STAT 5020: Topics in Multivariate Analysis

Assignment 2 (Due date: 20-Mar-2019)

Academic year 18/19, 2n term

1. A nonlinear SEM is defined as follows:

$$
\begin{aligned}
y_{1i} &= \mu_1 + a_1 * c_i + \eta_i + \epsilon_{1i}, \\
y_{2i} &= \mu_2 + a_2 * c_i + \lambda_{21} * \eta_i + \epsilon_{2i}, \\
y_{3i} &= \mu_3 + a_3 * c_i + \lambda_{31} * \eta_i + \epsilon_{3i}, \\
y_{4i} &= \mu_4 + a_4 * c_i + \xi_{1i} + \epsilon_{4i}, \\
y_{5i} &= \mu_5 + a_5 * c_i + \lambda_{52} * \xi_{1i} + \epsilon_{5i}, \\
y_{6i} &= \mu_6 + a_6 * c_i + \lambda_{62} * \xi_{1i} + \epsilon_{6i}, \\
y_{7i} &= \mu_7 + a_7 * c_i + \xi_{2i} + \epsilon_{7i}, \\
y_{8i} &= \mu_8 + a_8 * c_i + \lambda_{83} * \xi_{2i} + \epsilon_{8i}, \\
y_{9i} &= \mu_9 + a_9 * c_i + \lambda_{93} * \xi_{2i} + \epsilon_{9i}, \\
\eta_i &= b * d_i + \gamma_1 * \xi_{1i} + \gamma_2 * \xi_{2i} + \gamma_3 * \xi_{1i}^2 + \gamma_4 * \xi_{2i}^2 + \delta_i,
\end{aligned}
$$

   where the notations follow the lecture notes. Please conduct the Bayesian analysis:

   a. Write the model in a matrix form, draw the path diagram, and assign true values for the parameters in the model.

   b. Generate data from the model defined in (a) and conduct Bayesian analysis on the basis of 10 replications.

   c. Demonstrate how to check the convergence.

   d. Use Bias and RMS to summarize the estimation results.

   e. Show your prior inputs and check whether the Bayesian analysis is sensitive to the inputs.

2. Continue Q1, use Bayesian model comparison statistics and the 10 data sets generated in Q1 to answer the following questions:

   a. Compare the nonlinear SEM in Q1 with its linear SEM counterpart.

   b. Consider a new nonlinear SEM by modifying the structural equation in Q1 as follows:

$$\eta_i = b * d_i + \gamma_1 * \xi_{1i} + \gamma_2 * \xi_{2i} + \gamma_3 * \xi_{1i} * \xi_{2i} + \gamma_4 * \xi_{1i}^2 + \gamma_5 * \xi_{2i}^2 + \delta_i.$$

      Compare the nonlinear SEM in Q1 with the new SEM.