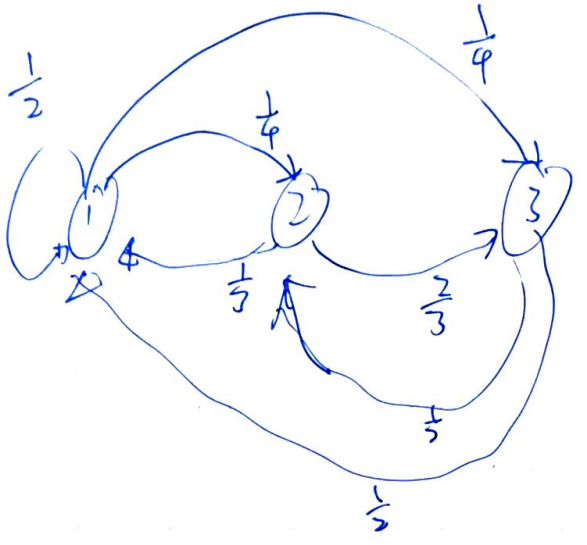
EAS595

Extra 1.



Find $P(X_1=3, X_2=2, X_3=1)$

$= P(X_1=3)\, P(X_2=2 \mid X_1=3)\, P(X_3=1 \mid X_2=2, X_1=3)$.

$= P(X_1=3)\, P(X_2=2 \mid X_1=3)\, P(X_3=1 \mid X_2=2)$ ← by markov's property.

$= \frac{1}{2} \times P_{32} \times P_{21}$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

Find $P(X_3=2)$

$= \frac{1}{4}$.

Find Steady State.

$$[X_1, X_2, X_3]^T \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix} = [\text{Steady}_1, \text{Steady}_2, \text{Steady}_3]$$

X_1 X_2 X_3

$X_1 + X_2 + X_3 = 1$

→

$$\frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{2}X_3 = X_1$$

$$\frac{1}{4}X_1 + \frac{1}{3}X_3 = X_2$$

$$\frac{1}{4}X_1 + \frac{1}{2}X_2 = X_3$$

$$\frac{1}{4}X_1 + \frac{1}{3}\left(\frac{1}{4}X_1 + \frac{1}{2}X_2\right) = X_2$$

$$\frac{9}{16}X_1 = X_2$$

$$\frac{1}{4}X_1 + \frac{2}{3}\left(\frac{1}{4}X_1 + \frac{1}{2}X_3\right) = X_3$$

$$X_1\frac{17}{24} = X_3$$

$$X_1 + \frac{9}{16}X_1 + \frac{17}{24}X_1 = 1$$

$$X_1 = 0.5$$

→ $X_3 = 0.22$. $X_2 = 0.28$

Steady state = (0.5 0.28 0.22)

Extra 2.

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$\rightarrow [X_1\ X_2\ X_3] \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{matrix} \frac{1}{4}X_1 + \frac{1}{3}X_2 + \frac{1}{2}X_3 = X_1 & (1) \\ \frac{1}{2}X_1 - X_2 & (2) \\ \frac{1}{4}X_1 + \frac{2}{3}X_2 + \frac{1}{2}X_3 = X_3. & (3). \end{matrix}$$

$$\frac{1}{2}X_3 = \frac{1}{4}X_1 + \frac{1}{3}X_2 \Rightarrow X_3 = \frac{14}{12}X_1$$

$$X_2 = \frac{1}{2}X_1$$

$$\therefore \frac{3}{4}X_1 = \frac{1}{3} \times \frac{1}{2}X_1 + \frac{1}{2} \times \frac{14}{12}X_1 \checkmark$$

$$\Rightarrow X_1 + \frac{1}{2}X_1 + \frac{14}{12}X_1 = 1$$

$$X_1 = \frac{6}{16}$$

$$X_2 = \frac{3}{16}$$

$$X_3 = \frac{7}{16}$$

$$\frac{1}{\frac{6}{16}} = \frac{16}{6}$$

$$E[R \mid X_0 = 1] = \frac{16}{6}.$$