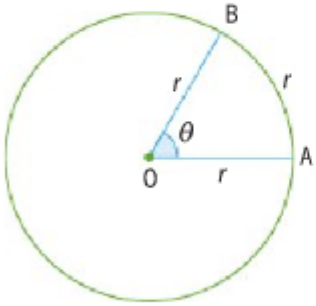
Radians

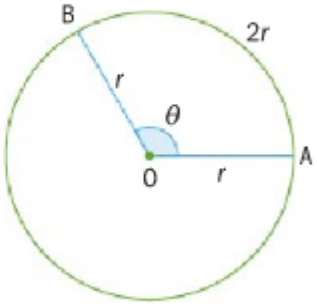
$Arc\ length = r\theta$

One radian is defined as the size of the central angle **subtended** by an arc which is the same length as the radius of the circle.



θ = 1 radian

Two radians is the size of the central angle subtended by an arc with a length equal to twice the radius of the circle.

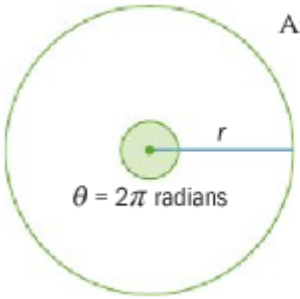


θ = 2 radians

One complete turn around the circle is subtended by an arc equal in length to the circumference of the circle.

$$\text{The circumference} = 2\pi r$$

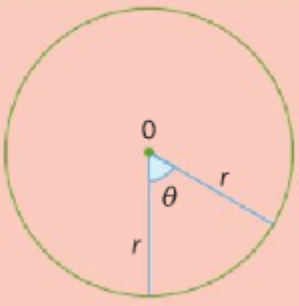
Therefore, the angle which subtends the circumference of the circle is 2π radians.



Arc length = circumference = $2\pi r$

→ $\text{Arc length} = \left(\frac{\theta}{2\pi}\right)(2\pi r) = r\theta$

where r is the radius and θ is the central angle measured in radians.



➔ To convert degrees to radians multiply by $\frac{\pi}{180}$

➔ To convert radians to degrees multiply by $\frac{180}{\pi}$

➔ Area of sector = $\left(\frac{\theta}{2\pi}\right)\left(\pi r^2\right) = \frac{\theta r^2}{2}$

where r is the radius of the circle and θ is the central angle, in radians.

Exercises

1.

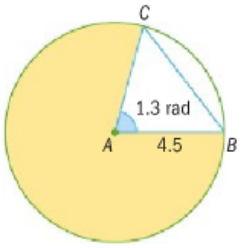
Find the length of the arc which subtends a central angle of 1.7 radians in a circle with radius 5.6 cm.

Find the length of the arc which subtends an angle of 3.25 radians at the center of a circle with diameter 24 cm.

2.

Convert these angles to radians: 30°, 45°, 60°
Give exact answers.
Convert these angles to degrees: $\frac{2\pi}{5}$ rad, $\frac{\pi}{9}$ rad
Give exact answers.

3 The diagram shows the circle, center A, radius 4.5 cm, and $B\hat{A}C = 1.3$ radians.

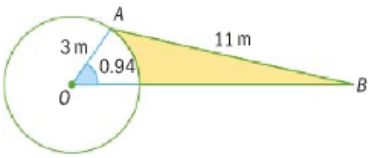


a Find the area of $\triangle ABC$.

b Find the length BC.

c Find the area of the shaded region.

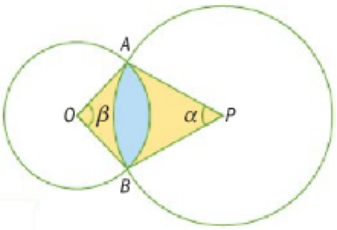
4 The diagram shows the circle, center O, with radius 3 m, $AB = 11$ and $A\hat{O}B = 0.94$ radians. Find the shaded area.



5 and 6.

In the circle with center P the arc QR subtends an angle of θ at the center. If the length of arc QR is 27.2 cm and the area of sector PQR is 217.6 cm^2, find θ and the radius of the circle.

Circle O has radius 4 cm, and circle P has radius 6 cm. The centers of the circles are 8 cm apart. If the circles intersect at A and B, find the blue shaded area in the diagram.



7.

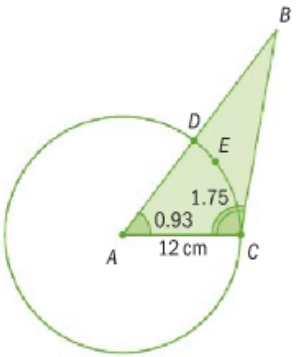
The diagram shows a circle with center A and radius 12 cm. Angle $DAC = 0.93$ radians, and angle $BCA = 1.75$ radians.

a Find BC.

b Find DB.

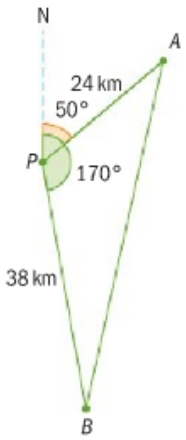
c Find the length of arc DEC.

d Find the perimeter of the region $BDEC$.



8.

Two ships sail from the same port P at the same time.
Ship A sails on a bearing of 050° for a distance of 24 km before droping anchor.
Ship B sails on a bearing of 170° for a distance of 38 km before droping anchor.
Find the distance between the two ships when they are stationary.

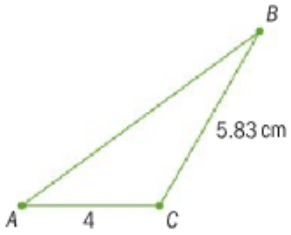


9.

This diagram shows the triangle ABC, which has an area of 10 cm^2.

a Find $A\hat{C}B$, given that it is an obtuse angle.

b Find AB.



10.

The diagram shows triangle PQR, with $\hat{Q} = 118°$, $PQ = 9.5$ m and $QR = 11.5$ m.

a Find PR.

b Find $\hat{P}$.