Rendering Equation in Water Column

Shehtab Zaman

December 2017

1 Model

From the general radiance equation,the reflected radiance is given by

$$L_r = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\frac{\pi}{2}} f_r L_s T(d) cos(\theta_i) sin(\theta_i) d\theta_i d\phi_i \tag{1}$$

Where, the f_r is Lambertian BRDF.

The transmission function $T(d)$ can be related to the depth of the reflecting surface.

Given an incident light of radiance, L_0, we have the decomposed radiance as

$$L_{sa} + L_s = L_0$$

where, L_{sa} is the absorbed light radiation.

According to the Beer-Lambert Law we have the relation,

$$Log_{10}\left(\frac{L_0}{L_s}\right) = Log_{10}(T(d)) \tag{2}$$

and,

$$T(d) = 10^{\frac{-\epsilon dC}{cos(\theta)}} \tag{3}$$

where, ϵ is the molar absorptivity, d is the depth, C is the concentration.

Since we want to specifically take into account the absorbance due to chlorophyll a, we limit the consideration to considering the radiance of light at $\lambda = 662nm$

Even though the incident light is poloychromatic, but since we are considering the specific wavelength and the incident light is so spread out, we can reasonably treat it as monochromatic. (?)

The reflectance in the particular frequency then becomes,

$$L_{r\lambda} = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\theta_c} f_r L_{s\lambda} 10^{\frac{-\epsilon dC}{cos(\theta)}} cos(\theta_i) sin(\theta_i) d\theta_i d\phi_i \tag{4}$$

Here we sum the incident angle upto some cutoff angle θ_c, as the higher incident lights have negligible effect on the reflected light.

We can further make an assumption that the secchi disk is perfectly matte with rotationally invariant reflectance. The Lambertian BRDF is thus,

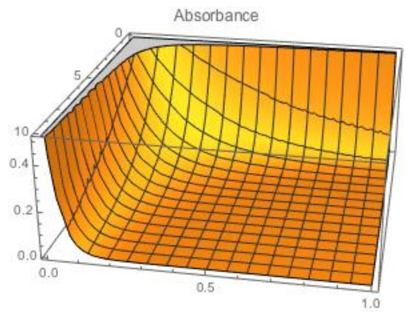
$$f_r = \frac{\rho_D}{\pi} \tag{5}$$

Combining (4) and (5) we get the reflectance off of the surface of secchi disk as,

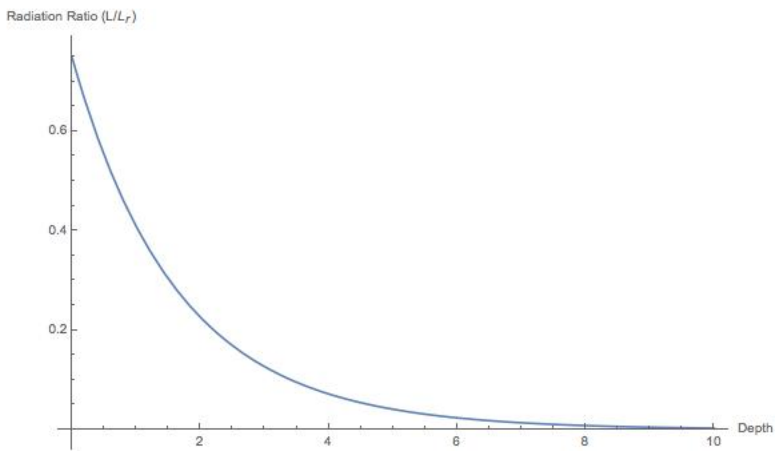
$$L_{r\lambda} = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\theta_c} \frac{\rho_D}{\pi} L_{s\lambda} 10^{\frac{-\epsilon dC}{cos(\theta)}} cos(\theta_i) sin(\theta_i) d\theta_i d\phi_i \tag{6}$$

Chlorophyll a has an extinction coefficient of $\epsilon_{662} = 0.088\frac{cm^{-1}}{g}$ at 663nm.

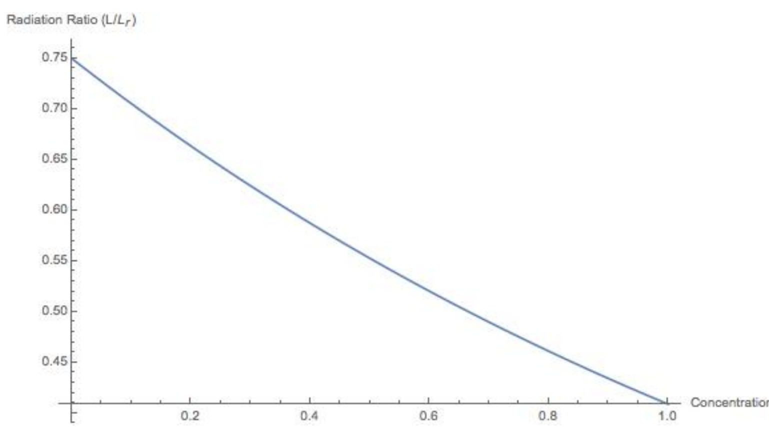
Plotting the function with varying depth and concentration (ranging form $0 \leq \epsilon C \leq 1$), we get,



Restricting the cutoff to $\theta_c = \frac{\pi}{3}$, holding the concentration consant, we obtain the relationship between $\frac{L_{r\lambda}}{L_{s\lambda}}$ and depth (not in cm as we are using $\epsilon = 1$ for modelling purposes)



Following the same procedure as before, keeping the depth constant, we have the relationship between the concentration and relative reflectance ($\frac{L_{r\lambda}}{L_{s\lambda}}$).



2 References

Harold H. Strain, Mary R. Thomas, Joseph J. Katz, Spectral absorption properties of ordinary and fully deuteriated chlorophylls a and b, In Biochimica et Biophysica Acta, Volume 75, 1963, Pages 306-311, ISSN 0006-3002, https://doi.org/10.1016/0006-3002(63)90617-6. (http://www.sciencedirect.com/science/article/pii/0006300263906176)