Two Body Motion

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1 Background

Mehcanics of two particle that experience a mtual central conservative interaction.

2 Lagrangian of Two-Body Problem

$$\mathcal{L} = \frac{1}{2}m_1\dot{r_1}^2 + \frac{1}{2}m_2\dot{r_2}^2 - U\left(|\vec{r_1} - \vec{r_2}|\right) \tag{1}$$

Lagrangian is has translation and rotational inavirance.

Rotational invariance is due to the potential only haveing magnitude of the vectors and not direction.

3 Solving Lagrangian

To decouple the lagrangian and the EL equations, change lagrangian to cener of mass and relative mass frame. Translational invariance allows for the transformation.

For center of Mass, radius $\vec{R}$ and relative mass, radius $\vec{r}$, we have

$$\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1 + m_2} \qquad \vec{r} = \vec{r_1} - \vec{r_2} \tag{2}$$

Using μ as the effective mass of

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad M = m_1 + m_2 \tag{3}$$

We can decompose the into frames,

$$\mathcal{L} = \mathcal{L}_{CM} + \mathcal{L}_{Rel} \tag{4}$$

Uing equations 2,3 to rewrite 1 we have

$$\mathcal{L} = \mathcal{L}_{CM} + \mathcal{L}_{Rel} = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu(\dot{r})^2 - U(|\vec{r}|) \tag{5}$$

Where we have,

$$\mathcal{L}_{CM} = \frac{1}{2}M\dot{R}^2 \qquad \mathcal{L}_{Rel} = \frac{1}{2}\mu\dot{r}^2 - U(|\vec{r}|) \tag{6}$$