Projective Geometry for Computer Vision

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3D Computer Vision

Classical Problem:

Given a collection of 2D images, build a model of the 3D world.

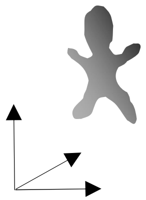
Example Applications:

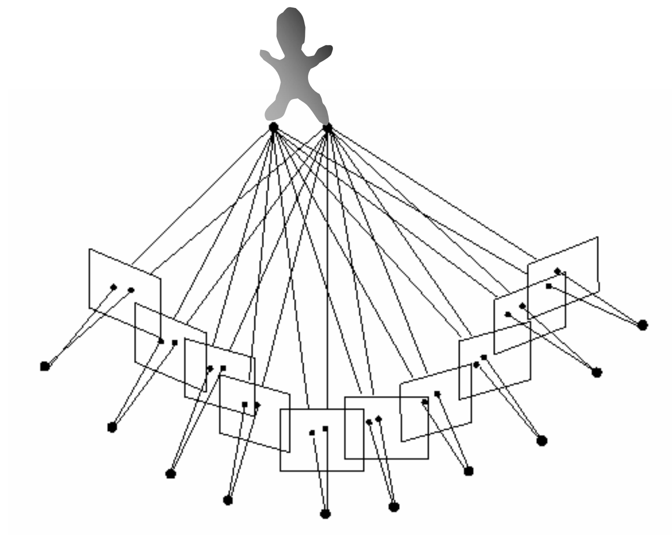
- virtual/immersive environments
- robotics & autonomous vehicles
- minimally invasive surgery

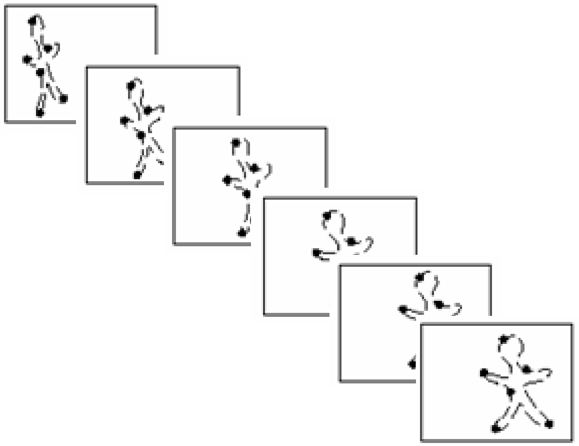
Outline

1. Projective Geometry Overview
2. Minimal Projective Parameters
3. Projective Parameter Estimation
4. Motion Boundary Detection
5. Conclusion

Image Formation

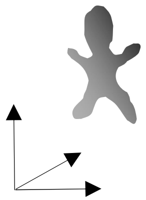


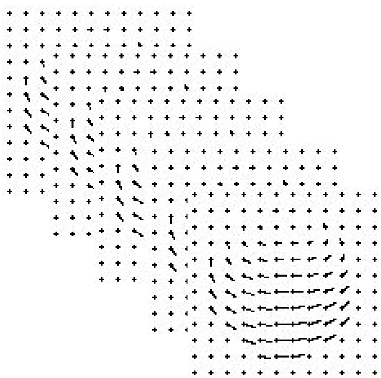


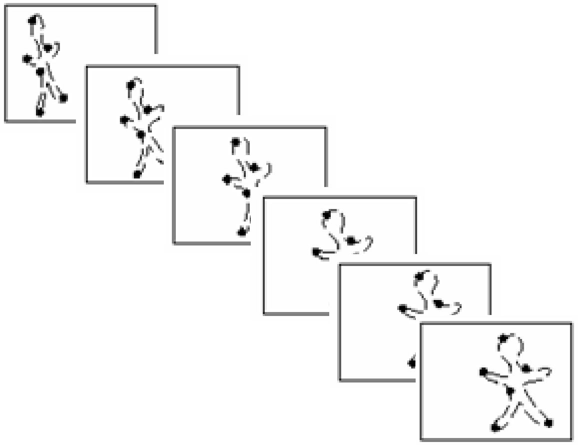


3D scene → imaging → 2D images

Computer Vision







3D scene model

data

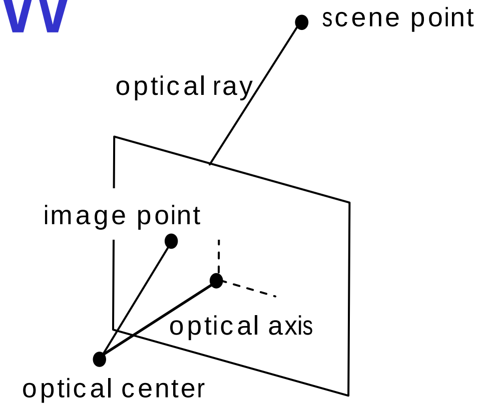
2D images

analysis

measurement

Camera Geometry: Single View

pinhole model of perspective projection



unknown depth at each point

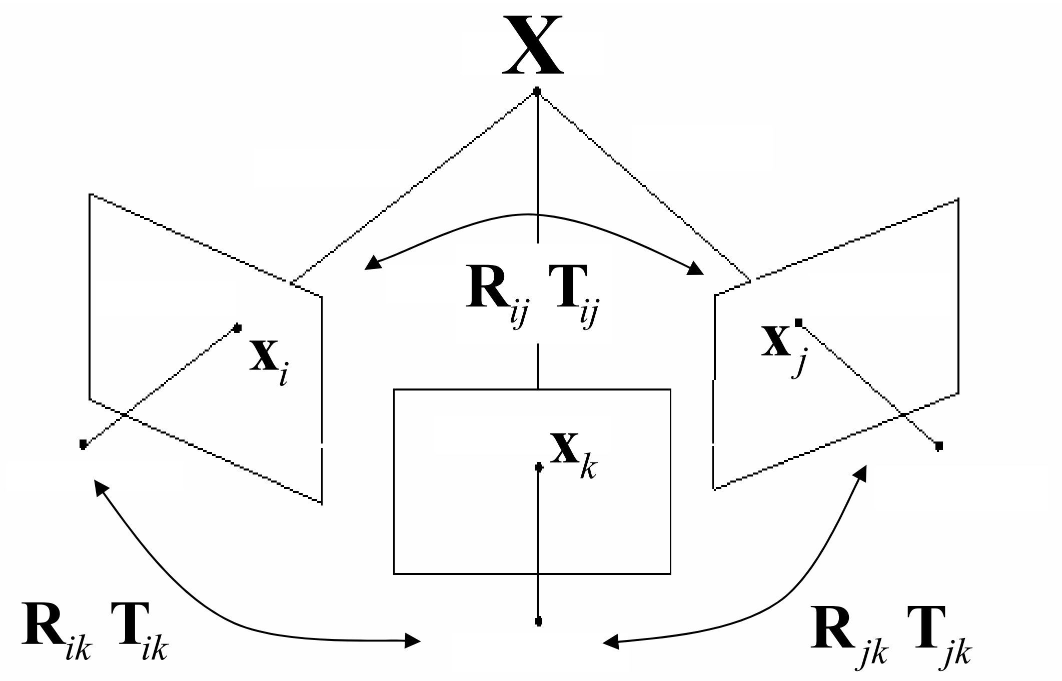
$$x = \frac{X}{Z} \qquad y = \frac{Y}{Z}$$

unknown internal camera parameters

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} f_x & s \\ 1 & f_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix}$$

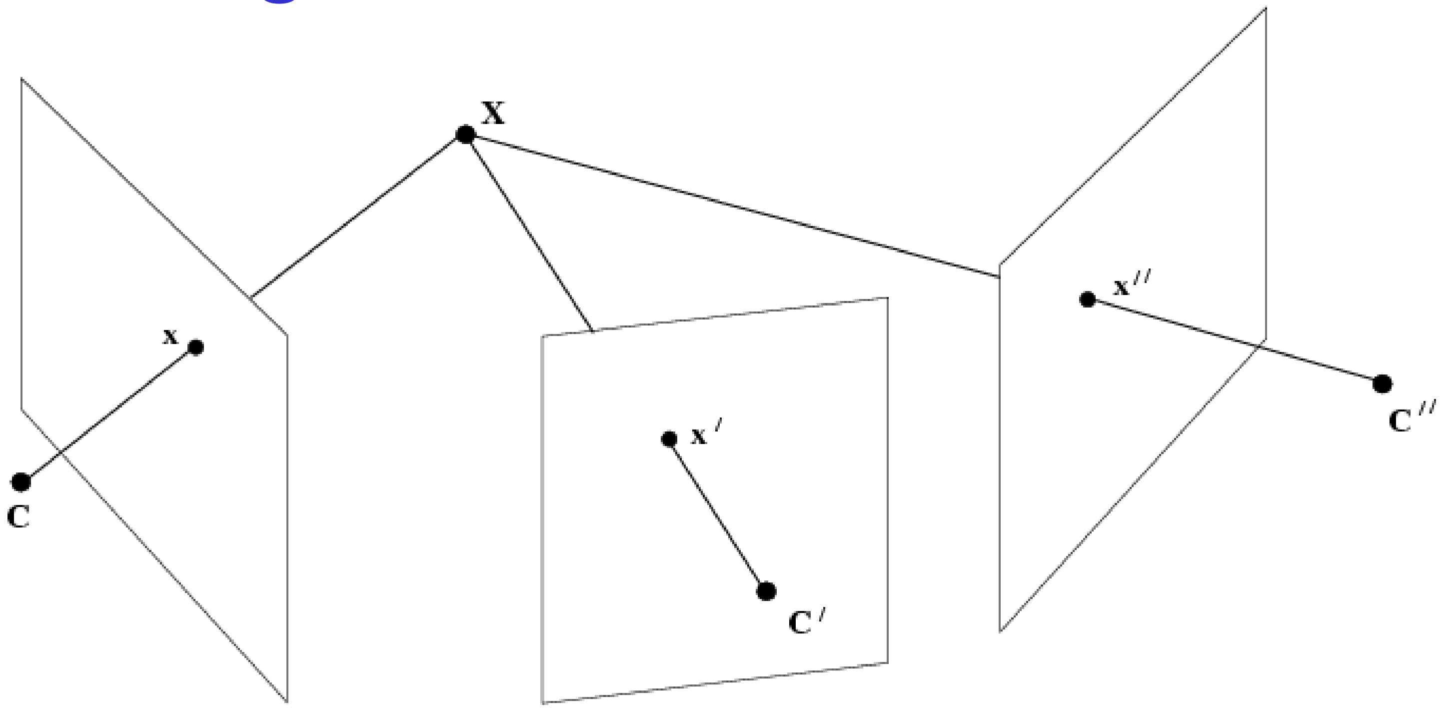
Camera Geometry: Multiple Views

unknown rotations and translations



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \mathbf{T}$$

Measured Data: Image Points and Lines



geometric constraint: optical rays intersect in 3D

projective geometry: express constraint in terms of measured 2D image features

Projective Camera Model

- linear model of image formation
- depth-independent expression for optical ray intersections
- multilinear relations among point and line matches

Bilinear Constraints

(Longuet-Higgins ,1981, Faugeras, 1992; Hartley, 1992)

$$\mathbf{X} = \lambda_i \mathbf{x}_i$$

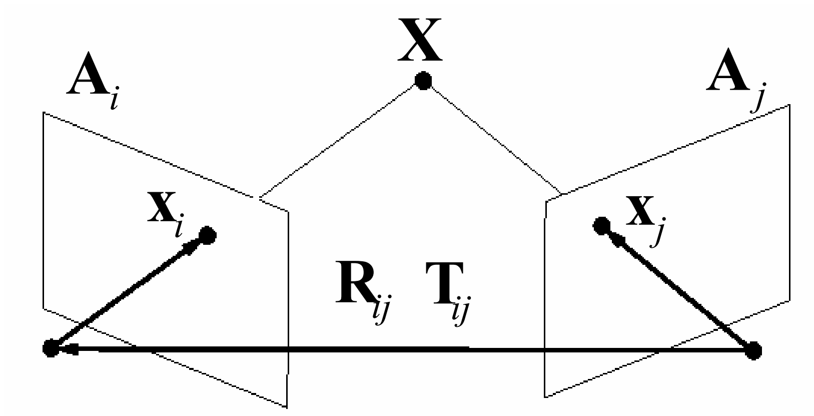
$$\lambda_j \mathbf{x}_j = \lambda_i \mathbf{R}_{ij} \mathbf{x}_i + \mathbf{T}_{ij}$$

$$\mathbf{x}_j^T [\mathbf{T}_{ij}]_\times \mathbf{R}_{ij} \mathbf{x}_i = 0$$

$$\mathbf{x}_i \rightarrow \mathbf{A}_i^{-1} \mathbf{x}_i$$

$$\mathbf{x}_j \rightarrow \mathbf{A}_j^{-1} \mathbf{x}_j$$

$$\mathbf{x}_j^T \underbrace{\mathbf{A}_j^{-T} [\mathbf{T}_{ij}]_\times \mathbf{R}_{ij} \mathbf{A}_i^{-1}}_{\mathbf{F}_{ij}} \mathbf{x}_i = 0$$

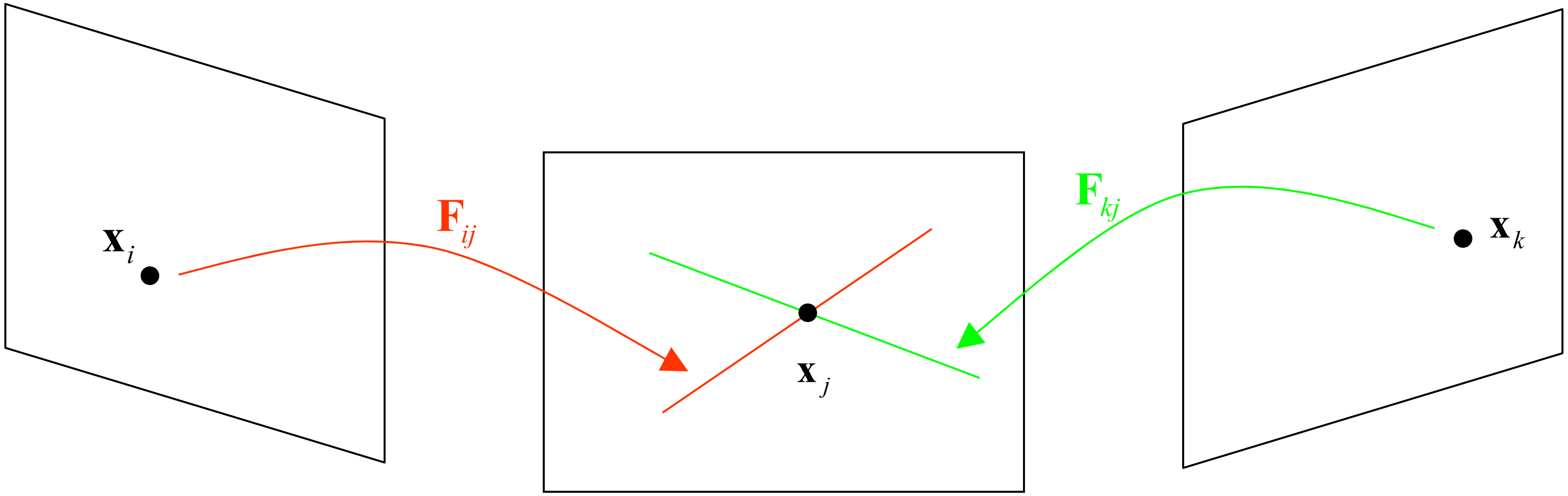


$$\mathbf{x}_j^T \mathbf{F}_{ij} \mathbf{x}_i = 0$$

fundamental matrix

Fundamental Matrix

Maps a point in one image to a line in the other image that contains its match



Given matching points in two views, predict the matching point in a third image.

Projective Models in Practice

- View synthesis and interpolation: point transfer function for dense point correspondences
- Self-calibration: automatic recovery of internal camera parameters from fundamental matrices
- Bundle adjustment initialization: initial rotation and translation for nonlinear Euclidean optimization

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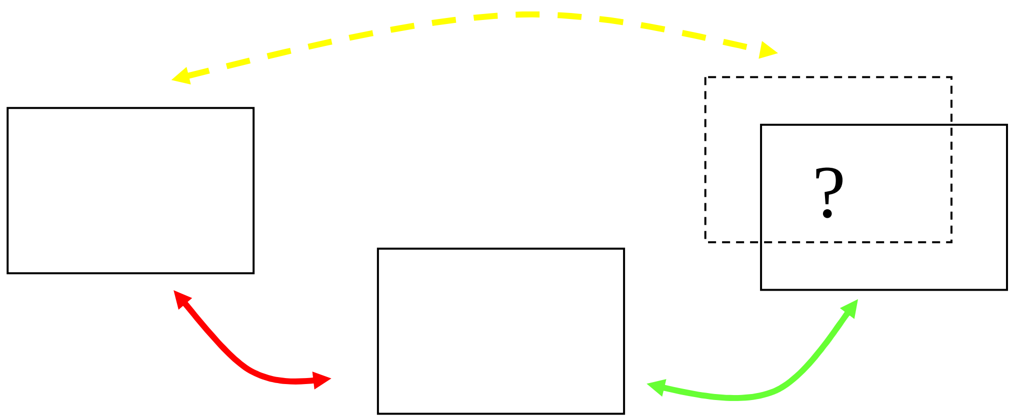
Practical Problem

- Few point matches between some views.
- Unstable for estimating geometric relationships.

Geometric Consistency

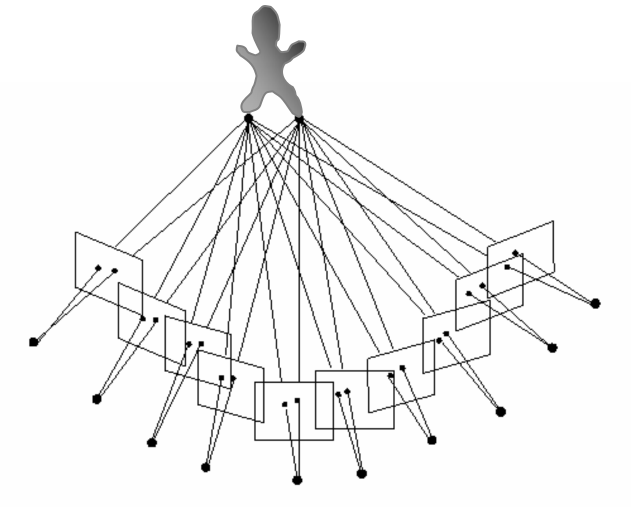
Pairwise geometric relations may be inconsistent.



Goals

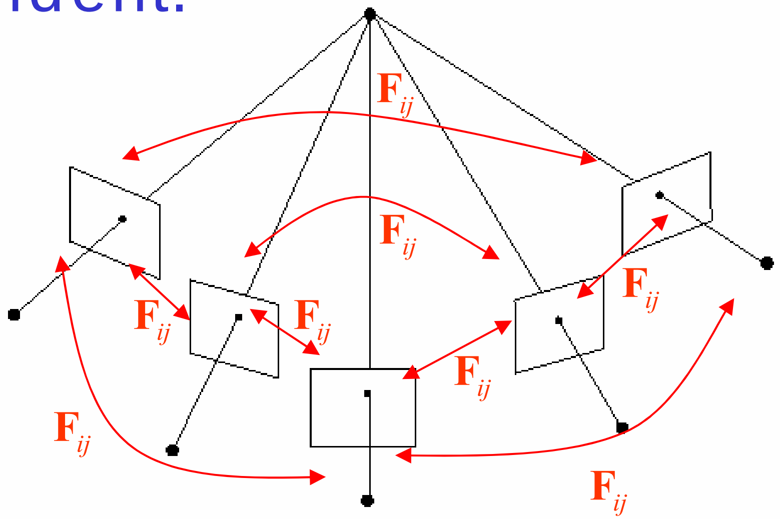
- Impose algebraic geometric constraints on stationary points seen in arbitrarily many views.

- Avoid estimating too many parameters: depths, rotations, translations



Geometric Dependencies

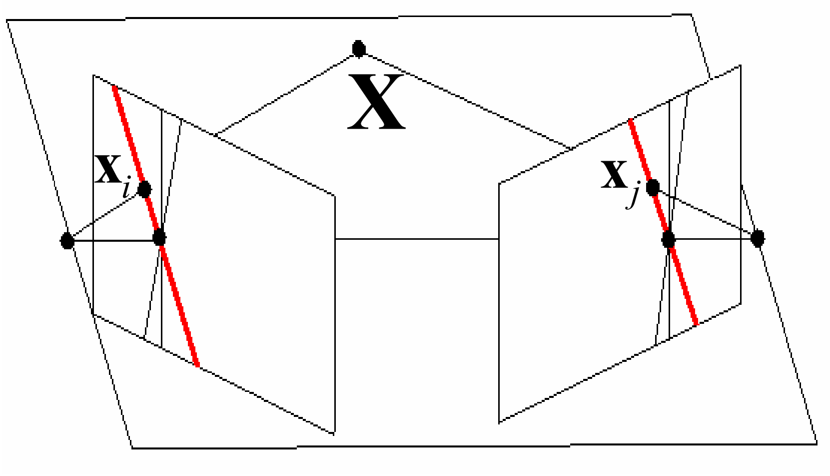
- Pairwise projective geometric relations are interdependent.



- Approach: define projective dependencies and restrict solutions to be globally consistent

Projective Bilinear Parameters

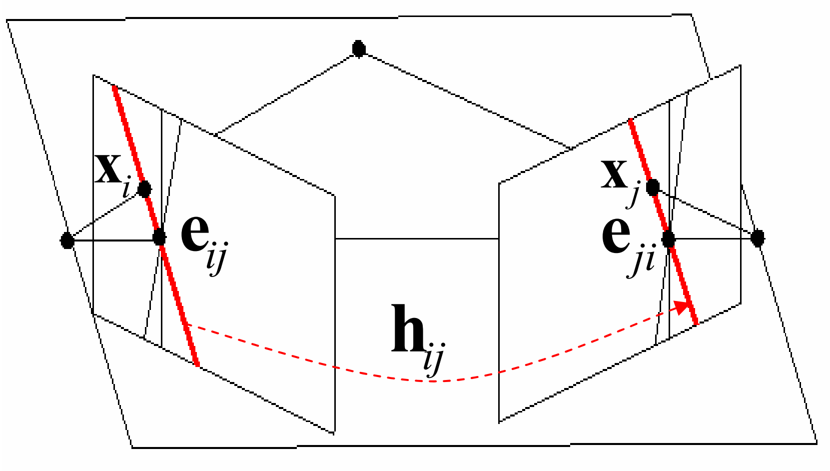
$$\mathbf{x}_j^T \mathbf{F}_{ij} \mathbf{x}_i = 0$$



$$\mathbf{F}_{ij} = \mathbf{A}_j^{-T} \left[\mathbf{T}_{ij}\right]_{\times} \mathbf{R}_{ij} \mathbf{A}_i^{-1}$$

Projective Bilinear Parameters

$$\mathbf{x}_j^T \mathbf{F}_{ij} \mathbf{x}_i = 0$$



epipoles

$\mathbf{e}_{ij}$ $\mathbf{e}_{ji}$

epipolar collineation

$\mathbf{h}_{ij}$

$$\mathbf{F}_{ij} \cong [\mathbf{e}_{ji}]_\times [\mathbf{p}_j \quad \mathbf{q}_j] \mathbf{h}_{ij} \begin{bmatrix} \mathbf{q}_i^T \\ -\mathbf{p}_i^T \end{bmatrix} [\mathbf{e}_{ij}]_\times$$

imaged 3D translation & rotation

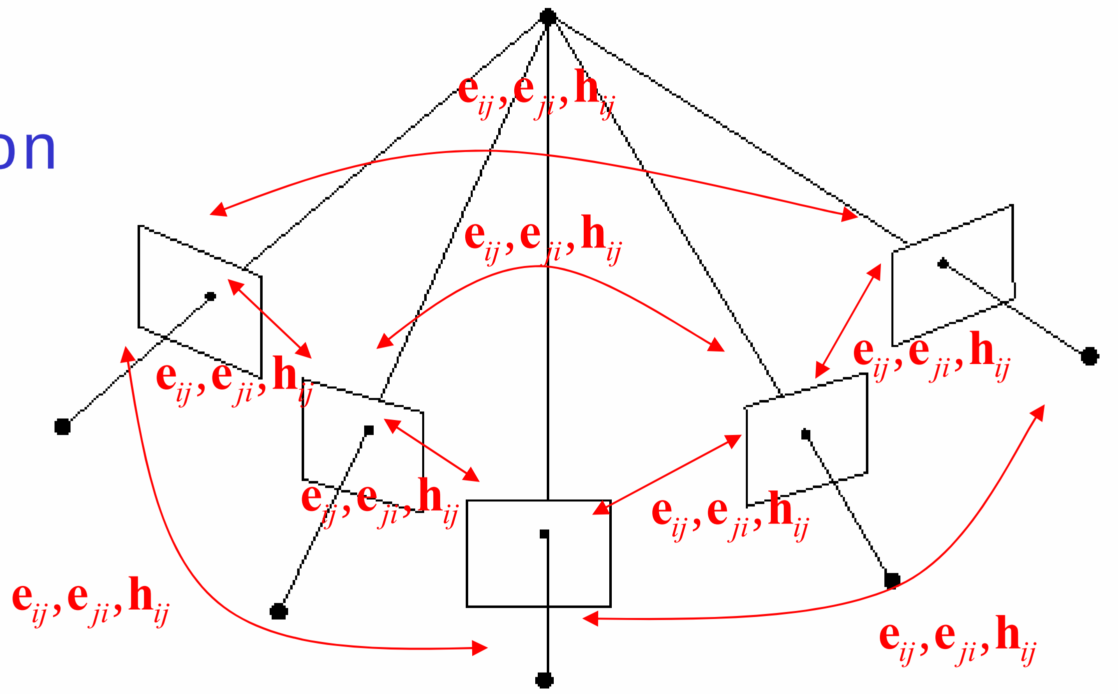
(Csurka, et.al., 1997)

Projective Parameters

- provide a complete projective model of camera configuration

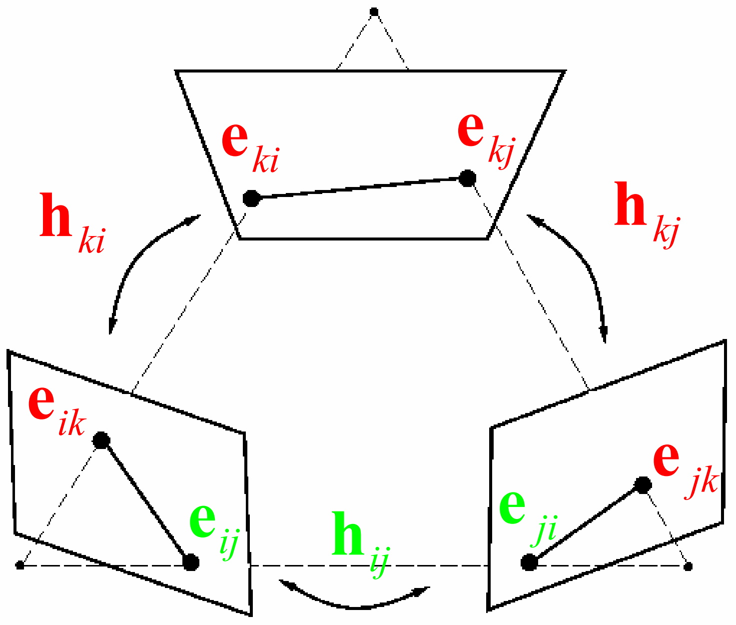
But...

- set of all pairwise parameters are still redundant
- not all images have sufficient overlap



Trifocal Dependencies

- derive dependencies among three fundamental matrices
- correctly models degrees of freedom in camera configuration
- geometrically consistent parameterized model of view triplets



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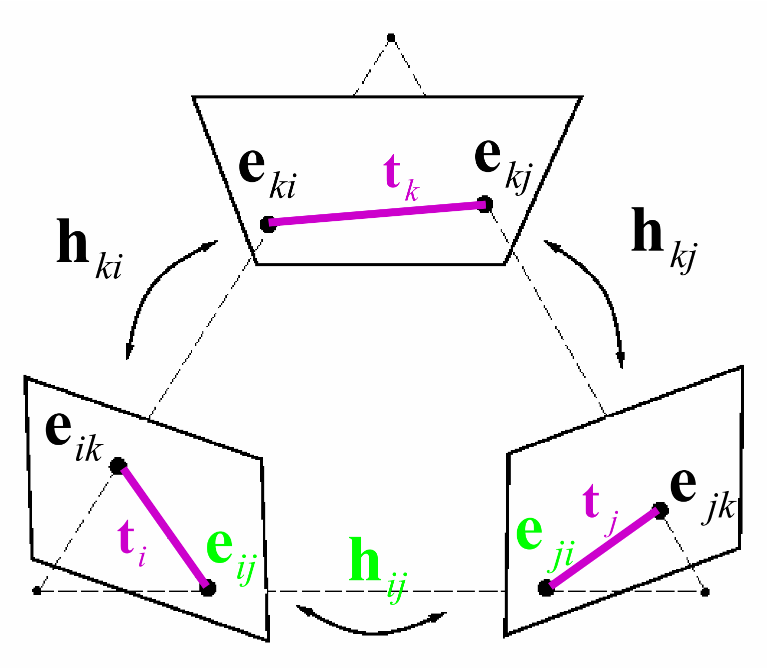

trifocal lines available from two fundamental matrices

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Recovering Camera Geometry

view i

view k

view j

← few correspondences →

Linear Initialization

8-point Algorithm

(Hartley, 1995)

Minimize $\sum_{\{(\mathbf{x}_i, \mathbf{x}_j)\}} \left(\mathbf{x}_j^T \mathbf{F}_{ij} \mathbf{x}_i \right)$ over all matching point pairs.

Rewrite bilinear constraints as

$$\begin{bmatrix} x_i x_j & y_i x_j & x_j & x_i y_j & y_i y_j & y_j & x_j & y_j & 1 \end{bmatrix} \mathbf{f}_{ij} = \mathbf{0}$$

where

$$\mathbf{f}_{ij} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{21} & f_{22} & f_{23} & f_{31} & f_{32} & f_{33} \end{bmatrix}^T$$

and solve linear system

$$\mathbf{A}\mathbf{f}_{ij} = \mathbf{0}$$

Projection to Parameter Space

Map linear estimate of fundamental matrix
to projective parameter space:

$$\mathbf{F}_{ij} \rightarrow \mathbf{p}_7^{ij} = \{\mathbf{e}_{ij}, \mathbf{e}_{ji}, \mathbf{h}_{ij}\} \rightarrow \mathbf{p}_4^{ij} = \{\boldsymbol{\gamma}_i, \boldsymbol{\gamma}_j, \mathbf{h}_{ij}\}$$

- parameterization requires choice of projective basis
- basis affects shape of error surface for nonlinear optimization

Geometric Objective Function

point-to-epipolar-line distance ~ image reprojection error

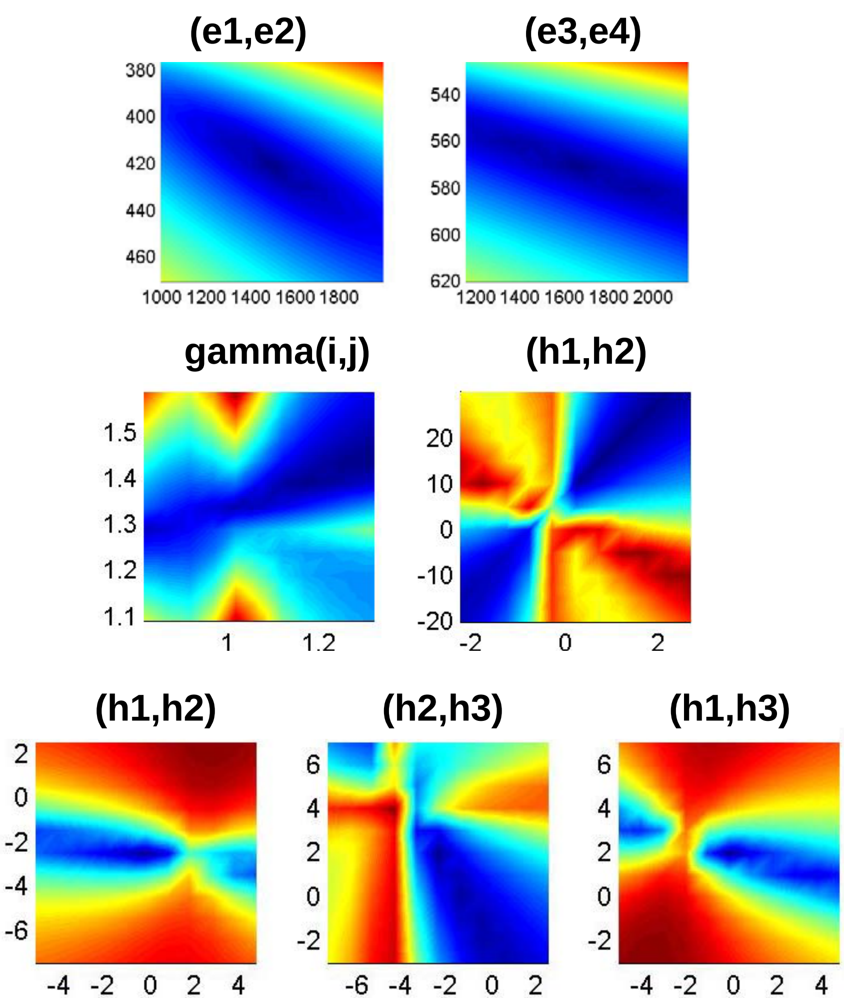
weighted residual of bilinear constraint

$$E(\mathbf{x}_i, \mathbf{x}_j; \mathbf{p}_7^{ij}) = w_{ij}\ \mathbf{x}_j^T \mathbf{F}_{\mathbf{p}_7^{ij}} \mathbf{x}_i$$

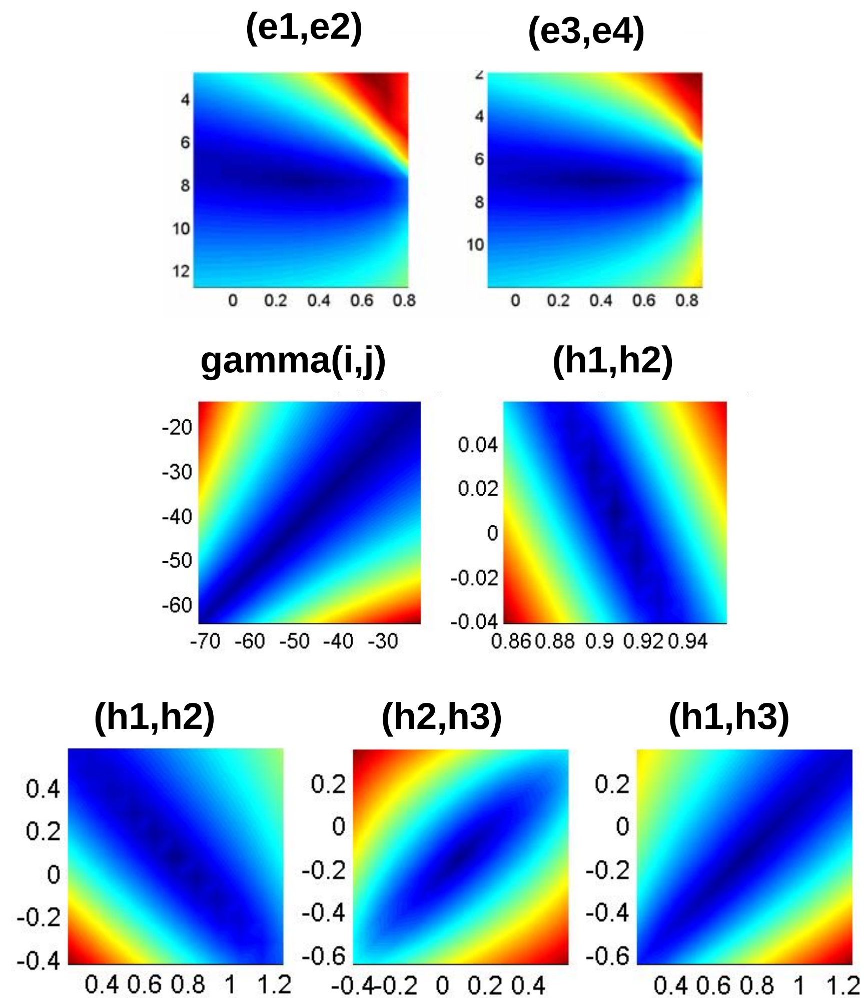
$$w_{ij} = \frac{1}{(\mathbf{F}_{ij}\mathbf{x}_i)_1^2 + (\mathbf{F}_{ij}\mathbf{x}_i)_2^2} + \frac{1}{(\mathbf{F}_{ij}^T\mathbf{x}_j)_1^2 + (\mathbf{F}_{ij}^T\mathbf{x}_j)_2^2}$$

Error Surface Depends on Basis

canonical basis



geometrically defined basis

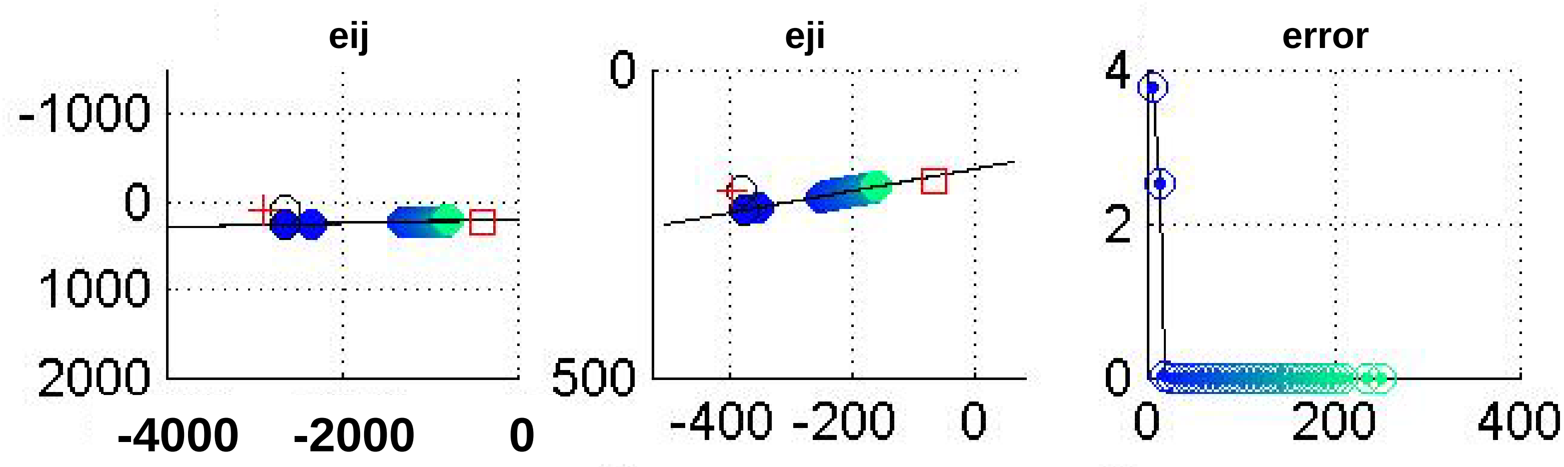


Nonlinear Trifocal Estimation

i	k	j

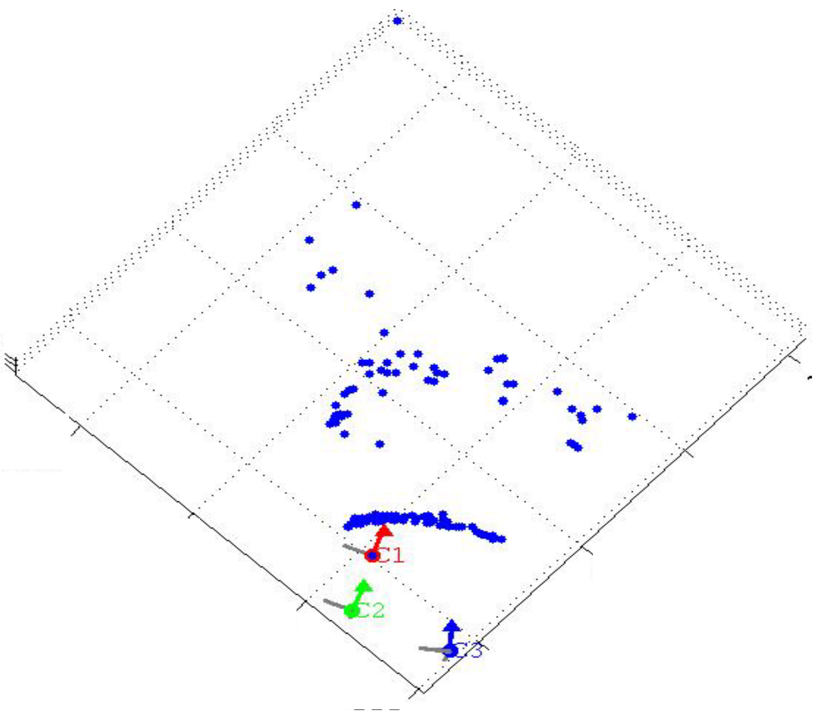
1. Initialize epipolar geometry
   - 8-point algorithm: linear solution to fundamental matrix for all view pairs
   - extract epipoles and epipolar collineations
2. 7D nonlinear minimization: bifocal parameters for view pairs (i,k) (j,k)
3. Trifocally constrained estimation for view pair (i,j)
   - compute trifocal lines
   - project parameters to trifocally constrained space
   - 4D nonlinear minimization for bifocal parameters

Convergence

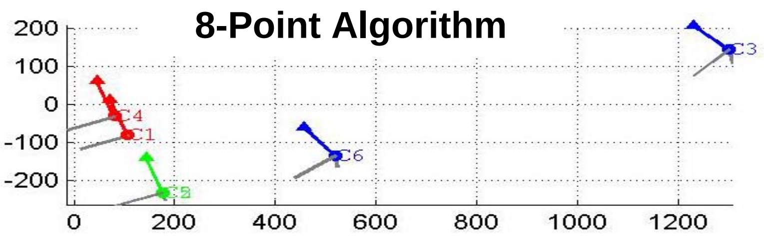


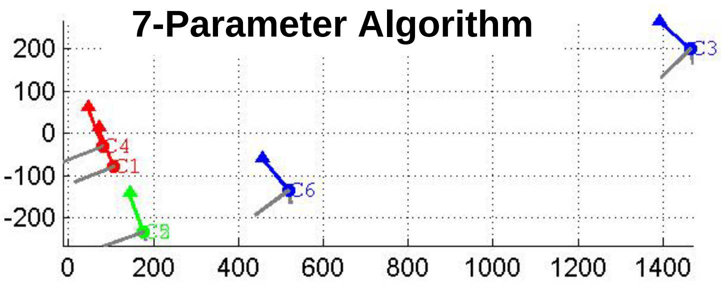
Ground Truth
8-point Algorithm
7-Parameter Search
Trifocal Projection
4-Parameter Search

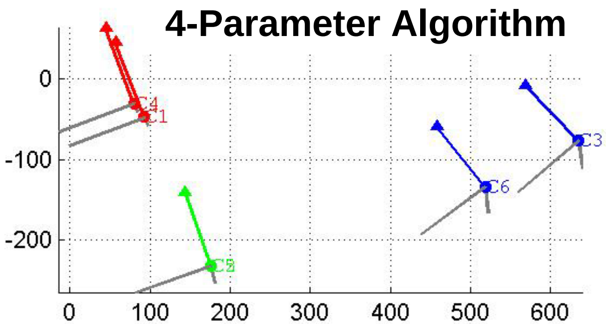
Ground Truth



Results





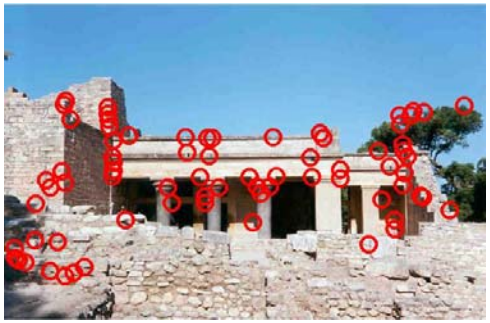


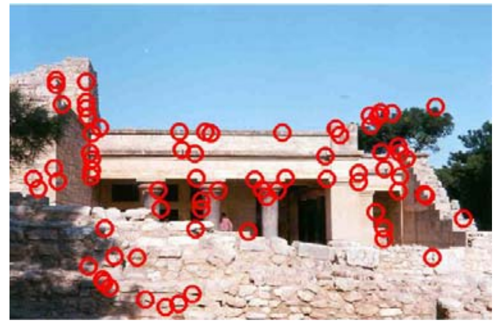
knossos sequence

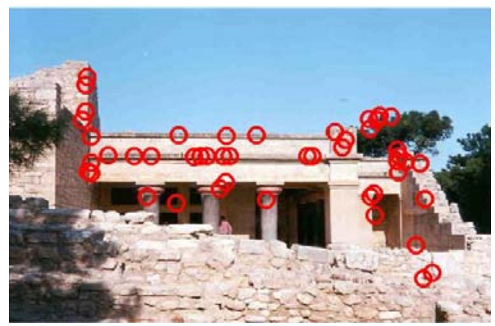
view i

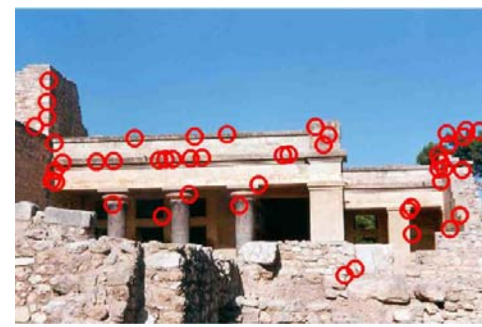
view k

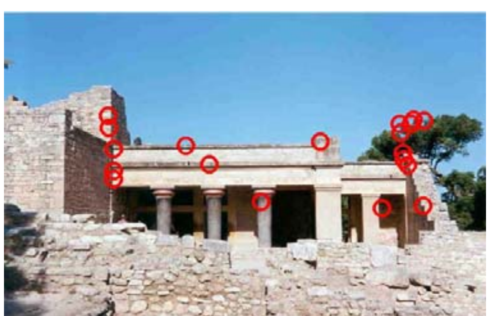
view j



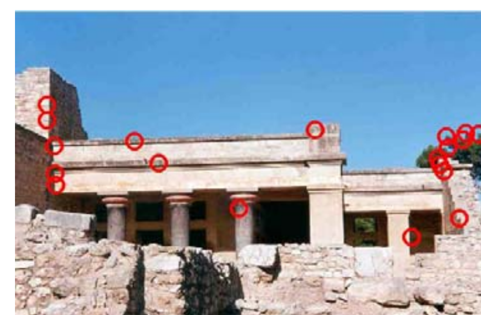




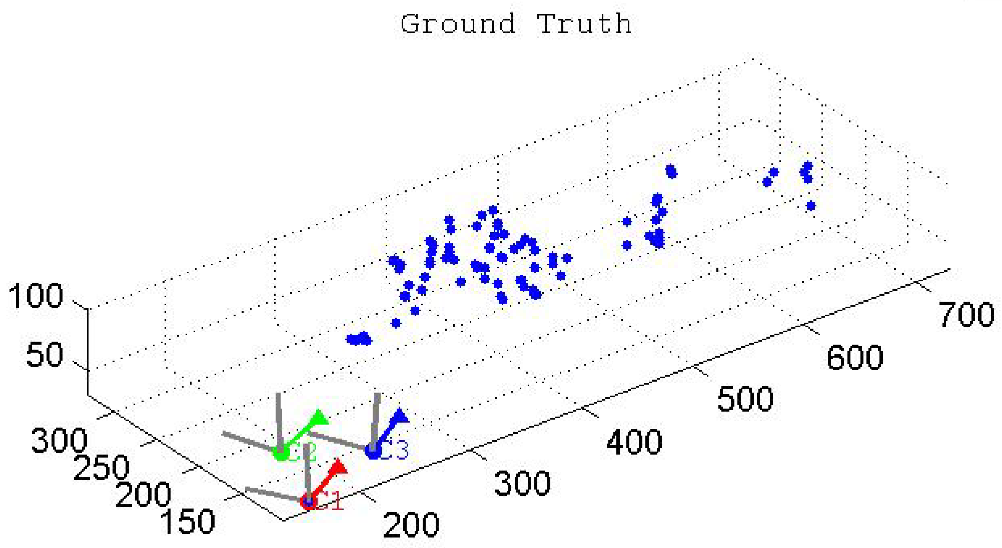




few
correspondences

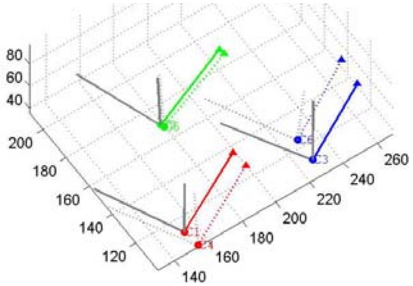


Ground Truth

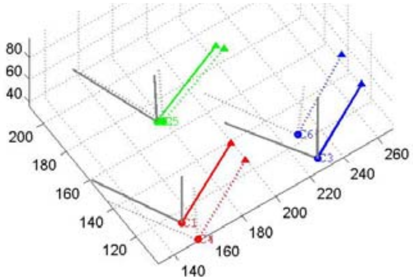


Results

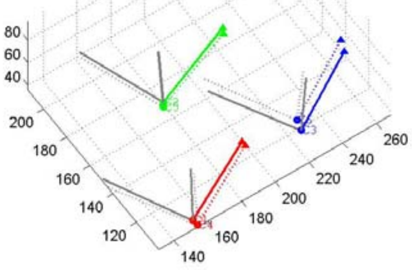
8-Point Algorithm



7-Parameter Algorithm



4-Parameter Algorithm



Summary

- Imposing projective constraints on camera geometry corrects the estimation of epipolar geometry
- Resulting camera configuration for multiple cameras is globally consistent

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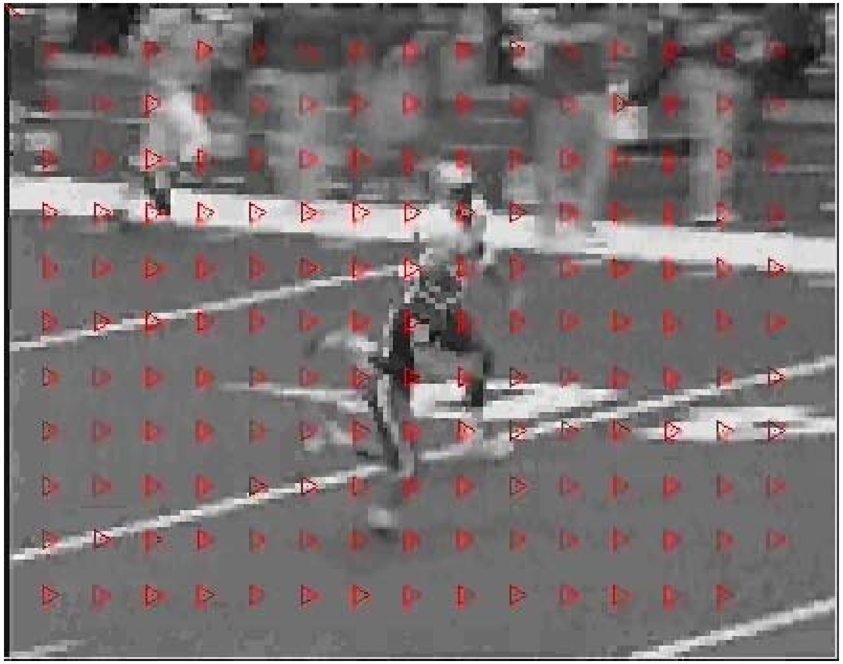
Camera and Scene Motion

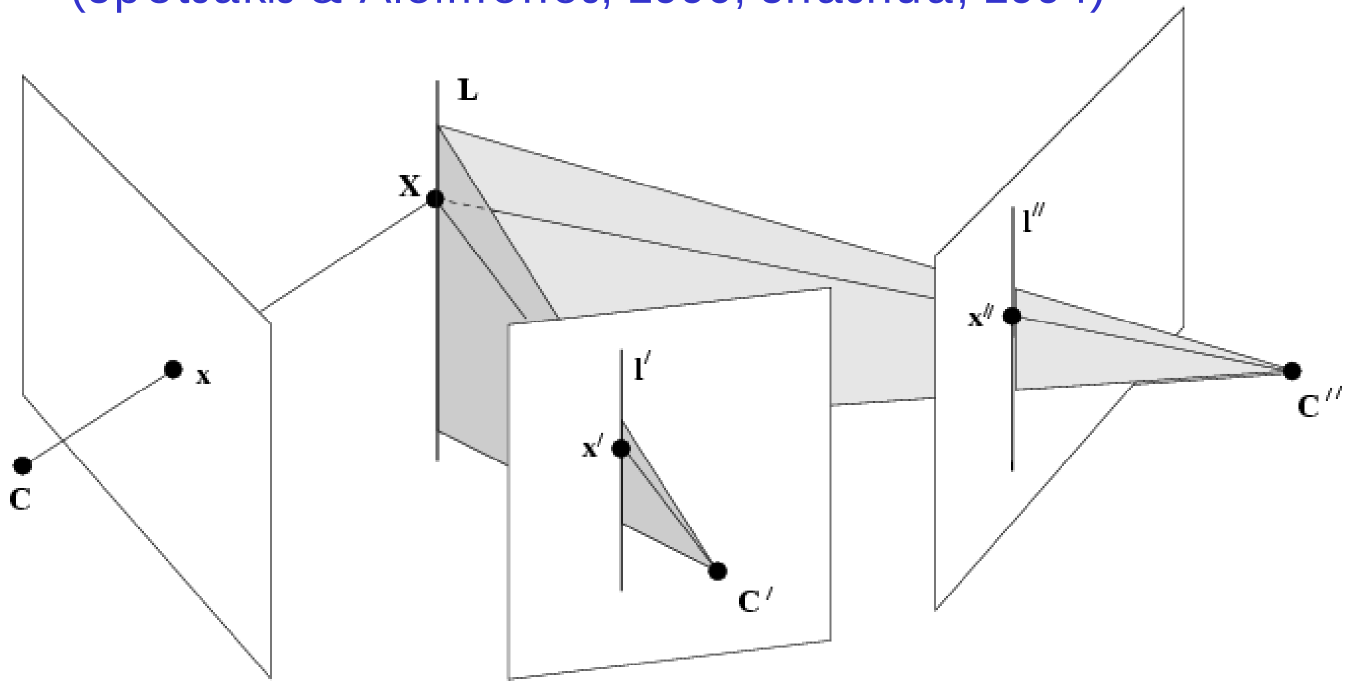


Combining Intensity and Geometry

trifocal tensor

projective linear form relating a point-line-line

(Spetsakis & Aloimonos, 1990; Shashua, 1994)



$$T(\mathbf{x}_i, \mathbf{l}_j, \mathbf{l}_k) = 0$$

Tensor Brightness Constraint

(Shashua & Hannah, 1995; Shashua & Stein, 1997)

$$u\,I_x + v\,I_y + I_t = 0$$

$$u = x - x_0 \qquad v = y - y_0$$

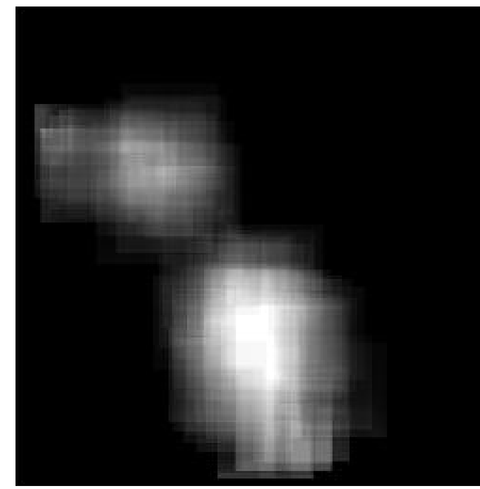
$$ax + by + c = 0$$

$$(a,b,c)^T \cong \begin{bmatrix} I_x \\ I_y \\ I_t - x_0 I_x - y_0 I_y \end{bmatrix}$$

- Horn-Schunk brightness constraint is linear in point coordinates
- Defines line in each image containing matching point
- Spatiotemporal gradient at every pixel provides test of rigid motion

Motion Boundary Detection

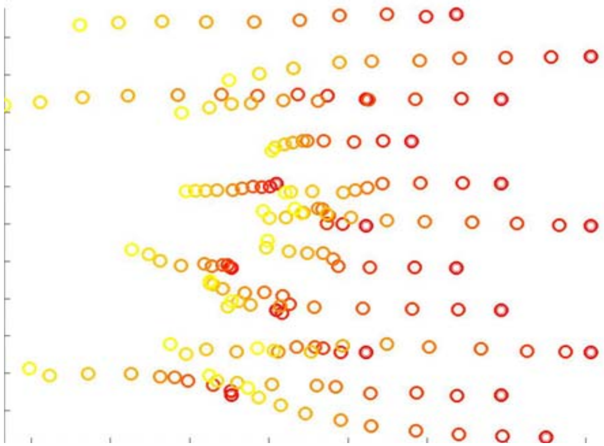
- Partition image into windows and solve for trifocal tensor coefficients.
- Only regions with rigid 3D motion have a good fit.
- Sum residual error of tensor solution.



- High residuals indicate regions that cross a motion boundary.

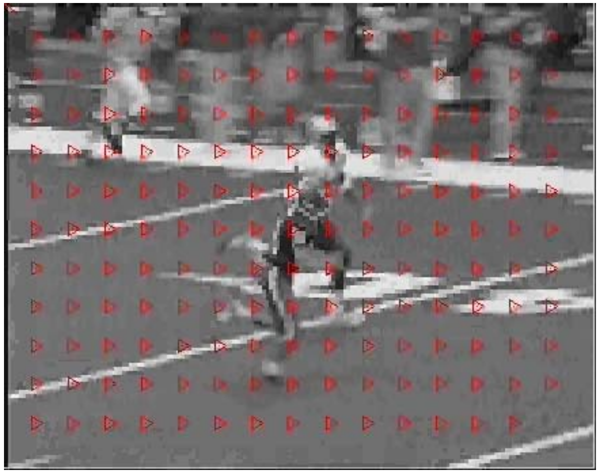
Multiple Frame Flow

- Track points over many frames



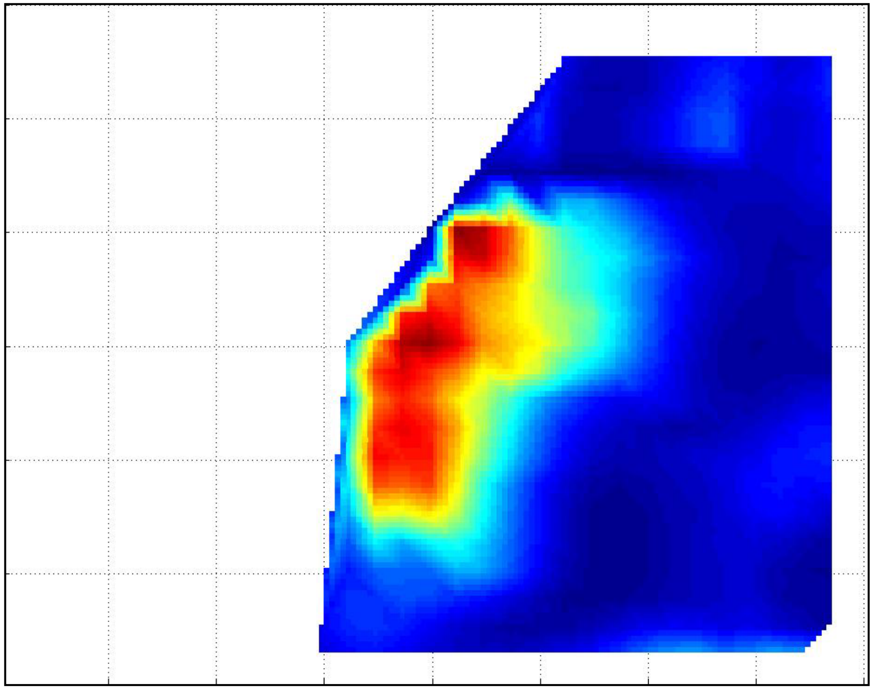
- Multi-frame tracks fall into separable classes

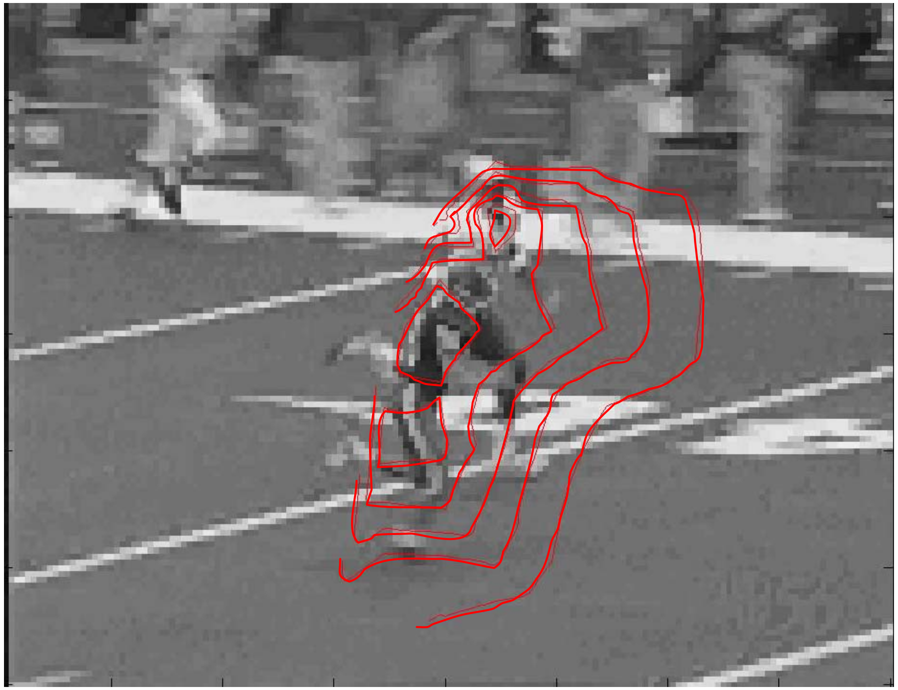
- Robustly fit tracks to linear approximation of instantaneous planar motion



$$\mathbf{x}(t) = \mathbf{x}_0 + t\,[\mathbf{A}\mathbf{x}_0 + \mathbf{b}]$$

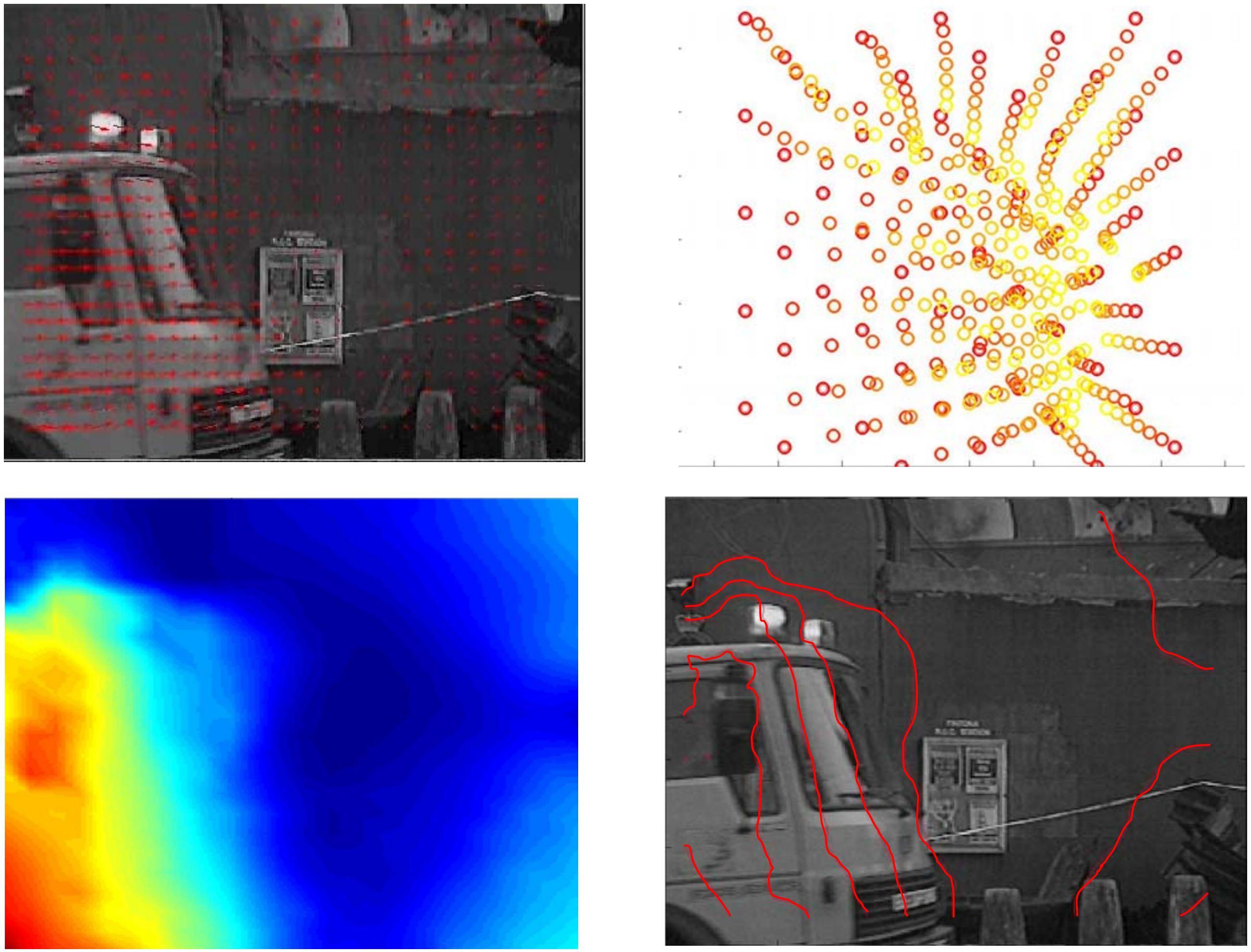
Detecting Independent Motions





Residual error of estimated motion model on all point tracks

Complexity of Motion Model



Conclusions

When possible, use domain and task knowledge to choose model:

- What type of information is needed
- What aspects of the imaging conditions are known or controlled
- What types of uncertainty can be modeled and compensated for

Future Needs

Role of learning in motion analysis:

- Supervised learning of geometric motion classes
- Data-driven model selection by flow classification
- Robust estimation of appropriate motion model
- Adaptive, time-varying estimation

END