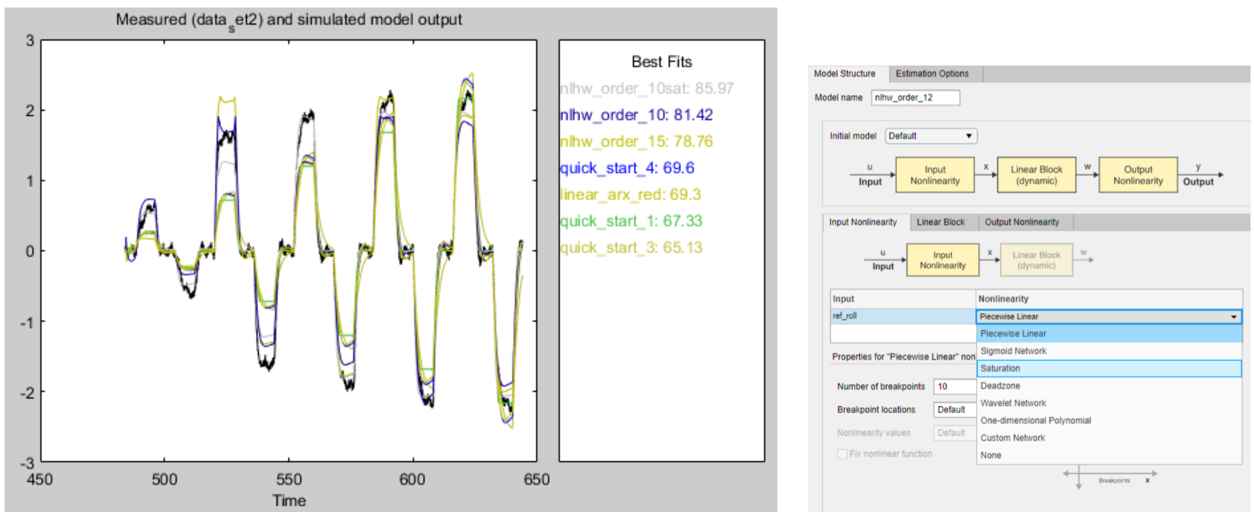

Figure 4: Hammerstein-Wiener model prediction tuning

From above, one should remark that the model using the previous order parameters (n_a, n_b and n_c) and the saturation mode has a best prediction than the one with an input linear mode and the one with a higher number of breakpoints (called nlhw_order_15). The final chosen parameters will therefore be n_a= 10 n_b=2 n_k=2 and a non-linearity saturation.

Question 5

Let us choose this time a nonlinear ARX model. We can choose various type of regressors and output functions let us state that many experiments were carried out (regarding the residual and performance analysis) and they resulted in the following choices :

- Regressor vector type=linear
- non-linear output function type = Wavelet network

It is important to mention that the wavelet network is a function that uses a sum of a linear function and a nonlinear function. The number of wavelets used by the function is configured to be chosen automatically equal to 10. As seen in Figure 5, let us remark that after having applied a dichotomy methodology a tuned parameter is finally 10, allowing the model's prediction to reach 85,61% of prediction focus.

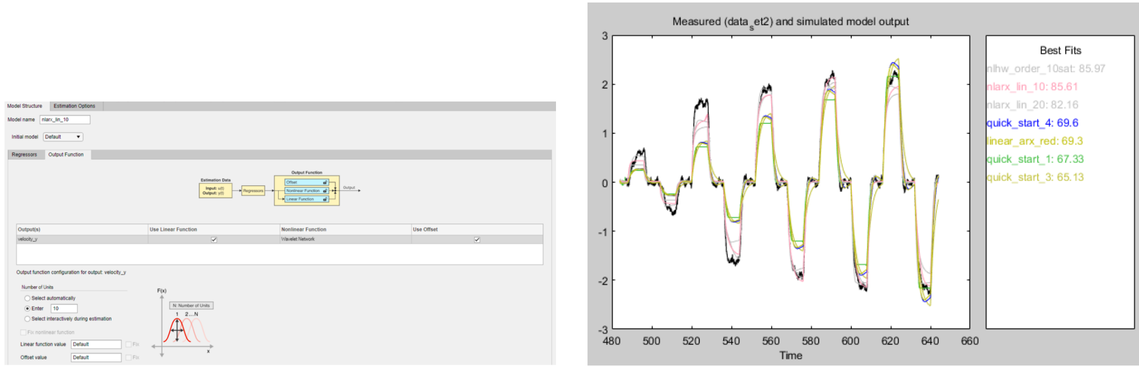

Figure 5: Non-linear ARX model, entry of the parameters and estimation

Question 6

The four models of Quick_start[4] 4,3,2 and 1 are respectively an impulse step response, a spad-frequency response, a fourth order autoregressive (ARX) model and a state-space model.

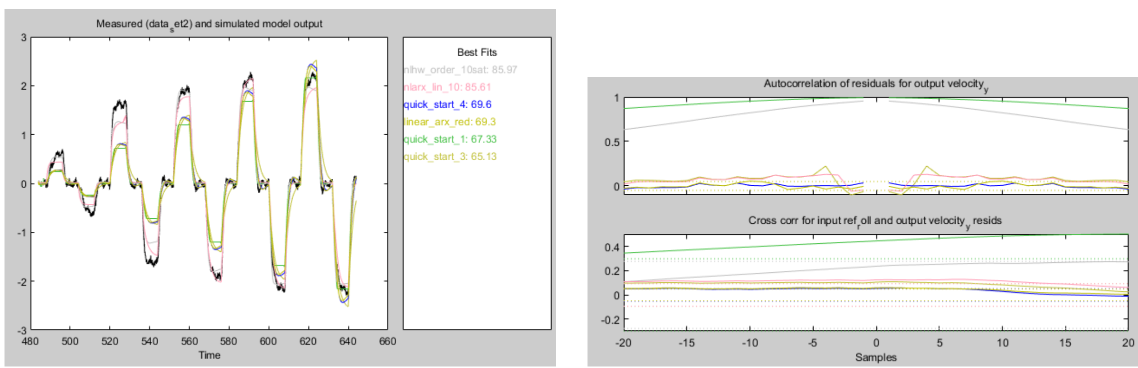

Figure 6: Performance (left picture) and residual (right picture) analysis, all model's comparison

From Figure 6, the simplest model that best describes the dynamics and successfully simulates or predicts the output for different inputs would be either the non_linear Hammerstein-Wiener model (best fit) or either the non-linear ARX model (2^{nd} best fit and lower residuals). Let us look at other criteria in order to make the most relevant choice.

[4]They were noted as n4s2, imp, arxqs and spad in the previous section

Table 6.1 FPE, MSE, AIC and BIC criterion, models comparison

Model	*FPE*	*MSE*	*AIC*	*BIC*
non-linear Hammer+saturation model	0,0253	0,02	-3,8857	-2,6e(+03)
non-linear ARX model	0,00165	0,00163	-6,4069	-9,43e(+03)
linear ARX model	0,001517	0,001485	-6,4969	-9,6e(+03)
quick_start_4	0,001498	0,001489	-6,5036	-9,73e(+03)
quick_start_3	0,001892	0,001876	-6,2699	-9,1e(+03)
quick_start_2	-	-	-	-
quick_start_1	-	-	-1,8164	3,26e(+03)

Remark 2. *Have in mind that to get Table 6.1, all models were exported to the workspace.*

One should know the following information:

- According to Akaike's theory, the most accurate model has the smallest AIC and BIC.
- The prediction error (FPE) needs to be the closest possible to 0
- The mean square error (MSE) needs to be the closest possible to 0

According to Table 6.1, the choice of the model is a tedious task, indeed depending on the criterion we favour it could be either the non-linear ARX or Hammerstein-Wiener model or either the linear ARX model or either the quick_start_4 model[5]. Now, from [1], let us state that the FPE criteria estimates multiple Output-Error (OE) models to pick the one with optimal trade-off. However, in our case most models are derived from ARX models, it should therefore not be prioritized. Moreover, from [3], the MSE criterion may match the irregularities of the measured data instead of the desired target function. Finally, from [2], the "BIC" criteria has the drawback of under selecting in finite samples, which is why AIC is often preferred in applications. Regarding all and prioritizing the "AIC" criteria, let us finally choose the non-linear Hammerstein-Wiener model. To conclude with, let us validate the chosen model comparing all the models with the validation data denoted as data_set_3 in the Figure 1.

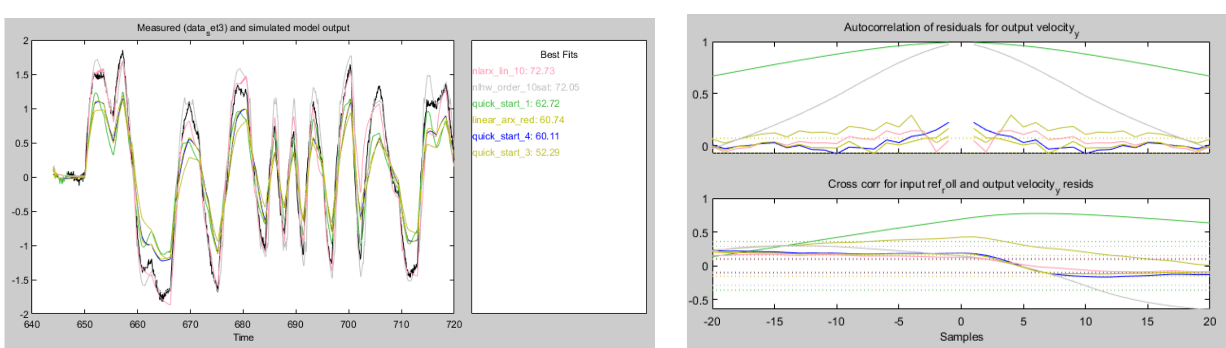

Figure 7: Performance (left picture) and residual (right picture) analysis, final's model choice validation

[5]Note that the quickstart models are not plot to be considered in the final choice but to help in the understanding of how the predictions work

According to Figure 7, the non-linear ARX model should, in the end, be the one to consider.

Remark 3. *Keep in mind that other criteria such as AICc, nAIC, CAIC and HQC could have been used, but this would have required a more in-depth study and therefore additional conclusions and comparisons, which would have added a few pages to this report which should be synthetic. However when one uses all the derived AIC criteria (AICc,nAIC,...), it results in an analogous percentage difference order as for the "AIC" criterion.*

Question 7

One should have noticed that in Question 4, the non-linear output parameter was not tuned, leading maybe to have a less accurate Hammerstein-Wiener model. Have in mind that, after carrying out some experiments, one should consider a saturation non-linear output instead of a linear one. Indeed, it should be able to map accurately the output of the linear block taking, as well as the input, into account the saturation. When using the performance and residual analysis one should know that the non-linear Hammerstein-Wiener model has now the best fit (over 86,81%) with an "AIC" criteria of -3,62. However, as one can see in the Figure 8, when one wants to choose one model among the three proposed model comparing them to the data-set 3, it is again the non-linear ARX model that offers the best fit (considering the performance and residual analysis). No other parameters were found to be adjustable due to a thorough initial methodology.

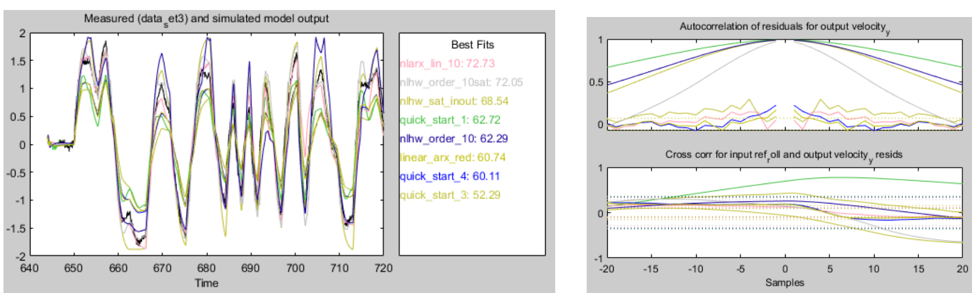

Figure 8: Performance (left picture) and residual (right picture) analysis, all model's comparison

References

[1] MATLAB identification toolbox

[2] https://stats.stackexchange.com/questions/219747/what-selection-criteria-to-use-and-why-aic-rmse-mape-all-possible-model-s

[3] *Model Order Selection Techniques for the Loop Filter Design of Virtual String Instruments*, Author: Cumhur ERKUT, Helsinki University of Technology