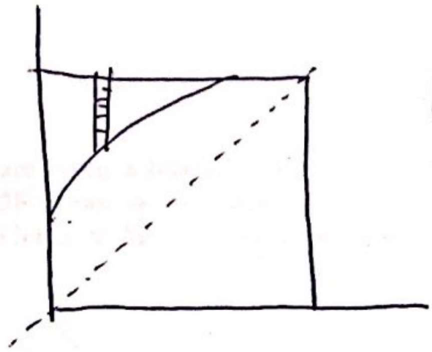
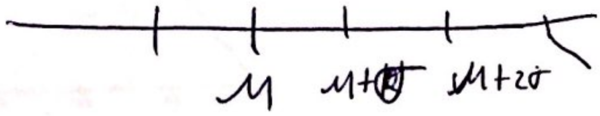
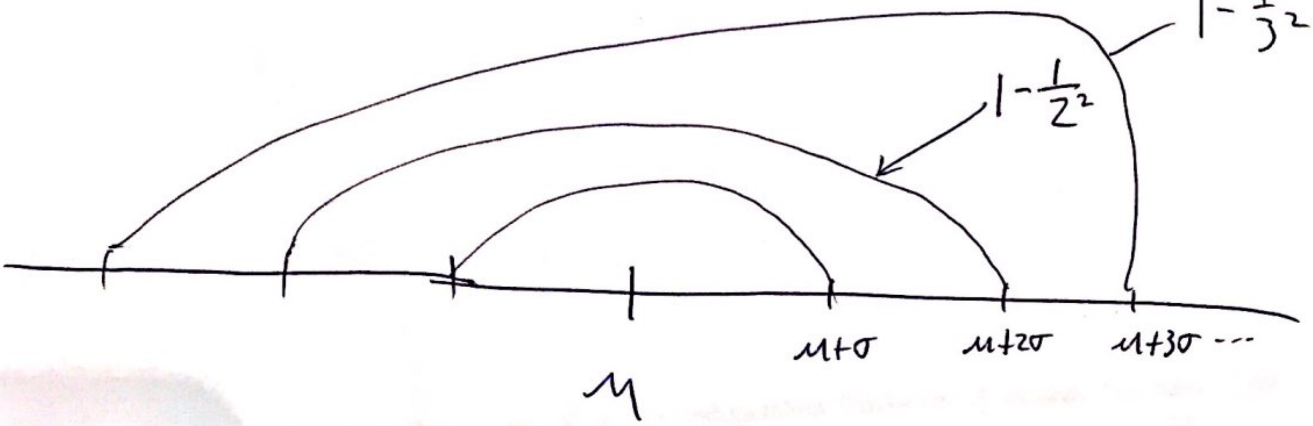
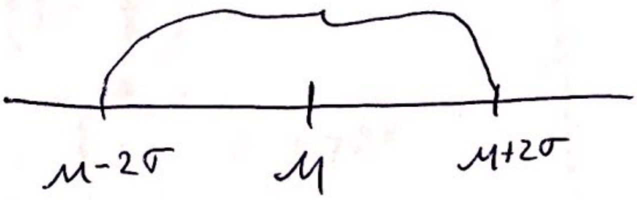


$$P(Z \le z) = P(X - Y^2 \le z)$$
$$= P(X \le z + Y^2)$$
$$= P(\sqrt{X - z} \le Y)$$





At least $1-\frac{1}{2^2} = .75$



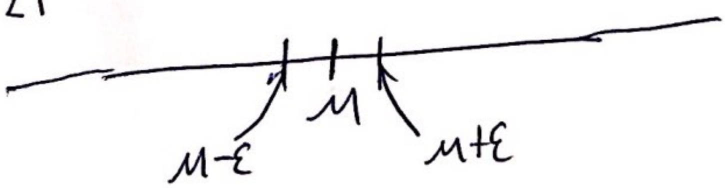
CLT

$\bar{X}$

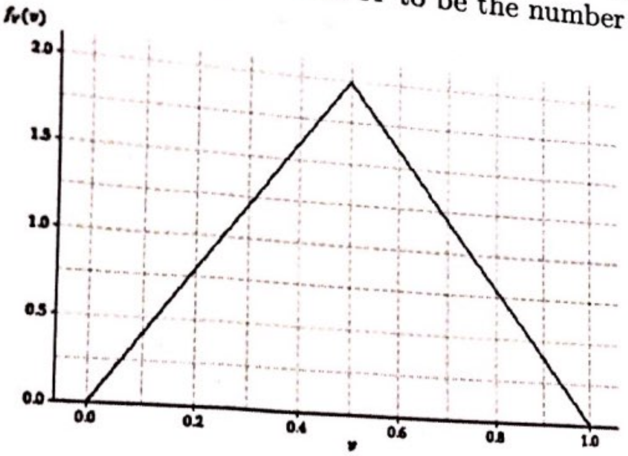
LofLN

You pick $\varepsilon > 0$

You pick Prob < 1



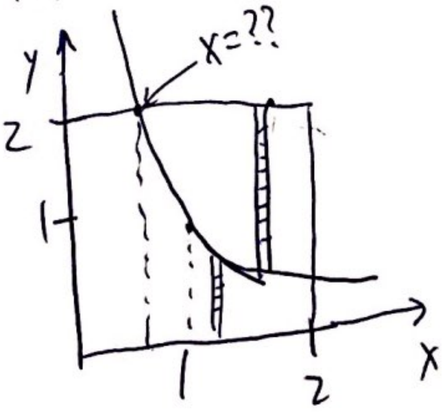
3. **(7 points)** We are given a biased coin. Probability(head)=V is a random variable itself with a PDF shown in the following figure. We toss the coin a fixed number of n times and we let X to be the number of heads obtain. What is $E[E[X|V]]$?



4. **(7 points)** Let X and Y be two independent Uniform(0,2) random variables. Find $P(XY < 1)$.

$$= P\left(Y < \frac{1}{X}\right)$$

$$= 1 - \int_{x=\frac{1}{2}}^{x=2} \int_{\frac{1}{x}}^{y=2} \left(\frac{1}{4}\right) dy\, dx$$



$V[\bar{X}] = E[\bar{X}^2] - E^2[\bar{X}]$ Know $E[\bar{X}] = \mu$

$V[\bar{X}] = \frac{\sigma^2}{n}$

5. (7 points) $X_1, X_2, \ldots, X_n$ are iid with mean μ and variance σ^2. What is the expected value of $Y = \frac{1}{n}\sum_{i=1}^{n} X_i^2$?

$V[\frac{1}{n} \sum X_i] = \frac{\sigma^2}{n}$

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$0 < Y < 8$

Problems – 65%

6. (25 points) X is a uniform random variable over the interval of (0 2). Y

- What is the PDF of $Y = X^2 + 2X$.
- What is the correlation of X and Y [Hint: You don't need to solve part 1 to solve this question].

$F_Y(y) = P(Y \le y) = P(X^2 + 2X \le y)$

$= P(X^2 + 2X + 1 \le y + 1)$

$= P[(X+1)^2 \le y+1] = P(-(y+1) \le (X+1)^2 \le y+1)$

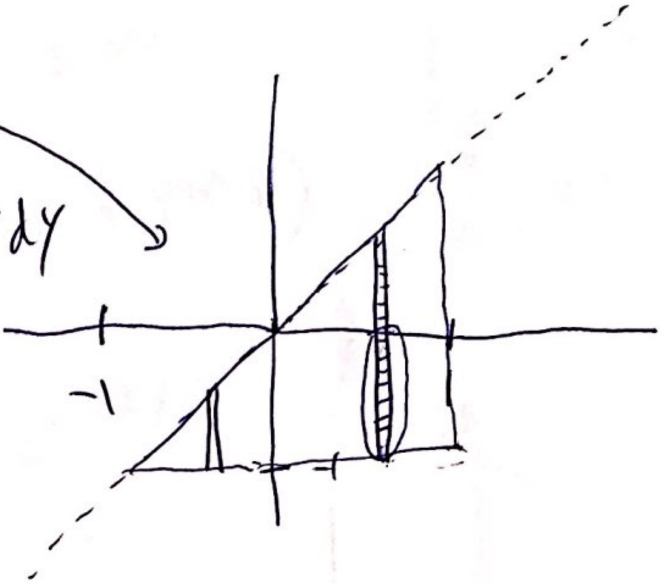
$|X| =$ $c|xy|$ $-cx|y|$ $cx|y|$

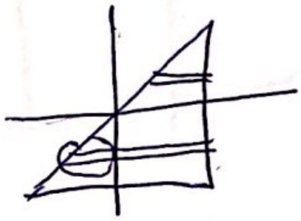
7. (20 points) Consider random variables X,Y with joint PDF.

$$f_{X,Y} = \begin{cases} cx|y| & x \in [-1\ 1],\ -1 \le y \le x \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

- Find $f_X(x), f_Y(y)$
  (Hint: You have to consider the positive and negative values separately)
- For extra credit find c

$$f_X(x) = \begin{cases} \int_{y=-1}^{y=0} -xy\,dy + \int_{y=0}^{y=x} xy\,dy & 0<x<1 \\ \int_{y=-1}^{y=x} xy\,dy & -1<x<0 \\ 0 & \text{otherwise} \end{cases}$$





$$f_Y(y) = \begin{cases} \int_{x=y}^{x=0} xy\,dx + \int_{x=0}^{x=1} -xy\,dx & -1<y<0 \\ \int_{x=y}^{x=1} xy\,dx & 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

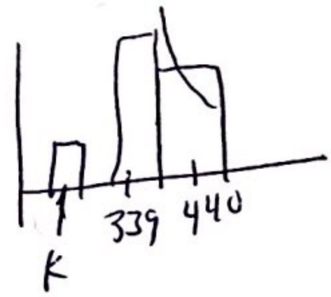
8. (20 points) A factory produces X_n gadgets on day n, where X_n are iid random variables with mean 5 and variance 9.

- Find an approximation to the probability that the total number of gadgets produced in 100 days is less than 440.
- Find the largest value of n such that $P(X_1 + \dots + X_n > 200 + 5n) \leq 0.05$

$\sum_{sum=0}^{n} \cancel{X}_n$

$P(S \leq 440)$

$P(\text{Norm} \leq 440.5)$



$P(S=0) + P(S=2)$
$+ \cdots P(S=440)$