##EAS596 HW4 Matthew Sah #.

1. a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

when $b_1 = b_2 = b_3$, system is stable.

$x_1 + 4x_2 + \cancel{3x_3}\ 2x_3 = b_1$

$2x_1 + 8x_2 + 4x_3 = b_2 \rightarrow x_1 + 4x_2 + 2x_3 = \frac{1}{2}b_2$

$-1x_1 + -4x_2 + -2x_3 = b_3 \rightarrow x_1 + 4x_2 + 2x_3 = -b_3$

b) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

when $b_1 = b_3$ system is stable.

$x_1 + 4x_2 = b_1$

$2x_1 + 9x_2 = b_2$

$-1x_1 + -4x_2 = b_3 \rightarrow x_1 + 4x_2 = -b_3$

2. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{cases} an_1 + bn_2 = 0 \\ cn_1 + dn_2 = 0. \end{cases}$

$\alpha\begin{bmatrix} a \\ c \end{bmatrix} + \beta\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} \alpha a + \beta b \\ \alpha c + \beta d \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \alpha a + \beta b \\ \alpha c + \beta d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

$a(\alpha a + \beta b) + b(\alpha c + \beta d) \Rightarrow c(\alpha a + \beta b) + d(\alpha a + \beta d) = 0$

$c(\alpha a + \beta b) + d(\alpha c + \beta d) \Rightarrow (a - c)(\alpha a + \beta b) + (b - d)(\alpha c + \beta d) = 0$

if $k = 1$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

let $\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} n_1 \\ 0 \end{bmatrix}$

$\rightarrow an_1 = 0$. $\quad a = c = 0$.

$cn_1 = 0 \quad \alpha c + \beta d = 0 \quad d = 0$.

$\begin{bmatrix} 0 & k \\ 0 & 0 \end{bmatrix}$ can be a matrix that nullspace = column space.

3. If null space = column space

then $\dim Null(A) + \dim Col(A) = n$. $\dim Null(A) = \dim Col(A)$.

n will not be odd number.

no 3x3 matrix will have null space the equals it's column space.

4. a) $\begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ rank = 1

nullity = 4-1=3

b) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -4 & -6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. a) $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 4 & 8 & 16 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 9 & -4.5 \\ 1 & 3 & -1.5 \\ 2 & 6 & -3 \end{bmatrix}$ c) $\begin{bmatrix} a & b \\ c & \frac{c}{a}b \end{bmatrix}$

6. a) $\begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}$

when $c=0, d=2$

rank = 2

b) $\begin{bmatrix} c & d \\ d & c \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

rank = 2

7. a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

b) let $B = (a, b, c)$

$a+b+c=0$

$2a+b=0$

let $a=1$

$b=-2$ $c=1$

$B = (1, -2, 1)$.

c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{matrix} 0 \\ 0 \end{matrix}$

8. a) $r \le m,\ r \le n$

since there is no solution

r will not equal m

$r < m,\ r \le n$

b)

$|N(A^T)| = m - r > 0.$

9 $a = (a_1 . a_2),\ b^T = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$a \otimes b^T = \begin{bmatrix} a_1 b_1 & a_2 b_1 \\ a_1 b_2 & a_2 b_2 \\ a_1 b_3 & a_2 b_3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 & a_2 b_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

row 2 $= \frac{b_1}{b_2}$ row 1 row 3 $= \frac{b_1}{b_3}$ row 1 ... row $(i) = \frac{b_1}{b_3}$ row 1

will always be rank 1.