

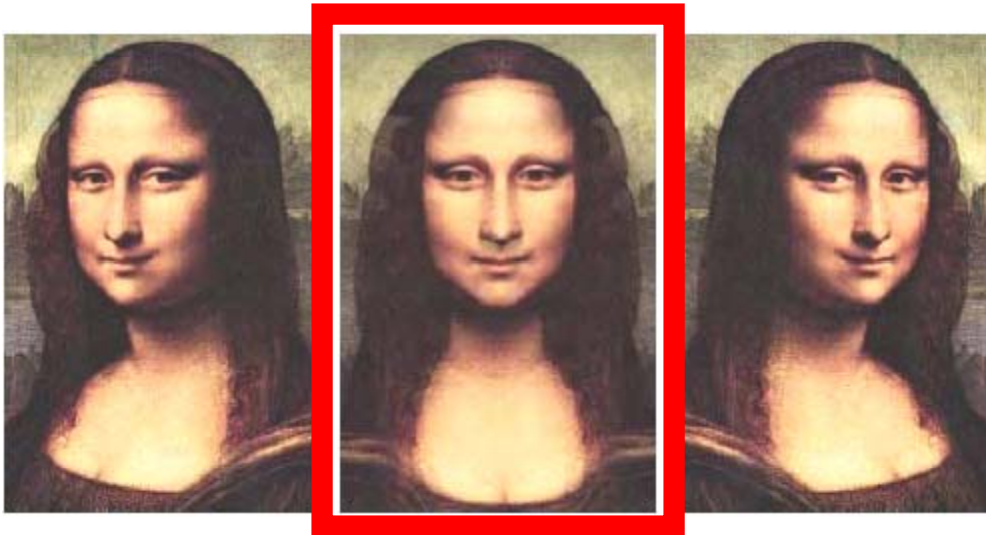


# Image Morphing and Warping

CS635 Spring 2010

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Department of Computer Science  
Purdue University

# Motivation – Rendering from Images



- Given
  - left image
  - right image
- Create intermediate images
  - simulates camera movement



## Related Work

- Panoramas ([Chen95/QuicktimeVR], etc)
  - user can look in any direction at few given locations but camera translations are ***not*** allowed...



# Topics

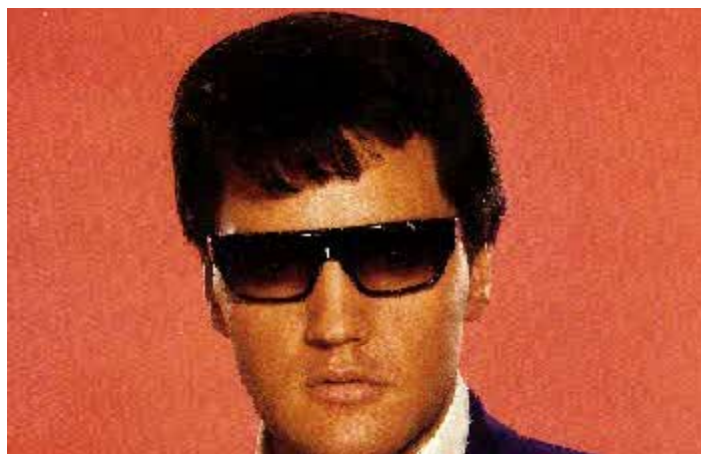
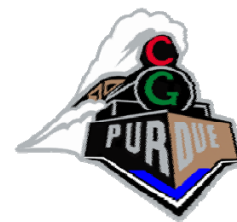
- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)



# Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

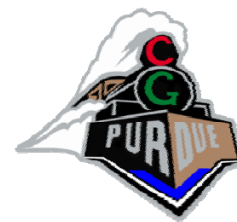
# Image Morphing





# Image Morphing

- Identify correspondences between input/output image
- Produce a sequence of images that allow a smooth transition from the input image to the output image

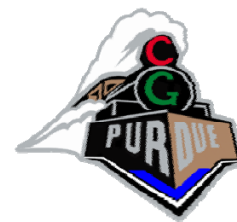


# Image Morphing

## 1. Correspondences







# Image Morphing

## 1. Correspondences

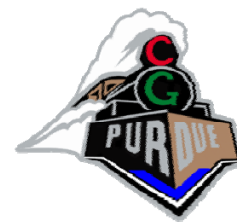




# Image Morphing

## 1. Correspondences





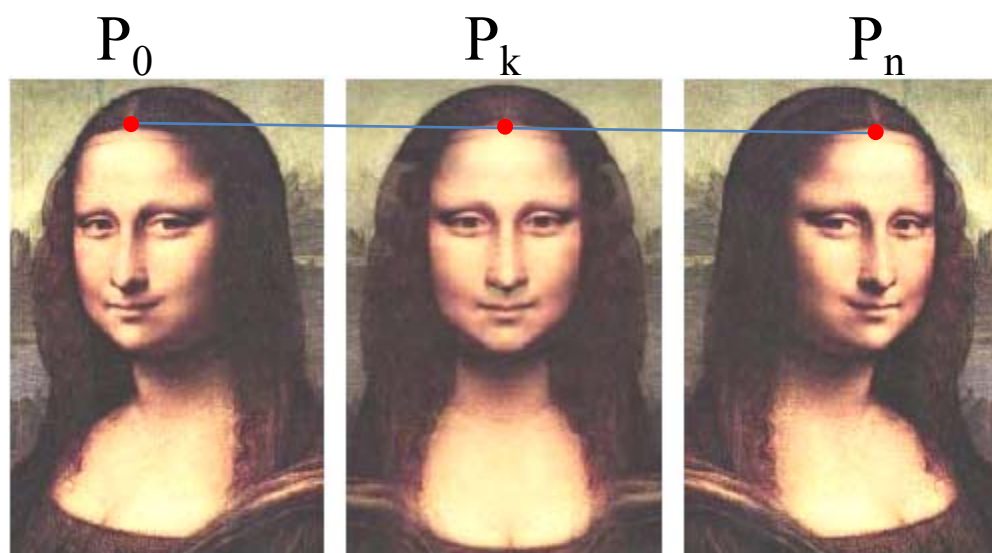
# Image Morphing

## 1. Correspondences





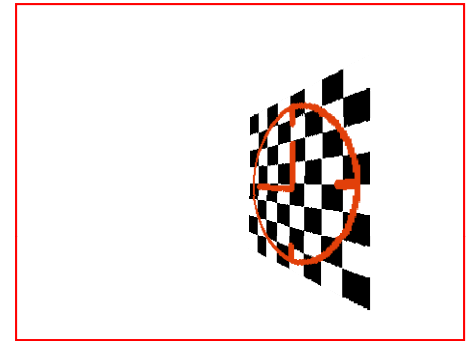
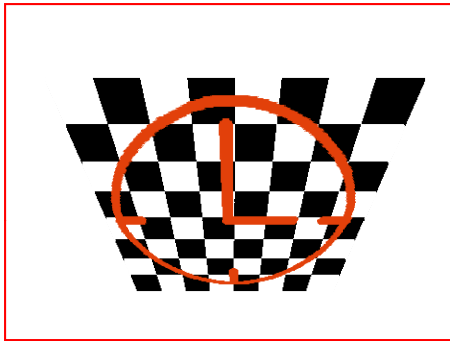
# Image Morphing



1. Correspondences
2. Linear interpolation

$$P_k = \left(1 - \frac{k}{n}\right)P_0 + \frac{k}{n}P_n$$

# Image Morphing

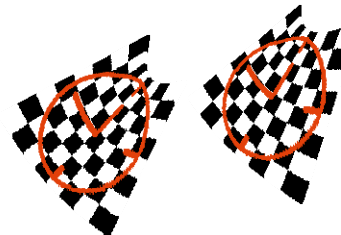
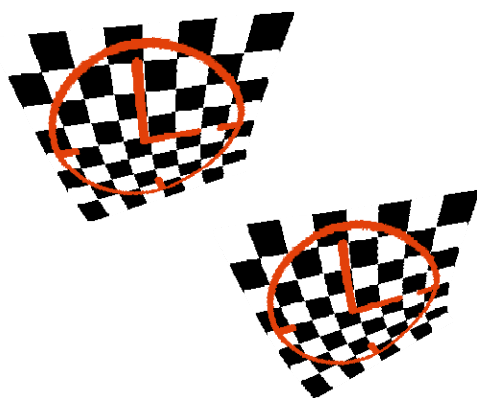
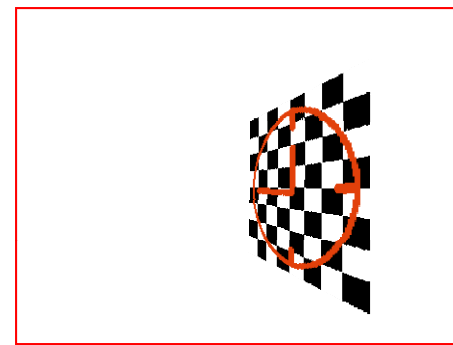




# Image Morphing



Image morphing is not  
shape preserving





# Topics

- Image morphing (2D)
- **View morphing (2D+)**
- Image warping (3D)

# View Morphing

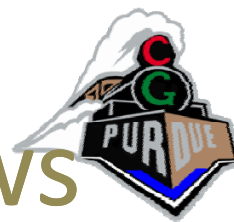




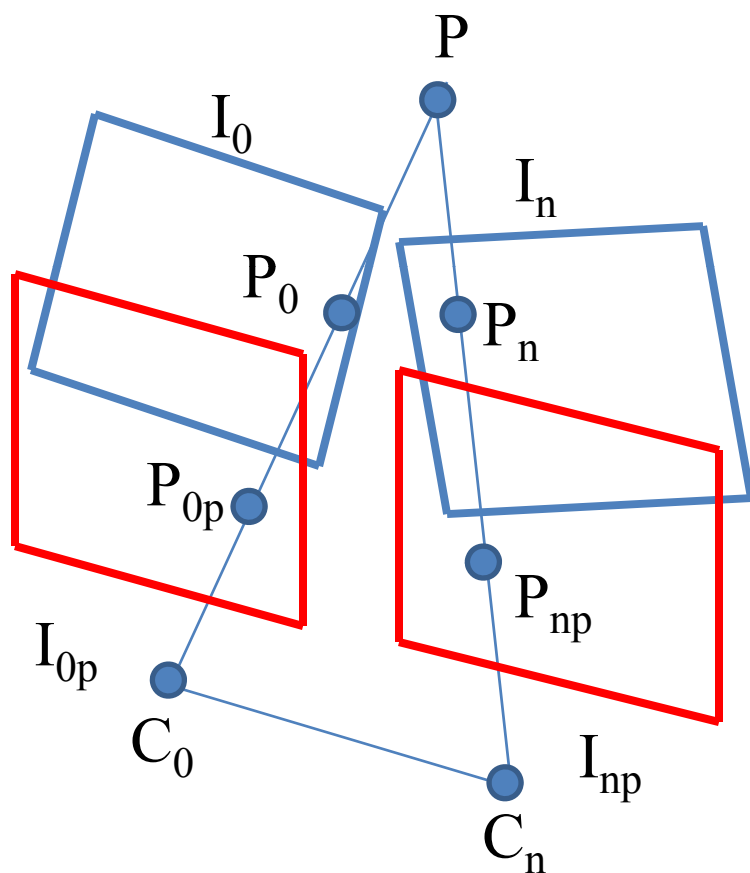


# View Morphing

- Shape preserving morph
- Three step algorithm
  - Prewarp first and last images to parallel views
  - Image morph between prewarped images
  - Postwarp to interpolated view

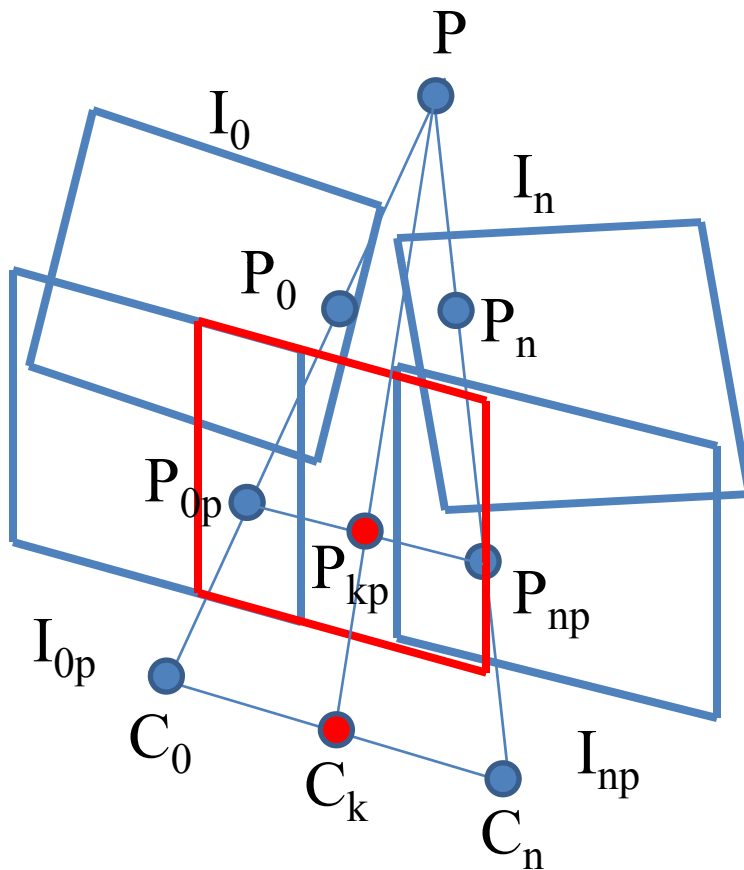


# Step 1: prewarp to parallel views



- Parallel views
  - same image plane
  - image plane parallel to segment connecting the two centers of projection
- Prewarp
  - compute parallel views  $I_{0p}$ ,  $I_{np}$
  - rotate  $I_0$  and  $I_n$  to parallel views
  - prewarp correspondence is  $(P_0, P_n) \rightarrow (P_{0p}, P_{np})$

## Step 2: morph parallel images

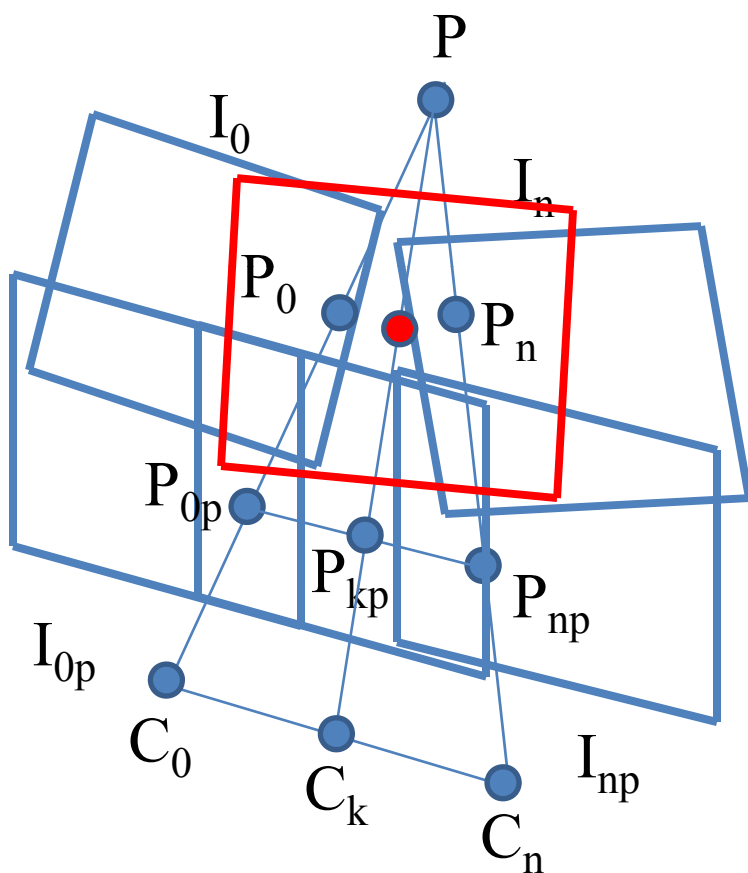


- Shape preserving
- Use prewarped correspondences
- Interpolate  $C_k$  from  $C_0$   $C_n$



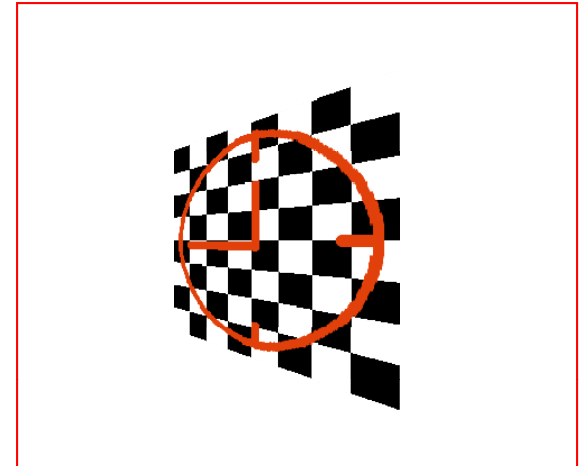
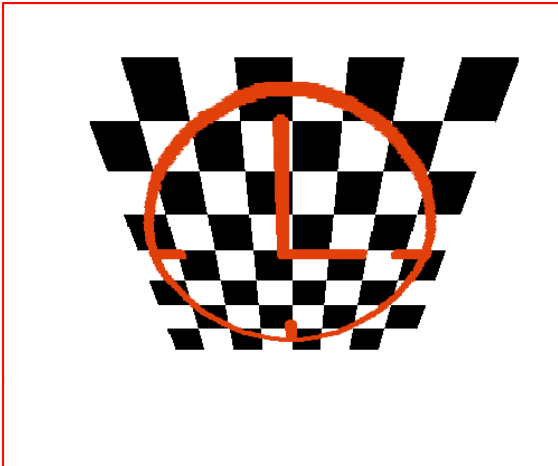
## Step 3: postwarp image

- Postwarp morphed image
  - create intermediate view
    - $C_k$  is known
    - interpolate view direction and tilt
  - rotate morphed image to intermediate view





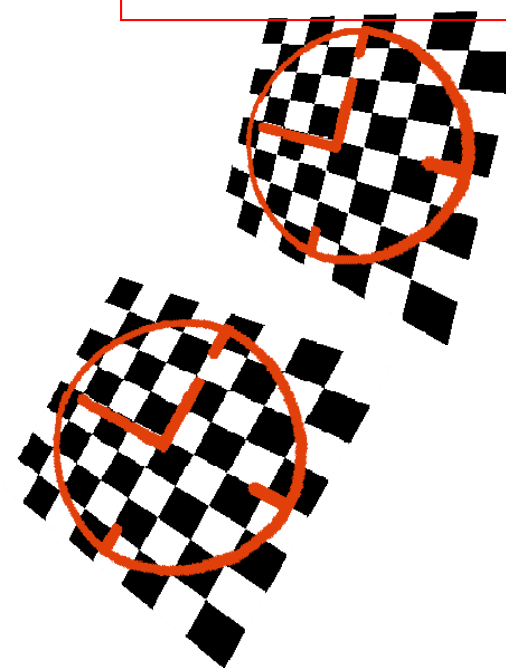
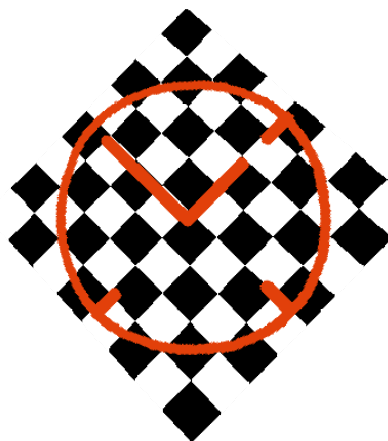
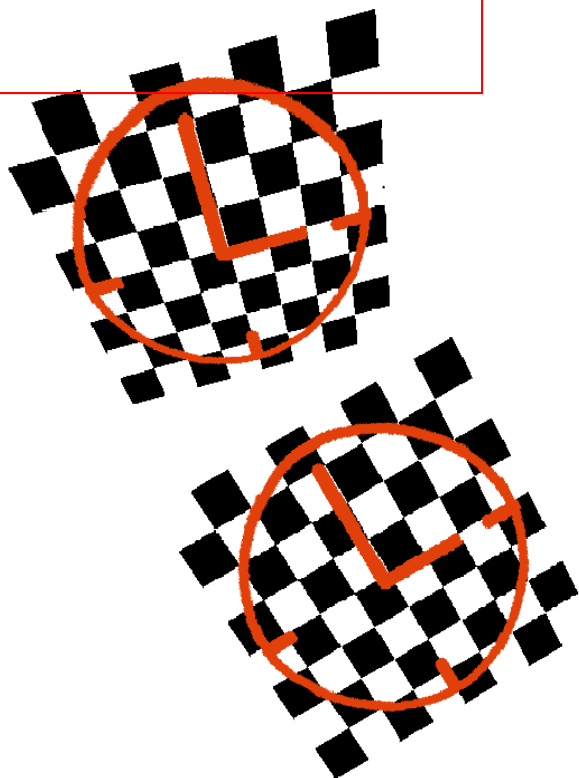
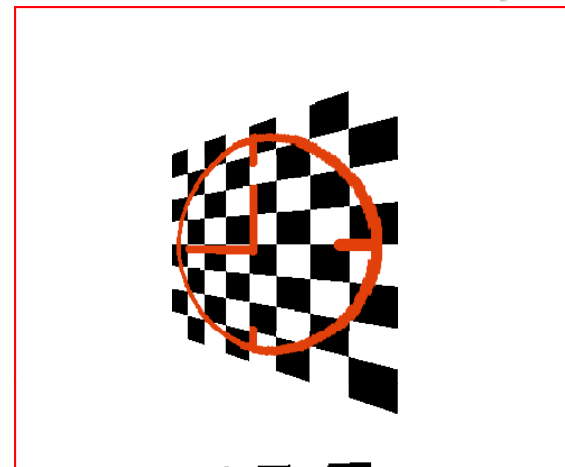
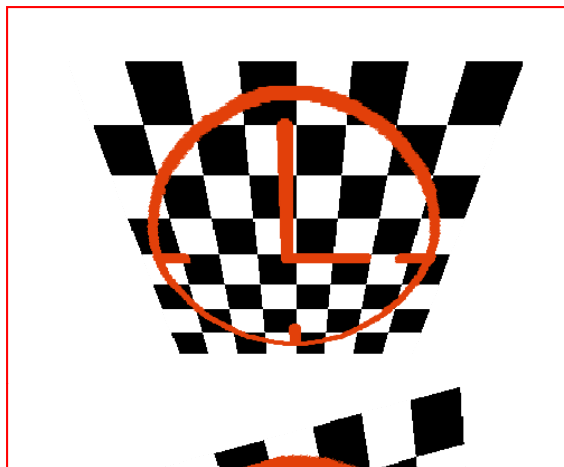
# View morphing





# View morphing

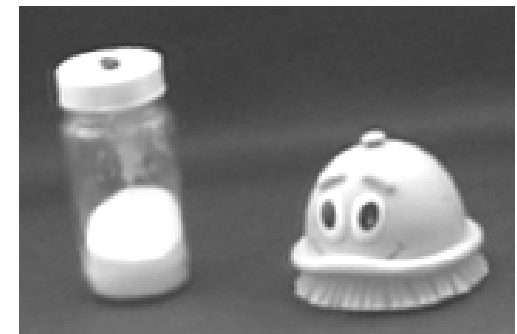
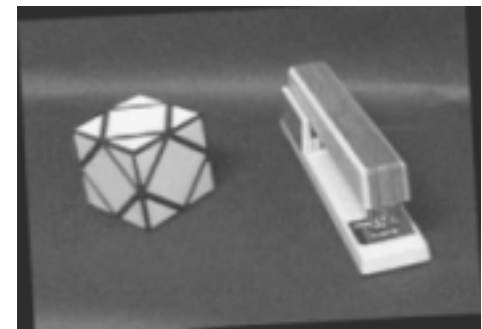
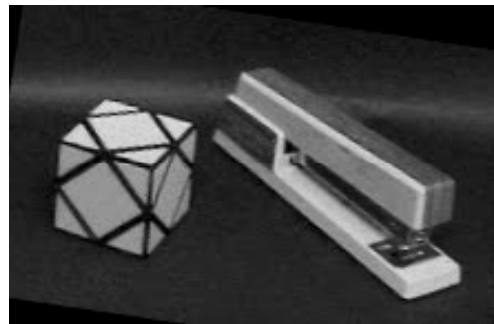
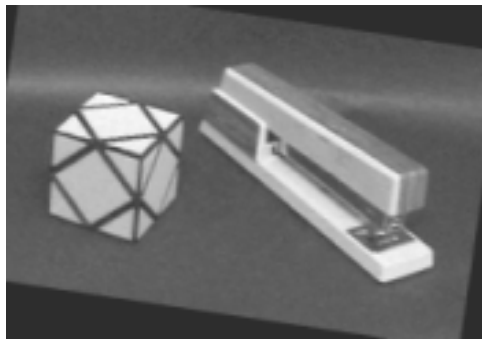
- View morphing is shape preserving



# View Morphing Examples



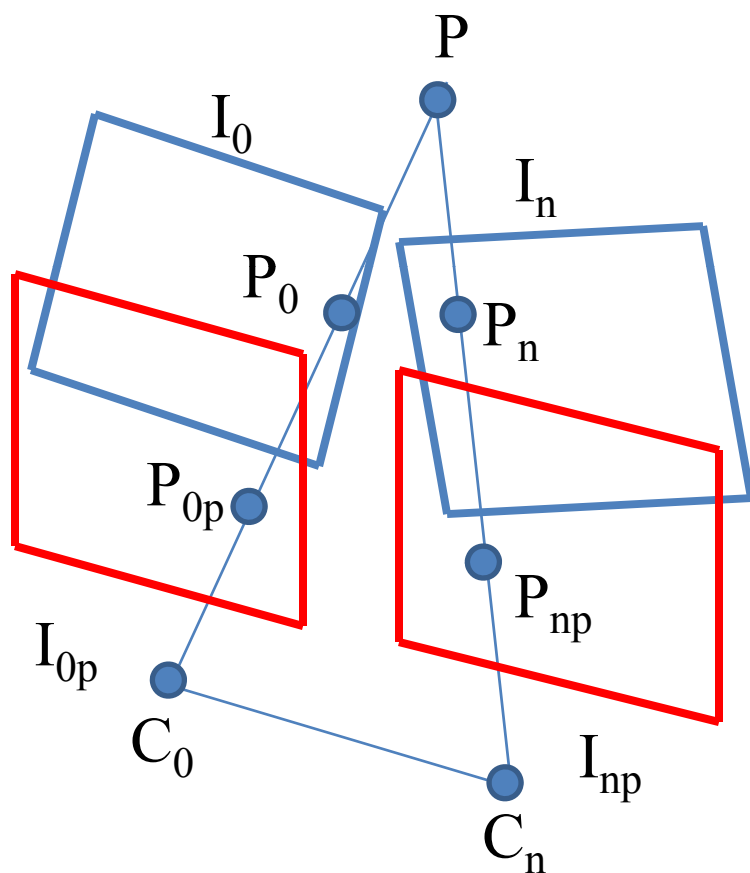
- Using computer vision/stereo reconstruction techniques





# Image Transformations

- Intuitively, how do you compute the matrix  $M$  by which to transform  $P_0$  to  $P_{0p}$ ?







# Image Transformations

- A geometric relationship between input  $(u,v)$  and output pixels  $(x,y)$ 
  - Forward mapping:
$$(x,y) = (X(u,v), Y(u,v))$$
  - Inverse mapping:
$$(u,v) = (U(x,y), V(x,y))$$



# Image Transformations

- General matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and operates in the “homogeneous coordinate system”.



# Affine Transformations

- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and accommodates translations, rotations, scale, and shear.

- How many unknowns? How to create matrix?



# Affine Transformations

- Transformation can be inferred from correspondences; e.g.,

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \Leftrightarrow \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

- Given  $\geq 3$  correspondences can solve for T



# Perspective/Projective Transformations

- Matrix form is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and it accommodates foreshortening of distant line  
and convergence of lines to a vanishing point;  
also, straight lines are maintained but not their  
mutual angular relationships, and  
only parallel lines parallel to the projection plane  
remain parallel



# Perspective/Projective Transformations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- How many unknowns?
- How many correspondences are needed?



# Perspective/Projective Transformations

- Solve

$$\begin{pmatrix} u_0 & v_0 & 1 & 0 & 0 & 0 & -u_0v_0 & -v_0x_0 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1x_1 & -v_1x_1 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -u_2x_2 & -v_2x_2 \\ u_3 & v_3 & 1 & 0 & 0 & 0 & -u_3x_3 & -v_3x_3 \\ 0 & 0 & 0 & u_0 & v_0 & 1 & -u_0y_0 & -v_0y_0 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1y_1 & -v_1y_1 \\ 0 & 0 & 0 & u_2 & v_2 & 1 & -u_2y_2 & -v_2y_2 \\ 0 & 0 & 0 & u_3 & v_3 & 1 & -u_3y_3 & -v_3y_3 \end{pmatrix}$$

$$A = b$$

where  $A$  is the vector of  
unknown coefficients  $a_{ij}$



# Topics

- Image morphing (2D)
- View morphing (2D+)
- Image warping (3D)

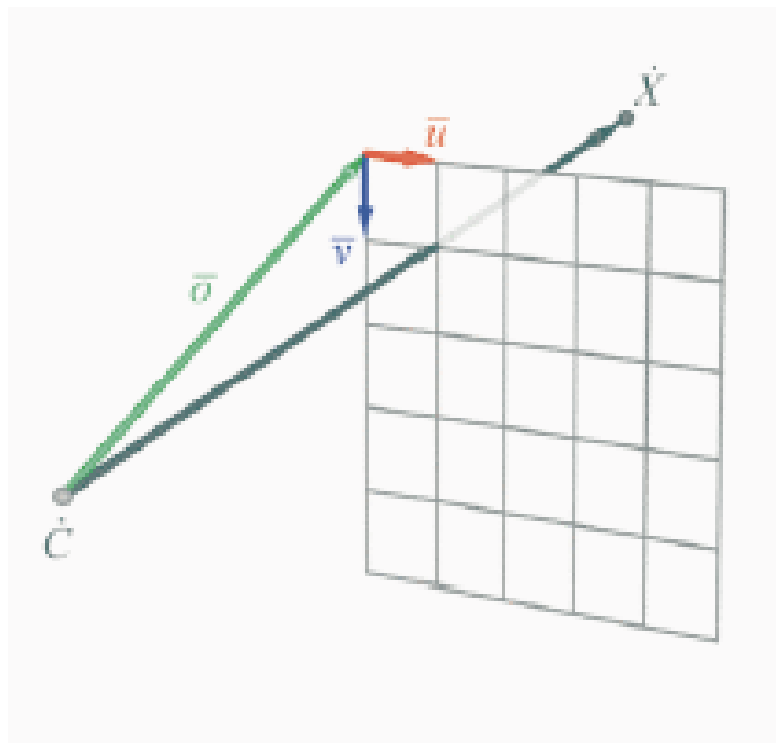




# 3D Image Warping

- Goal: “warp” the pixels of the image so that they appear in the correct place for a new viewpoint
- Advantage:
  - Don’t need a geometric model of the object/environment
  - Can be done in time proportional to screen size and (mostly) independent of object/environment complexity
- Disadvantage:
  - Limited resolution
  - Excessive warping reveals several visual artifacts (see examples)

# 3D Image Warping Equations

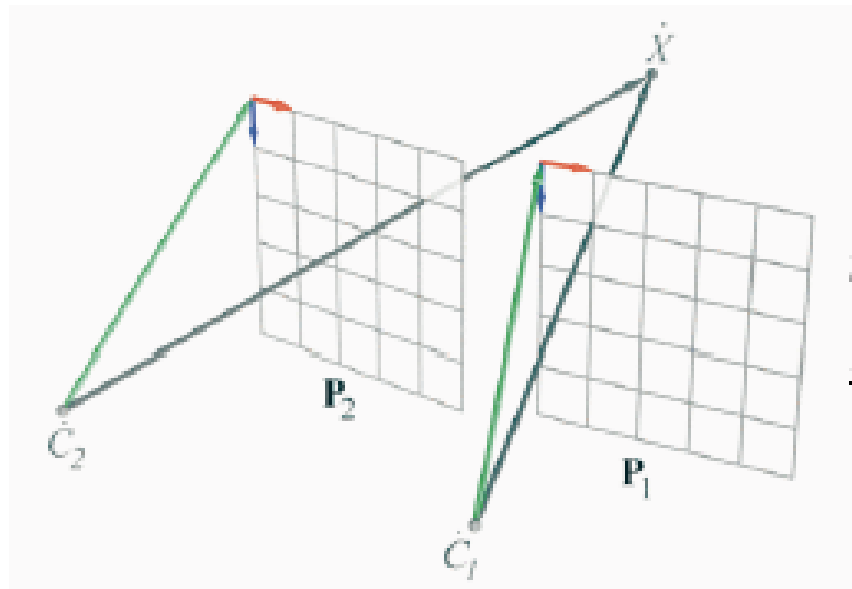


$$P = \begin{bmatrix} u_x & v_x & O_x \\ u_y & v_y & O_y \\ u_z & v_z & O_z \end{bmatrix}$$

$$\vec{X} = \vec{C} + t P \vec{x}$$

Some pictures courtesy of SIGGRAPH '99 course notes  
(Leonard McMillan)

# 3D Image Warping Equations



$$\dot{C}_2 + t_2 P_2 \bar{x}_2 = \dot{C}_1 + t_1 P_1 \bar{x}_1$$

$$t_2 P_2 \bar{x}_2 = \dot{C}_1 - \dot{C}_2 + t_1 P_1 \bar{x}_1$$

$$t_2 \bar{x}_2 = P_2^{-1} (\dot{C}_1 - \dot{C}_2) + t_1 P_2^{-1} P_1 \bar{x}_1$$

$$\frac{t_2}{t_1} \bar{x}_2 = \frac{1}{t_1} P_2^{-1} (\dot{C}_1 - \dot{C}_2) + P_2^{-1} P_1 \bar{x}_1$$

$$\bar{x}_2 \doteq \underbrace{\frac{1}{t_1} P_2^{-1} (\dot{C}_1 - \dot{C}_2)}_{\delta} + \underbrace{P_2^{-1} P_1}_{H_{21}} \bar{x}_1$$

# 3D Image Warping Equations



McMillan & Bishop Warping Equation:

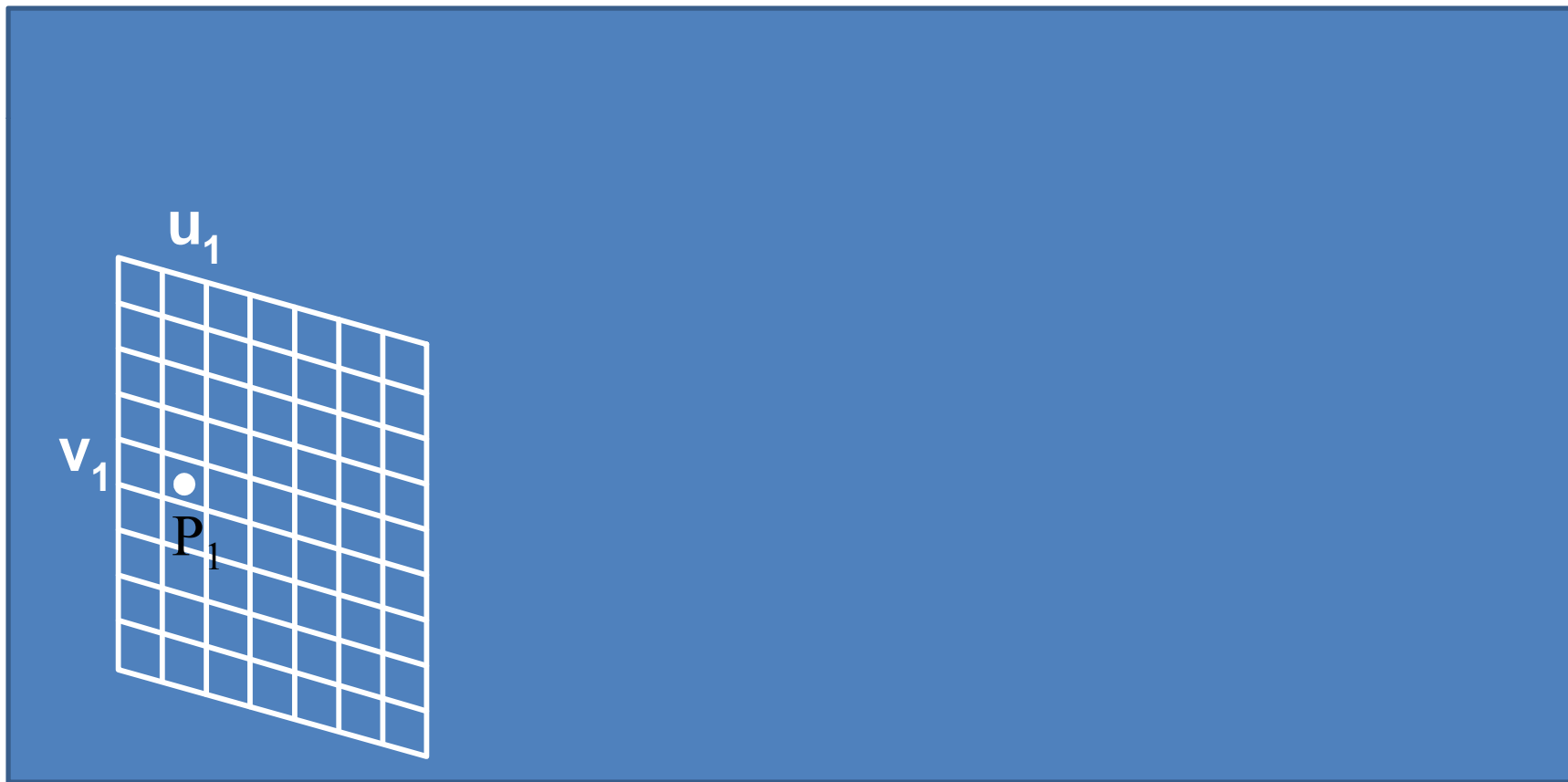
$$x_2 = \underbrace{\delta(x_1) P_2^{-1} (c_1 - c_2)}_{\text{Move pixels based on distance to eye}} + \underbrace{P_2^{-1} P_1 x_1}_{\sim \text{Texture mapping}}$$

- Per-pixel distance values are used to warp pixels to their correct location for the current eye position

# 3D Image Warping Equations



- Images enhanced with per-pixel depth  
[McMillan95]

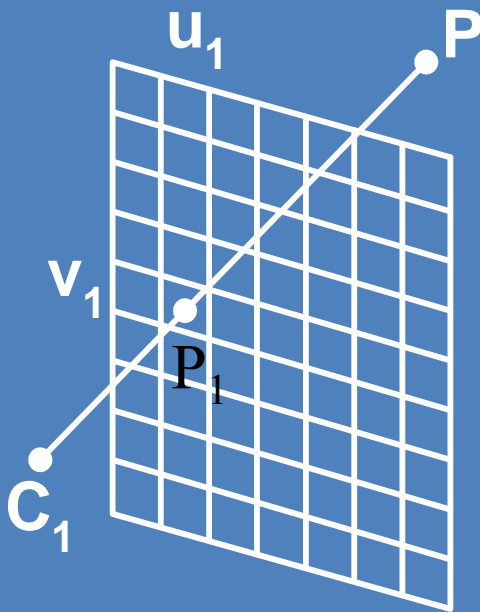


# 3D Image Warping Equations



$$\dot{P} = \dot{C}_1 + (\bar{c}_1 + u_1 \bar{a}_1 + v_1 \bar{b}_1) w_1$$

$$w_1 = \frac{C_1 P}{C_1 P_1}$$



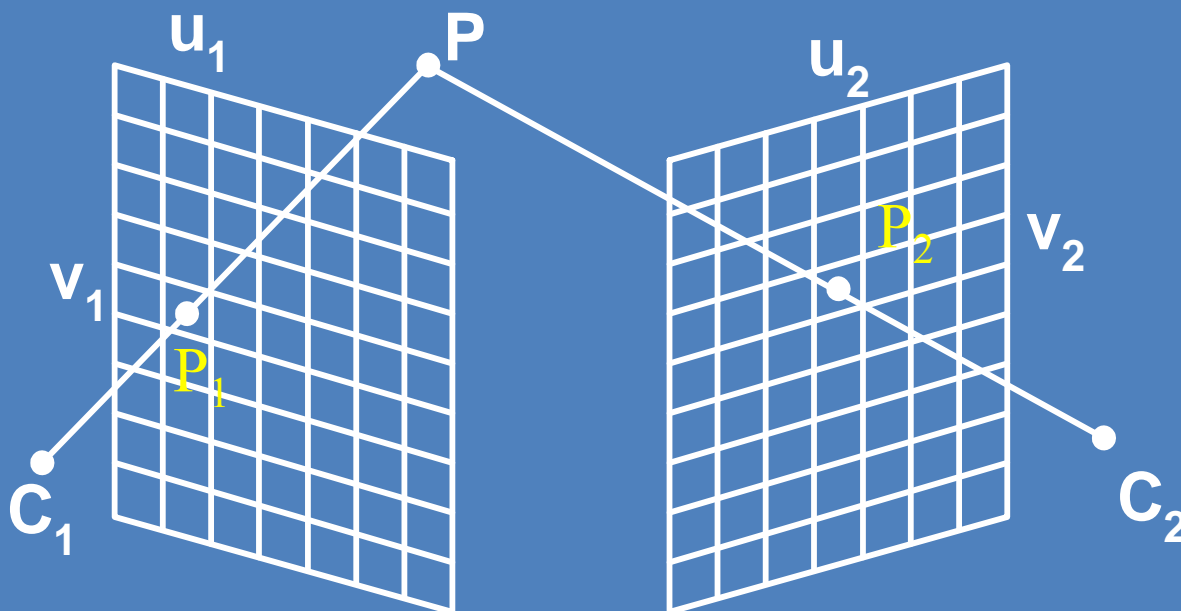
- $1/w_1$  also called generalized disparity
- another notation  $\delta(u_1, v_1)$

# 3D Image Warping Equations



$$\dot{P} = \dot{C}_1 + (\bar{c}_1 + u_1 \bar{a}_1 + v_1 \bar{b}_1) w_1$$

$$\dot{P} = \dot{C}_2 + (\bar{c}_2 + u_2 \bar{a}_2 + v_2 \bar{b}_2) w_2$$

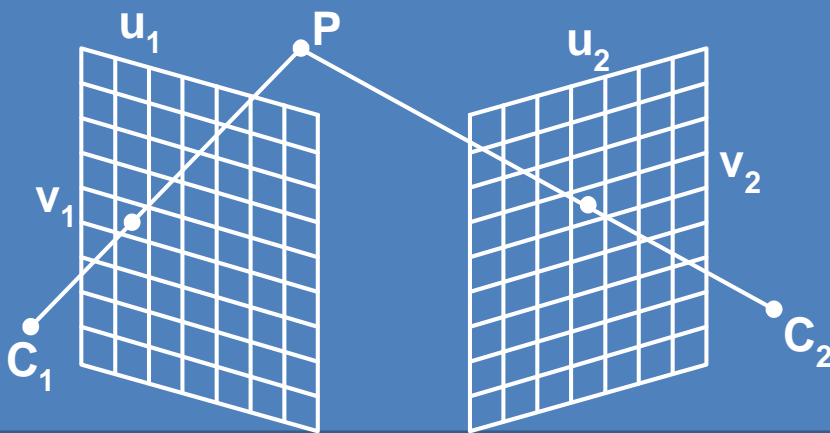


# 3D Image Warping Equations



$$u_2 = \frac{w_{11} + w_{12} \cdot u_1 + w_{13} \cdot v_1 + w_{14} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$

$$v_2 = \frac{w_{21} + w_{22} \cdot u_1 + w_{23} \cdot v_1 + w_{24} \cdot \delta(u_1, v_1)}{w_{31} + w_{32} \cdot u_1 + w_{33} \cdot v_1 + w_{34} \cdot \delta(u_1, v_1)}$$





# 3D Image Warping Example



# 3D Image Warping Example



- DeltaSphere
  - Lars Nyland *et al.*

# 3D Image Warping Example

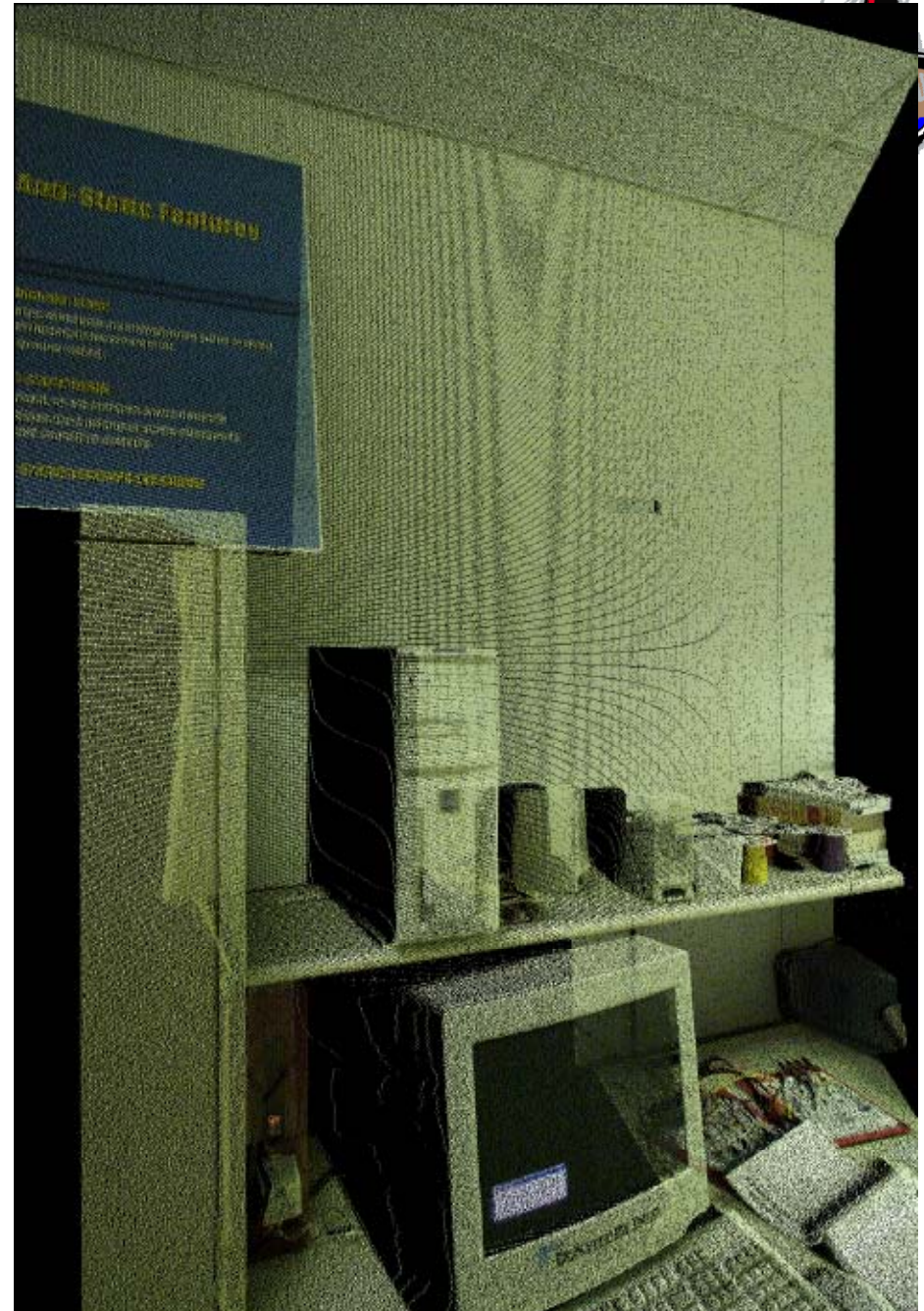


# 3D Image Warping Example

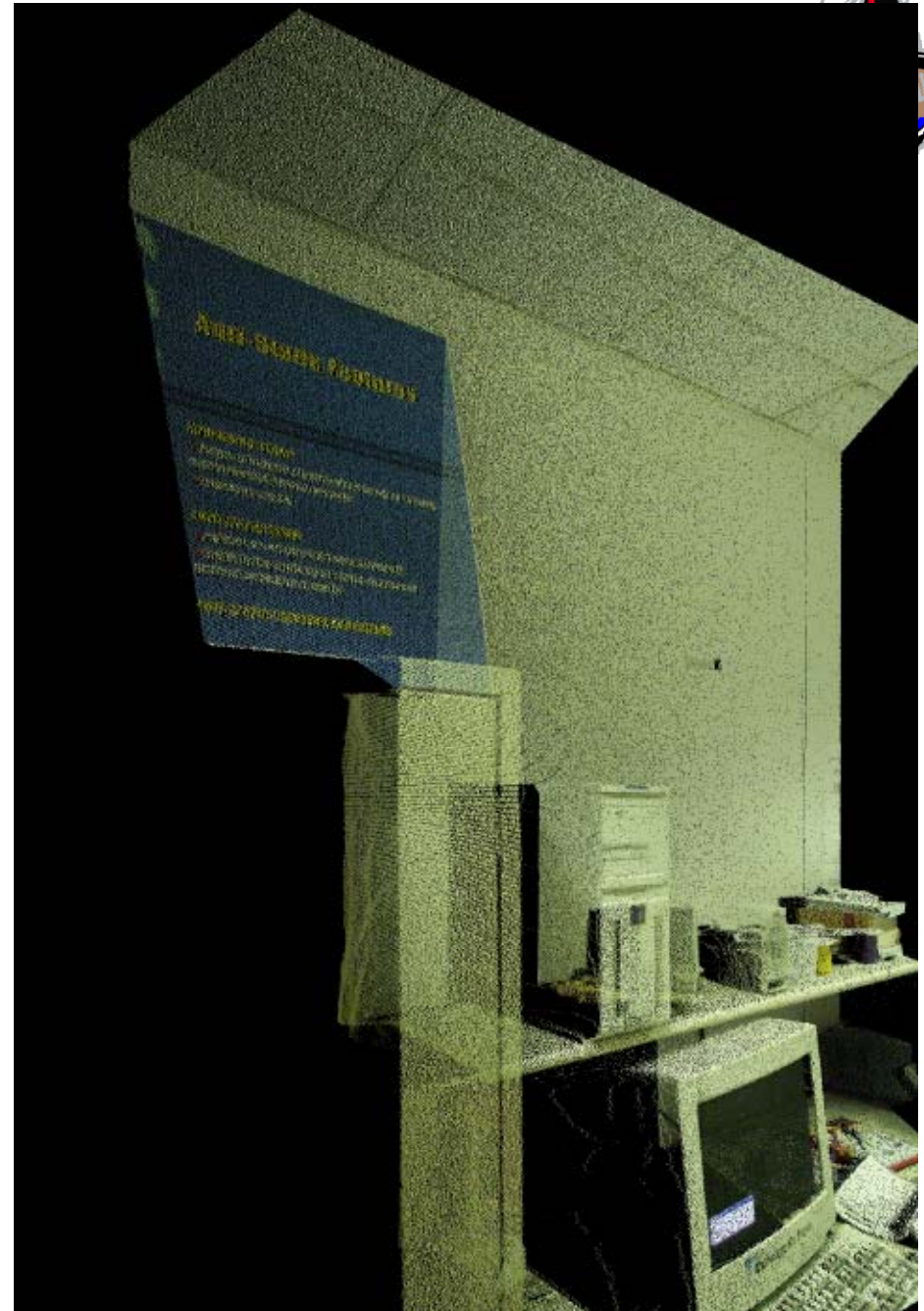




# 3D Image Warping Example



# 3D Image Warping Example





# Disocclusions

- Disocclusions (or exposure events) occur when unsampled surfaces become visible...



*What can we do?*

# Disocclusions



- Bilinear patches: fill in the areas



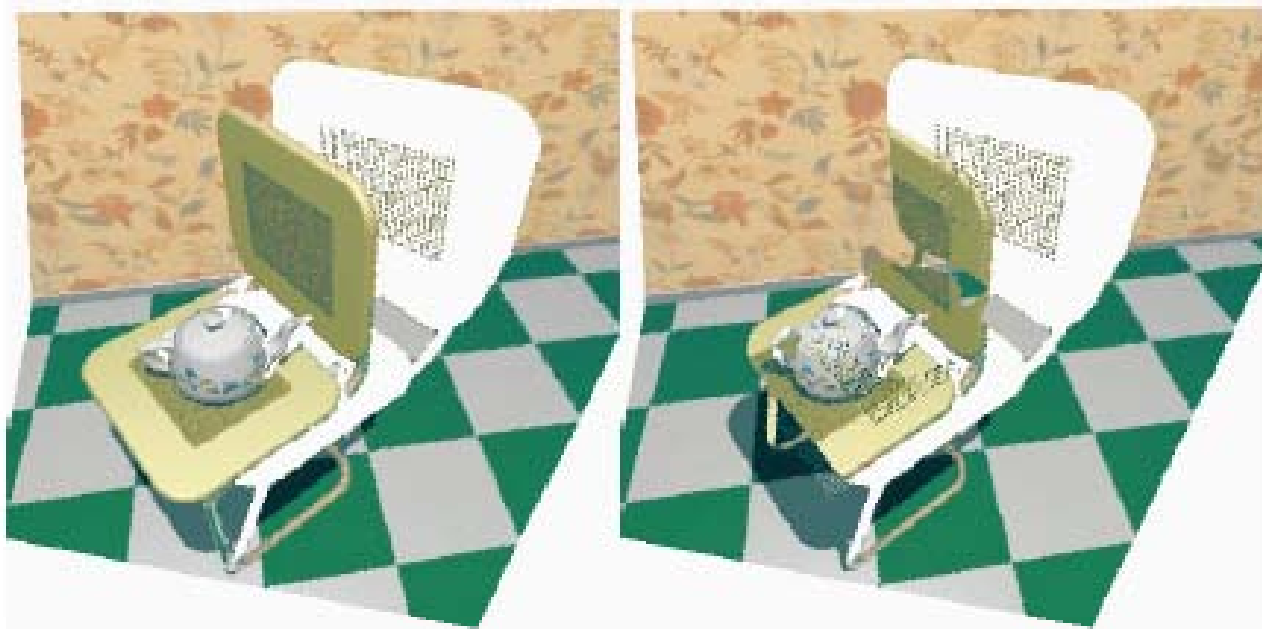
*What else?*





# Rendering Order

✓ The warping equation determines where points go...

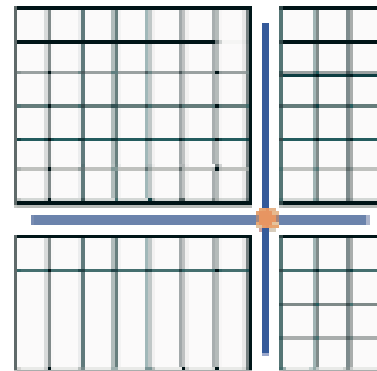
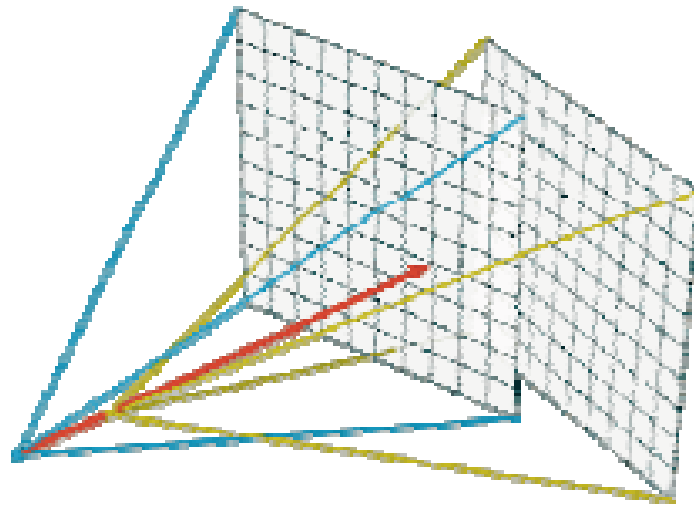


... but that is not sufficient

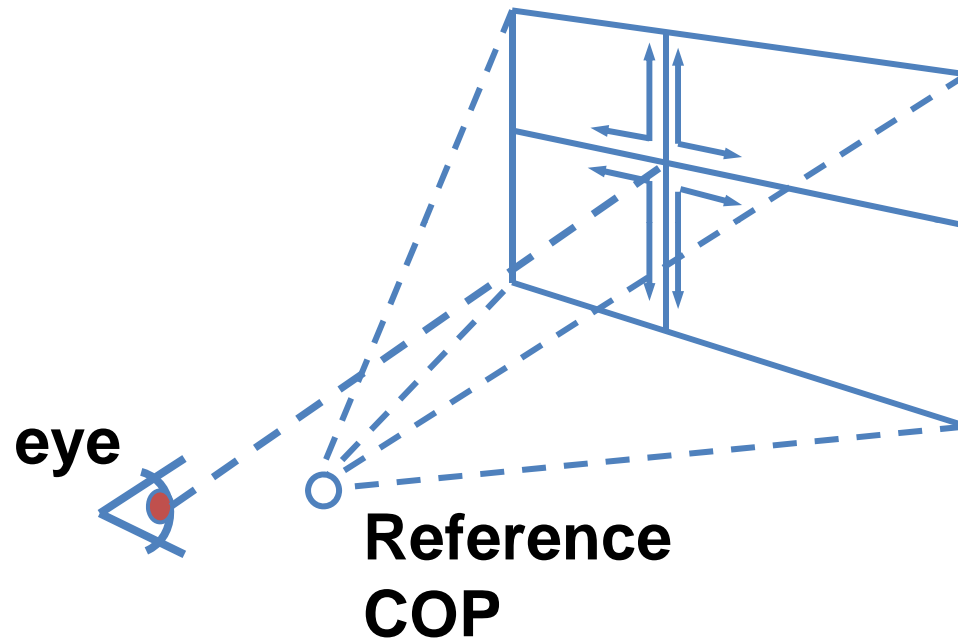
# Occlusion Compatible Rendering Order



- Remember epipolar geometry?
- Project the new viewpoint onto the original image and divide the image into 1, 2 or 4 “sheets”



# Occlusion Compatible Rendering Order



- A raster scan of each sheet produces a back-to-front ordering of warped pixels



# Splatting

- One pixel in the source image does not necessarily project to one pixel in the destination image
  - e.g., if you are walking towards something, the sample might get larger...
- A solution: estimate shape and size of footprint of warped samples
  - expensive to do accurately
  - square/rectangular approximations can be done quickly (3x3 or 5x5 splats)
  - occlusion-compatible rendering will take care of oversized splats
  - *BUT large splats can make the image seem blocky/low-res*

# Splatting

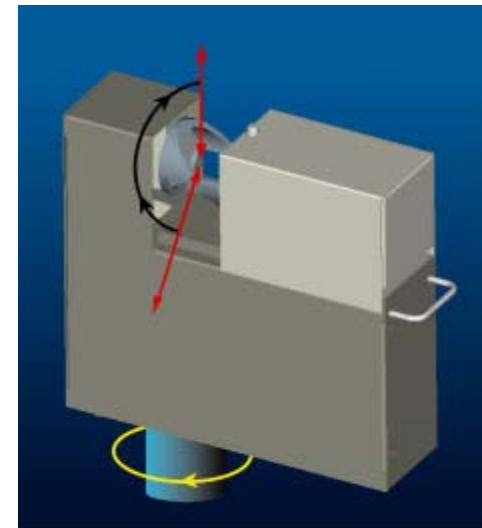


- QSplat Demo...

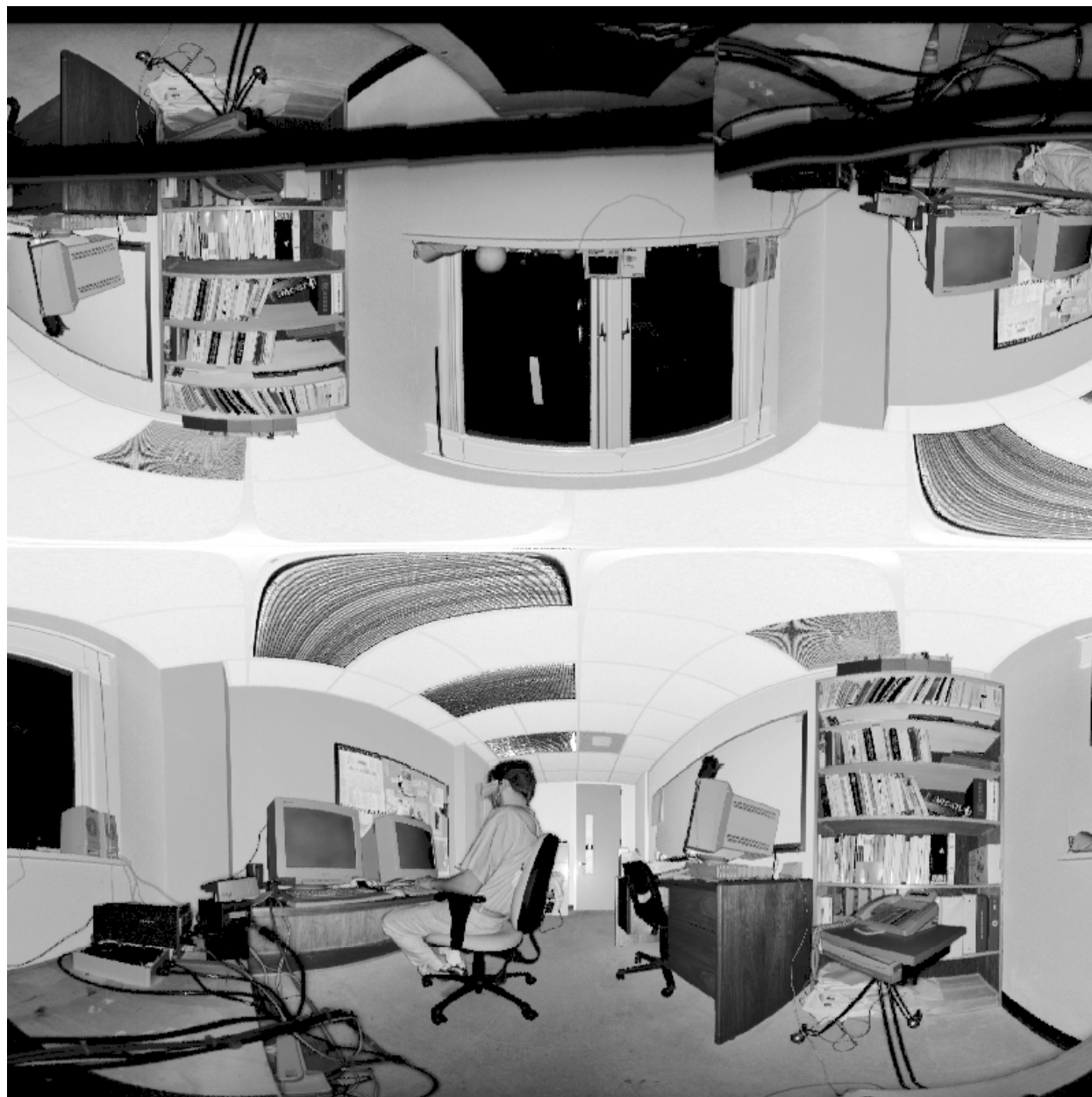
# More Examples Using the DeltaSphere



- Lars Nyland *et al.*



courtesy 3<sup>rd</sup> Tech Inc.



- 300° x 300° panorama
- this is the reflected light

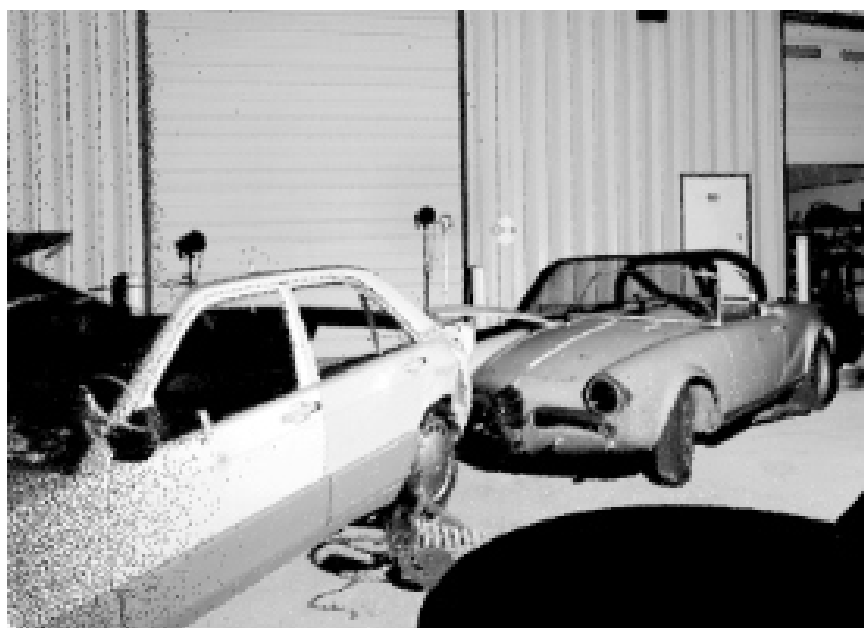
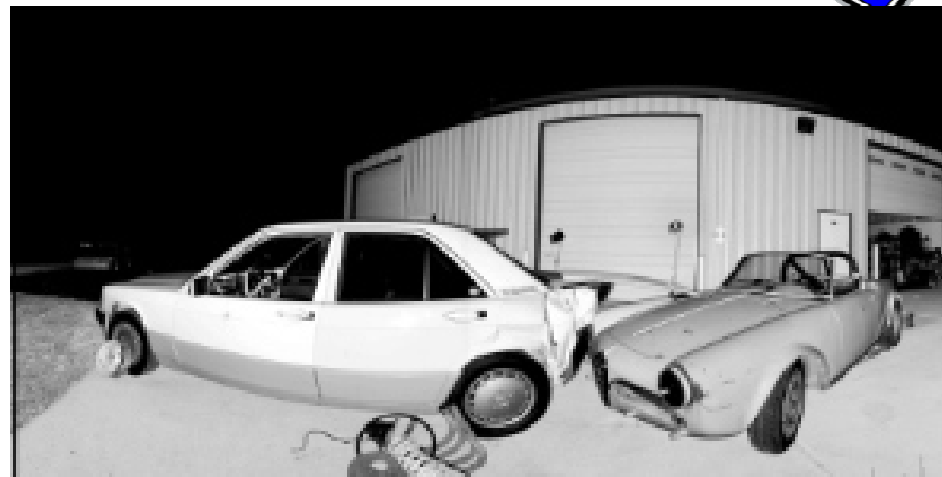


- 300° x 300° panorama
- this is the range light





spherical range panoramas



planar re-projection

*Courtesy 3<sup>rd</sup> Tech Inc.*



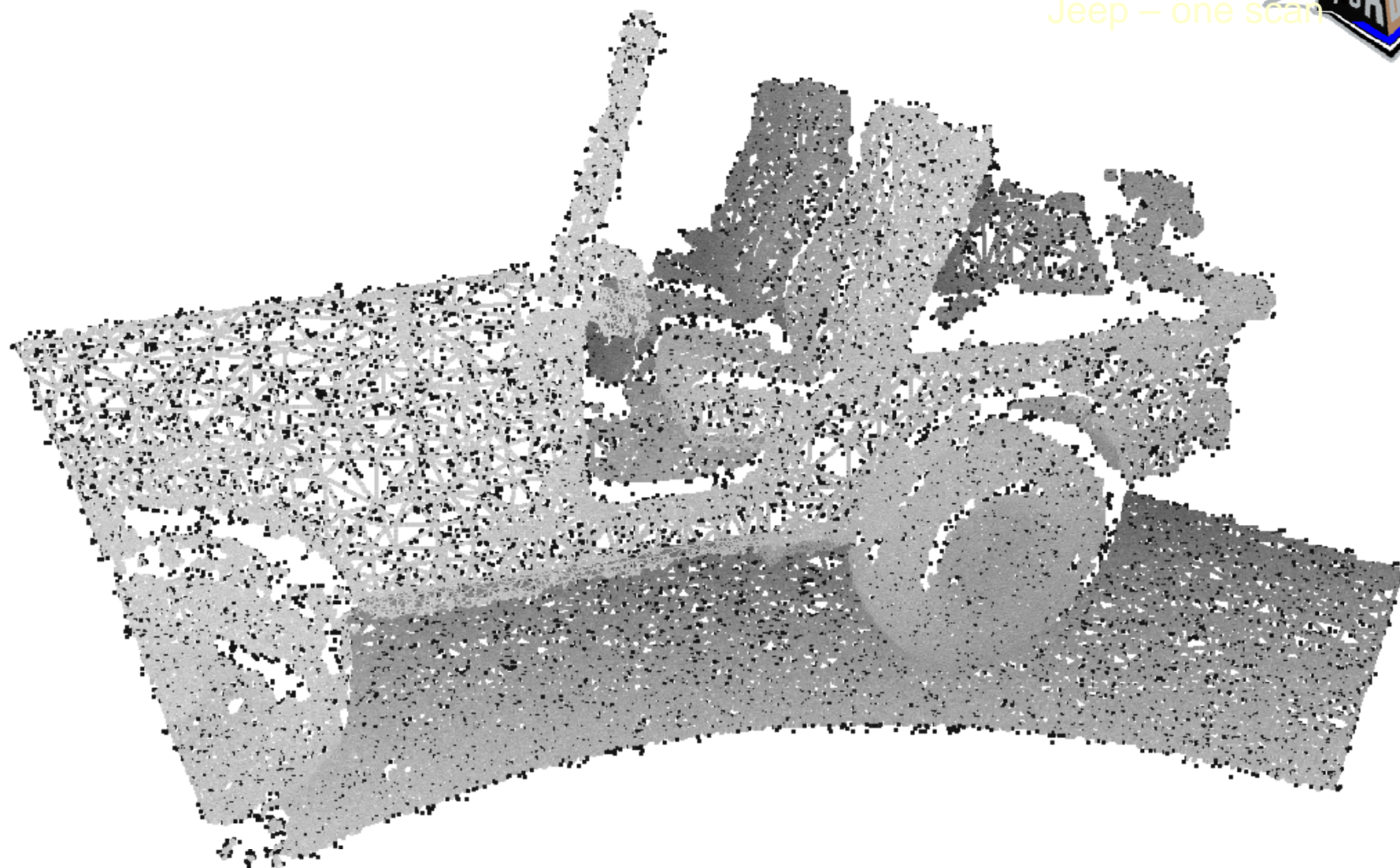
Jeep – one scan



*Courtesy 3<sup>rd</sup> Tech Inc.*



Jeep – one scan



*Courtesy 3<sup>rd</sup> Tech Inc.*



Complete Jeep model



*Courtesy 3<sup>rd</sup> Tech Inc.*

