Renormalization Group Notes

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November 2017

1 Background

a group in quantum mechanics is any transformation that leaves the physical system invariant.

In quantum electrodynamics, when computing physical quantity such as scattering rate between eletrons, a power series in the coupling constant α are given by integrals over all possible momenta k, which diverge.

We wish to consider the application of Renormalization Group methods to interacting non-relativistic fermions especially weyl fermions.

In condensed matter physics, it is still possible to use RG a even with given natural cutoffs (such as the inverse lattice spacing) and absense of ultraviolet infinities.

2 Use Cases

2.1 Cubic Lattice(Shankar)

Considering a d dimensional cubic lattice, with a real scalar field $\phi(\hat{n})$ at each site. The partition function is given by,

$$\mathbf{Z} = \int \prod_{\hat{n}} d\phi(\hat{n}) e^{S(\phi\hat{n})} \tag{1}$$

where S is the action.

The two-point function, the average of the correlation between the variables at two sites, is given by

$$\begin{aligned} G(\hat{n_1}, \hat{n_2}) &= G(\hat{n_1} - \hat{n_2}) && (2)\\ &= \langle \phi(\hat{n_1}) \phi(\hat{n_2}) \rangle && (3)\\ &= \frac{1}{\mathbf{Z}} \int \prod_{\hat{n}} d\phi(\hat{n}) \phi(\hat{n_1}) \phi(\hat{n_2}) e^{S(\phi\hat{n})} && (4) \end{aligned}$$

The Fourier transform of the lattice vector field is given by,

$$\phi(\hat{k}) = \frac{1}{V} \sum_{\hat{n}} e^{i\hat{k}-\hat{n}} \phi\hat{n} \tag{5}$$

So we can write the partition function from eq.1 as,

$$\mathbf{Z} = \int \prod_{|\hat{k}| \leq \frac{\pi}{a}} d\phi(\hat{k}) e^{S(\phi(\hat{k}))} \tag{6}$$

When considering problems of large seperations where $\hat{n}_1 - \hat{n}_2$ is large, in momentum space with small $\hat{k}$

So by using a cutoff momentum $\frac{\Lambda}{s}$, we can have

$$\phi_< = \phi(k)(\text{slow modes}) \tag{7}$$

$$\phi_{>} = \phi(k)(\text{fast modes}) \tag{8}$$

In order to perform the Renormalization Group technique, we must perform the integral over the $\phi_{<}$ fields only. So the first step for RG is to obtain an effective action ,$S^{'}(\phi_{<})$, such that $e^{S^{'}(\phi_{<})}$ produces all the slow momentum correlation function when integrated over the slow modes.

3 Critical Phenomenon

Macroscopic variables like specific heat, magnetic susceptibility or correlation length either diverge or approachzero as $T \to T_c$

4 Definitions

Scalar Field: ($\phi(\hat{n})$) A mathematical function that assigns a value to each point/site labeled by vector $\hat{n}$.
Reciprocal Space: $kx = 2\pi$
Weyl Fermion: Masslesss spin half quasi-particle with definite chirality.
Majorana Fermion: Neutral spin half quasi-particle which is it's own anti-particle.

5 References

Majorana, E., 1937, Il Nuovo Cimento (1924-1942) 14(4), 171.