

Fermionic Hubbard Model for Two Site System

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May 2017

1 Hubbard Model Hamiltonian for Fermion

$$\mathcal{H} = -t \sum_{\sigma} \left(f_{1\sigma}^{\dagger} f_{2\sigma} + f_{2\sigma}^{\dagger} f_{1\sigma} \right) + U (\hat{n}_{1\uparrow} \hat{n}_{1\downarrow} + \hat{n}_{2\uparrow} \hat{n}_{2\downarrow}) \quad (1)$$

Where for site i and spin σ , the creation, annihilation and number operators are $f_{i\sigma}^{\dagger}, f_{i\sigma}, \hat{n}_{i\sigma}$ respectively. The number operator is defined as

$$\hat{n}_{i\sigma} = f_{i\sigma}^{\dagger} f_{i\sigma}$$

2 Fock Space

2.1 Notes on Fock Space

2.2 Fock Space Basis

3 Two-Site Hubbard Model Matrix

$$\mathcal{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & U & -t & -t & 0 & 0 \\ 0 & -t & 0 & 0 & -t & 0 \\ 0 & -t & 0 & 0 & -t & 0 \\ 0 & 0 & -t & -t & U & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

4 Exact Solution