Implementation of Optimal control for ball and beam system

Anjali T.
Department of Electrical and Electronics Engineering
Mar Baselios College of Engineering and Technology
Thiruvananthapuram, India

Shyju Susan Mathew
Department of Electrical and Electronics Engineering
Mar Baselios College of Engineering and Technology
Thiruvananthapuram, India

***Abstract*— The ball and beam system is a widely used tool for learning classical as well as advanced control techniques. The system with highly non linear characteristics is an excellent tool to represent unstable systems. The paper presents an optimal control strategy for controlling the position of ball on the beam. The system is an open loop unstable system. The control problem is a challenging one as the ball position continuously changes with the beam angle. The control strategy includes PID control for both ball position control and servo position control. The control parameters are tuned using Genetic Algorithm. The results are verified experimentally using real time ball and beam system.**

Keywords—PID Tuning; Ball and Beam; Genetic Algorithm; cascade control; ball position control, servo position control

I. INTRODUCTION

Most of the industrial processes that we are dealing with today involves unstable processes. These are mostly dangerous ones and have to be dealt with carefully. Such processes can be analyzed and studied only in the laboratories. The ball and beam system is a widely used set up to study unstable systems. It consists of a ball rolling on top of a beam with one end fixed and the other end pivoted at the output shaft of a servomotor. The ball is free to roll on top of the beam. The system represents many of the complex dynamics associated with unstable systems. For example, the control problem of regulating the position of the ball on the beam is similar to the one that we encounter during the pitch control of an aircraft. It has proved to be an excellent benchmark in testing control algorithms.

The ball and beam system is an ideal set up for learning classical and modern control strategies. The system has remained a favorite of researchers since a long time for the testing of new control methodologies and algorithms. J.Hauser, S. Sastry and P.Kokotovic applied their approach for approximate input-output linearization on ball and beam system [1]. W.Yu and F. Ortiz implemented an asymptotically stabilising PD controller that ensures for a well defined set of initial conditions, the ball remains on any point on the beam [2]. J.S. Kim, Gyu-Man Park and Ho-Lim Choi proposed a sliding mode controller and experimentally validated the results [3]. Paul H. Eaton and Danil V. Prokhorov proposed intelligent controller based on neural networks for the control of ball on beam [4]. Several soft computing methods are also being used by the researchers for getting the desired results using the ball and beam system. D.Puangdownreong and A. Sakulin used meta heuristic technique current search method to obtain optimal PID parameters [5]. Y. H. Chang, Wei-Shou Chang and Chia-Wen Chang used T-S Fuzzy modelling to formulate the ball and beam system as a strict feedback form [6].

Control engineering has witnessed the evolution of a number of control algorithms and strategies over the years. Still thousands of industries worldwide make use of PID controllers for their processes. Generally the PID controllers are applicable to most of the control systems especially when the mathematical model of the plant is not known [7]. The simple structure and ease of implementation makes it the most preferred one. One biggest challenge with the use of PID controller is the tuning of controller parameters. Tuning is the process of selecting the controller parameters so that the performance specifications are satisfied. Overtuned or undertuned controllers may not give the desired results. Performance of the entire control system is dependent on the control parameters. Several empirical and analytical tuning methods are used for parameter tuning. Zeigler Nichols, Cohen Coon methods fall in the first category and they are best suited for simple process models. Frequency response method, Root locus method [7] etc fall under the second category and these are suitable for tuning controllers for higher order systems. The PID tuning problem still continues to be an area of active research. Evolutionary techniques like Genetic Algorithm, Particle Swarm Optimization etc are widely used for solving optimization problems. The paper presents Genetic Algorithm as an effective algorithm in tuning PID parameters.

The process under study is the position control of a ball and beam system. The ball and beam system is used extensively for demonstrating modern as well as classical control techniques. The system consists of a long beam mounted on the output shaft of a servomotor. A ball moves freely on top of the beam. The control problem is to regulate

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the position of the ball on the beam. Feedback control is required to achieve this as the system is unstable in open loop. The rest of the paper is organised as follows. Section II presents a review of PID controller structure followed by an overview of steps involved in optimising using Genetic Algorithm. Section IV and Section V includes mathematical modelling of ball and beam and servomotor respectively. Section VI discusses the control strategy followed by tuning of PID parameters in Section VII. Section VIII deals with implementation of control and Section IX concludes the paper.

II. PID CONTROLLER

The general structure of a PID controller [8] is as shown in Fig.1.

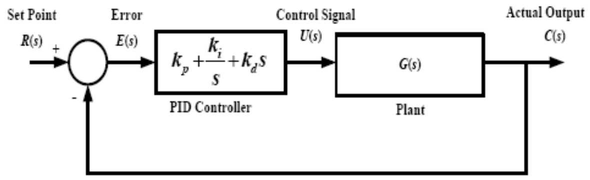

Fig. 1. General PID Structure

The controller input is the difference between the process output and the setpoint. Suitably designed control parameters Kp, Ki and Kd acts on the error signal and generates the control signal. The control signal is then applied to the plant and new process output is generated. The process continues till steady state is reached. The controller transfer function is represented as $\frac{U(s)}{E(s)} = Kp + \frac{Ki}{s} + sKd$.

III. GENETIC ALGORITHM

Genetic Algorithm (GA) is a stochastic evolutionary algorithm which is based on the principles and concepts of biological evolution. It was first proposed by Holland. Genetic Algorithm is found to be effective in solving complex optimization problems. The steps included in the optimisation algorithm are listed below.

1. First GA generates a random initial population. Each individual or the chromosomes in the population is then encoded into binary strings.
2. The binary strings are then decoded into real numbers and the objective function is evaluated. The value returned by the objective function is given to the GA and based on the value, it assigns a fitness value to the individuals.
3. Each population is then acted upon by the GA operators like selection, crossover, mutation etc and creates the next population. The new population replaces the initial one
4. The process continues till the user specified stopping criteria are met. Each iteration of the algorithm is termed as a generation.

GA is initialised with a randomly generated population typically consisting of 20-100 individuals. Each individual that represents the PID parameters are encoded into binary strings. Length of the string depends on the precision required [8]. The most important part in GA is the objective function. The performance of each individual is assessed by the objective function. Each individual is assigned a corresponding number called fitness value. This value forms the basis for the selection of individuals for reproduction. New generation of individuals are created using the operators selection, crossover and mutation [9].

Selection-Each chromosome or individual undergoes a selection process and the one with higher fitness values has more chances of getting selected.

Crossover-This is the basic operator that produces new chromosomes. The assumption is that the offspring will be superior to the parents if it takes the beast characteristics from both parents.

Mutation-This operator helps to maintain genetic diversity from one generation to the next. Mutation acts as a safety mechanism to recover good genetic material that may be lost due to crossover and selection.

One major drawback while optimising using Genetic Algorithm is the premature convergence of result. This happens when the diversity in the population is lost. Use of Elitism helps to maintain population diversity. The population size should also be selected appropriately.

IV. MODELLING OF BALL AND BEAM SYSTEM

Fig. 3 shows the schematic representation of the forces acting during the motion of ball on beam. The system under study consists of a beam supported at one end. The other end is coupled to the output gear system which is driven by a DC Servomotor.

Modelling of ball and beam system is done by using Euler Lagrange equations. It is assumed that the ball rolls without slipping. The equations are obtained as follows.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = \tau \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \tag{2}$$

α – beam angle

x – position of ball on beam

$$\ddot{\alpha}\left(J_b + m_b x^2\right) + 2 m_b x \dot{x} \dot{\alpha} + m_b g x Cos\alpha = \tau \tag{3}$$

$$\ddot{x} + \frac{5}{7}\left(gSin\alpha - x\left(\dot{\alpha}\right)^2\right) = 0 \tag{4}$$

The equation can be linearised around the operating point as,

$$\ddot{x}+\frac{5}{7}gSin\alpha=0 \qquad (5)$$

The motor gear angle and the beam angle can be related. Equating the arc distances from Fig. 3,

$$\alpha L_{beam}=\theta r_{arm} \qquad (6)$$

θ -servo angle

Rewriting (5) using (6) and taking Laplace Transforms, the ball and beam transfer function can be obtained as,

$$\frac{X(s)}{\theta(s)}=\frac{0.4182}{s^2} \qquad (7)$$

The ball and beam system specifications [10] are given in Table1.

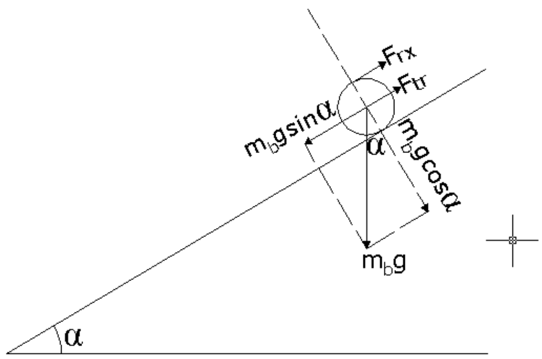

Fig.2 Forces acting during the motion of ball on beam

TABLE 1. BALL AND BEAM SYSTEM SPECIFICATIONS

	Parameter	Value
1.	Beam Length	0.4255m
2.	Distance between servo output gear shaft and coupled joint	0.0254m
3.	Radius of ball	0.0127m
4.	Mass of ball	0.064kg

V. MODELLING OF SERVOMOTOR

Fig. 3 shows the servomotor armature circuit and gear train.

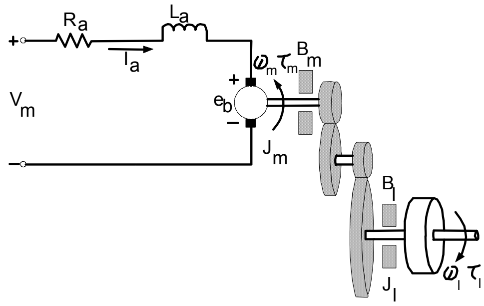

Fig. 3 Servomotor armature circuit and gear train

Applying Kirchoff's law to the armature circuit and neglecting armature winding inductance

$$V_m(t)-I_aR_a-k_m\omega_m(t)=0 \qquad (8)$$

k_m - back emf constant of motor

Equation of motion of load can be written as,

$$J_l\frac{d\omega_l(t)}{dt}+B_l\omega_l(t)=\tau_l(t) \qquad (9)$$

J_l- Moment of inertia of load

B_l- Load inertia coefficient

τ_l – total torque applied on load

τ_{ml} –resulting torque acting on the motor shaft

Motor shaft equation can be expressed as,

$$J_m\frac{d\omega_m(t)}{dt}+B_m\omega_m(t)+\tau_{ml}(t)=\tau_m(t) \qquad (10)$$

J_m – Motor shaft moment of inertia

Combining load and motor dynamics, the equation can be represented as,

$$J_{eq}\frac{d\omega_l(t)}{dt}+B_m\omega_l(t)=A_mV_m(t) \qquad (11)$$

Taking the Laplace Transforms, the motor transfer function can be obtained as,

$$\frac{\theta_l(s)}{V_m(s)}=\frac{A_m/B_m}{s(1+s\,J_{eq}/B_m)}=\frac{K}{s(1+\tau s)} \qquad (12)$$

The nominal parameters K and tau are obtained from the motor specifications.

VI. CONTROL STRATEGY FOR BALL AND BEAM SYSTEM

The ball and beam system is an openloop unstable system. Feedback control is required to bring the ball position to the desired one. The open loop response of the system is as shown in Figure 4.

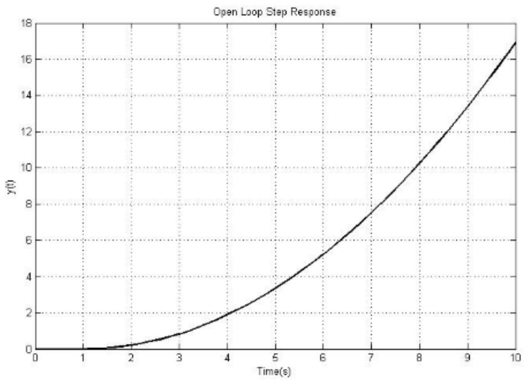

Fig.4 Open loop response of ball and beam system

The ball and beam system makes use of a cascade control strategy. The outer loop is the ball position control and the inner loop is the servo position control. The ball position control calculates the servo angle required to bring the ball position to the desired one. The inner loop control calculates the servo input voltage to bring the servo angle to the desired location. The paper presents PID control for both inner and outer loops. The control parameters are tuned using Genetic Algorithm.

VII. TUNING OF PID CONTROL PARAMETERS

A. PID Parameter tuning for ball position control

GA is initialized with a random population consisting of 30 individuals. Number of iterations required is 120. The GA parameters used for tuning are given in Table 2

TABLE 2 GA PARAMETERS FOR TUNING BALL POSITION CONTROLLER

Parameter	**Value**
Population Size	30
Number of generations	120
Crossover	Arithmetic
Selection	Tournament
Mutation Rate	0.01
Crossover Fraction	0.8

B. PID Parameter tuning for servo position controller

The GA parameters used for tuning servo position controller is given in Table 3. The solution converged after 80 iterations, and a population size of 40 was used.

TABLE 3 GA PARAMETERS FOR TUNING SERVO POSITION CONTROLLER

Parameter	**Value**
Population Size	40
Number of generations	80
Crossover	Arithmetic
Selection	Tournament
Mutation Rate	0.01
Crossover Fraction	0.8

The tuned controller parameters are given in Table 4 and Table 5.

TABLE 4 PID PARAMETERS FOR BALL POSITION CONTROL

Kp	**Ki**	**Kd**
2.7587	0.0080	1.5392

TABLE 5 PID PARAMETERS FOR SERVO POSITION CONTROL

Kp	**Ki**	**Kd**
9.6345	0.0394	0.1899

The results of simulation with the tuned PID parameters are shown in Fig 5 and Fig.6. The simulations have been done using the non linear plant model. The controller design has to take care of actuator saturation limits. The motor input voltage should not exceed +-10 Volts. The ball position response with the tuned PID parameters is shown in Fig.5. The motor input voltage response is shown in Fig.6.

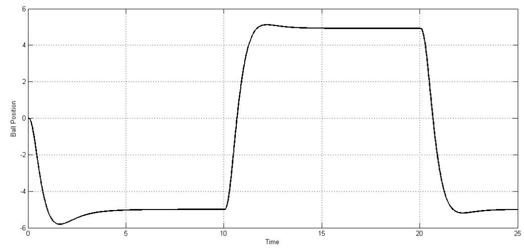

Fig. 5 Ball position response with PID-PID cascade control

The ball response is satisfactory with very less overshoot and fast settling with no steady state error. Also the input voltage to the motor is well within the saturation limits.

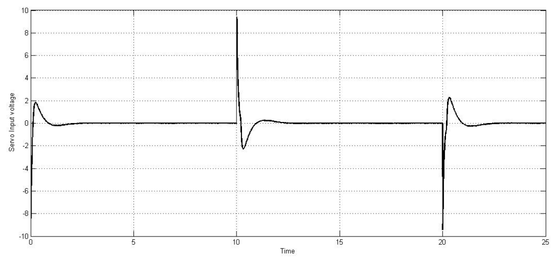

Fig. 6 Servo input voltage with PID-PID cascade control

VIII. IMPLEMENTATION OF PID-PID CASCADE CONTROL

A. *Experimental Setup.*

The tuned PID parameter values are experimentally verified using the real time ball and beam system. The plant used is the Quanser ball and beam system and the implementation is done through Quarc platform. Quarc generates real time code directly from Simulink designed controllers and runs the code in real time. The peripherals required include data acquisition board and voltage amplifier. The data acquisition board used is the Quanser Q8-USB and VoltPaq-X1 single channel linear voltage amplifier is used.

B. *Results*

The tuned PID parameters for servo positioning and ball positioning have been implemented in real time ball and beam system. The ball position response is shown in the Fig, 7. With the designed control parameters, the system is able to track the reference trajectory satisfactorily With the designed control parameters, the system is able to track the reference trajectory satisfactorily.

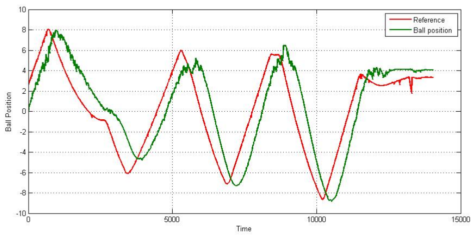

Fig. 7 Ball position response with PID-PID control implemented in real time plant

IX. CONCLUSION

Ball and Beam systems have been used widely for learning classical as well as modern control techniques. The paper presented a cascaded PID control strategy for position control of ball and beam system. PID parameters for servo position control and ball position control has been tuned using Genetic Algorithm. The tuned parameters are verified experimentally by using it to run a real time ball and beam plant. With the tuned PID parameters, the system was able to track the reference trajectory while satisfying the actuator saturation constraints.

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