

EA5596 #6 Matthew Sah.

Q1)  $Ax = b$

$QRx = b$  - QR decomposition

$Rx = Q^T b$  Orthogonalize

$x = R^{-1} Q^T b$  Forward Substitution

Q2) SVD

$$x = A^{-1} b + A^{-1} \sigma b$$

for least square estimation

use  $x$  with smallest  $|x|^2$  such that  $b$  is in the range  $A$

$x$  minimizes  $R = \|Ax - b\|$

(2)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{aligned} \rightarrow AA^T &= -\lambda^3 + 10\lambda^2 - 16\lambda = -\lambda(\lambda^2 - 10\lambda + 16) \\ &= -\lambda(\lambda - 8)(\lambda - 2) \end{aligned}$$

$$\lambda = 8, 2, 0.$$

Singular values are  $2\sqrt{2}, \sqrt{2}, 0$ .

$$\Sigma = \begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For  $\lambda = 8$ , eigenvector =  $(1, 2, 1)$

$$\rightarrow P_1 = \left( \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$$\lambda = 2 \rightarrow P_2 = \left( -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right)$$

$$\lambda = 0 \rightarrow P_3 = \left( \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \right)$$

Q4

VB.png  $\rightarrow$  50. At rank 10, there is still obvious noise in the photo, with the white not so white, but at 50 this issue is solved.

fururamer.png  $\rightarrow$  100

Being the most complicated photo of the set (most colorful), it reaches minimum rank at 100, at 50 there was still easily noticed noise all around,

square  $\rightarrow$  2.

Since this photo is mainly black, it's not too difficult to reach minimum rank. at rank = 1, there are ~~printing~~ printed out of the original box, for rank 2, the box becomes a solid square with nothing poking out