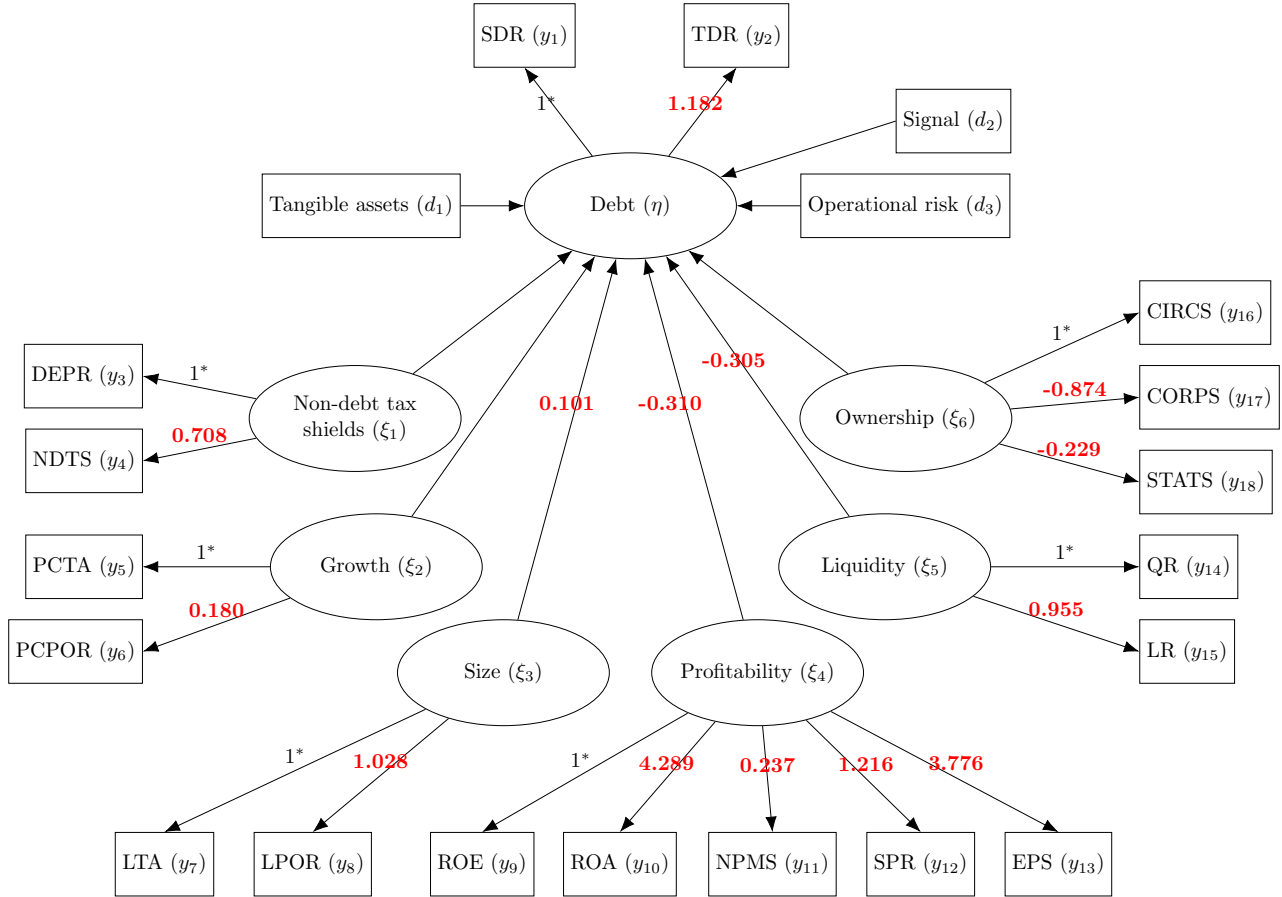


STAT 5020: Topics in Multivariate Analysis

Assignment 1 (Due date: 20-Feb-2019)

Academic year 18/19, 2n term

1. The path diagram of a structural equation model (SEM) is presented in the following figure, along with the estimates of important parameters (the estimates of non-significant parameters are omitted). Assume that the response variables are continuous.



- Write down the explicit form of the model.
- Interpret the model in terms of the variables provided in the rectangles and ellipses.
- Why is the model identified?
- List and interpret the fixed and unknown parameters in the model.
- Discuss the differences between SEMs and conventional regression models.

2. A non-linear SEM is defined as

$$\mathbf{y}_i = \boldsymbol{\mu} + \boldsymbol{\Lambda}\boldsymbol{\omega}_i + \boldsymbol{\epsilon}_i, \quad \boldsymbol{\eta}_i = \boldsymbol{\Pi}\boldsymbol{\eta}_i + \boldsymbol{\Gamma}\mathbf{F}(\boldsymbol{\xi}_i) + \boldsymbol{\delta}_i,$$

where

- $\boldsymbol{\xi}_i \sim N(0, \boldsymbol{\Phi})$, in which $\boldsymbol{\Phi}$ is a general covariance matrix,
- $\boldsymbol{\epsilon}_i \sim N(0, \boldsymbol{\Psi}_\epsilon)$, in which $\boldsymbol{\Psi}_\epsilon = \text{diag}(\psi_{\epsilon 1}, \dots, \psi_{\epsilon p})$,
- $\boldsymbol{\delta}_i \sim N(0, \boldsymbol{\Psi}_\delta)$, in which $\boldsymbol{\Psi}_\delta = \text{diag}(\psi_{\delta 1}, \dots, \psi_{\delta p})$,
- $\mathbf{F}(\boldsymbol{\xi}_i) = (f_1(\boldsymbol{\xi}_i), \dots, f_t(\boldsymbol{\xi}_i))'$ is a $t \times 1$ vector-valued function with necessary properties.

- In a Bayesian analysis, what prior distributions do we usually assign to the model parameters? Why?
- Derive the full conditional distributions for the unknown parameters.