6.a.

$$A = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & -4 & 0 \\ 1 & -\lambda & 2 \\ 0 & 2 & 5-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)\{(-\lambda)(5-\lambda)-4\} + 4[5-\lambda] = 0$$

$$=(5-\lambda)(-5\lambda + \lambda^2 - 8) = 0$$

if $\lambda = 5 \quad \lambda^2 - 5\lambda = 0.$

$\rightarrow$ therefore $\lambda = 0, 5.$

$\lambda$ 0, 5, 5 are eigenvalues

eigenvector at $\lambda = 0.$

$$\begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 2 \end{bmatrix} = 0 \qquad V_1 = \begin{bmatrix} -2 \\ -\frac{5}{2} \\ 1 \end{bmatrix}$$

eigenvector at $\lambda = 5$

$$\begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 2 \end{bmatrix} = 0 \qquad V_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

6.b

the matrix is not diagonable since it doesn't have 3 eigenvectors.