$$f(x|y) \propto \exp(-|x| - a|y-x|).$$

$$= \exp\left[\begin{cases} -x - a(y-x) = (a-1)x - ay. & 0 \le x \le y. \\ -(-x) - a(y-x) = (a+1)x - ay. & x \le \min(0, y) \\ -x - a(x-y) = (-1-a)x + ay & x \geq \max(0, y) \\ -(-x) + a(y-x) = (1-a)x + ay. & y \le x \le 0 \end{cases}\right]$$

$$\propto \exp\left[\begin{cases} (a-1)x & 0 \le x \le y \\ (a+1)x & x \le \min(0, y) \\ (-1-a)x & x \geq \max(0, y) \\ (1-a)x & y \le x \le 0 \end{cases}\right]$$

To use inverse cdf, then it needs to calculate the (relative) probability of each case.

$$P_1 \triangleq \int_0^y \exp(-|x| - a|y-x|)\,dx = \exp(-ay)\,\frac{\exp((a-1)y) - 1}{a-1} I(y \geq 0)$$

$$P_2 \triangleq \int_{-\infty}^{\min(0,y)} \exp(-|x| - a|y-x|)\,dx = \exp(-ay) \cdot \frac{\exp((a+1)\min(0,y))}{a+1}$$

$$P_3 \triangleq \int_{\max(0,y)}^{\infty} \exp(-|x| - a|y-x|)\,dx = \exp(ay) \cdot \frac{-\exp[(-1-a)\max(0,y)]}{-1-a}$$

$$P_4 \triangleq \int_y^0 \exp(-|x| - a|y-x|)\,dx = \exp(ay) \cdot \frac{1 - \exp[(1-a)y]}{1-a} I(y \le 0)$$

For these cases. we can use inverse cdf to sample.

$0 \le x \le y$: $C = \int_0^y \exp((a-1)x)\,dx = \dfrac{\exp((a-1)y)-1}{a-1}$

$$u = \frac{a-1}{\exp((a-1)y)-1} \quad \frac{\exp((a-1)x)-1}{a-1} = \frac{\exp((a-1)x)-1}{\exp((a-1)y)-1}$$

$$(a-1)x = \log\left[1 + u(\exp((a-1)y)-1)\right]$$

$$x = \frac{\log\left[1+u(\exp((a-1)y)-1)\right]}{a-1}$$

$x \le \min(0,y)$

$$C = \int_{-\infty}^{\min(0,y)} \exp[(a+1)x]\,dx = \frac{\exp((a+1)\min(0,y))}{a+1}$$

$$u = \frac{a+1}{\exp((a+1)\min(0,y))} \cdot \frac{\exp((a+1)x)}{a+1}$$

$$= \frac{\exp((a+1)x)}{\exp((a+1)\min(0,y))}$$

$$x = \frac{1}{a+1}\log\left[u \cdot \exp((a+1)\min(0,y))\right]$$

$x \geq \max(0, y)$

$$C = \int_{\max(0,y)}^{\infty} \exp((-1-a)x)\,dx = \frac{-\exp((-1-a)\max(0,y))}{-1-a}$$

$$u = \frac{a+1}{\exp(-(a+1)\max(0,y))} \cdot \frac{\exp(-(a+1)x)}{a+1} = \frac{\exp(-(a+1)x)}{\exp(-(a+1)\max(0,y))}$$

$$x = \frac{-1}{a+1} \log\left[u \cdot \exp(-(a+1)\max(0,y))\right]$$

$y \leq x \leq 0$

$$C = \int_{y}^{0} \exp((1-a)x)\,dx = \frac{1-\exp((1-a)y)}{1-a}$$

$$u = \frac{1-a}{1-\exp((1-a)y)} \cdot \frac{1-\exp((1-a)x)}{1-a} = \frac{1-\exp((1-a)x)}{1-\exp((1-a)y)}$$

$$x = \frac{1}{1-a} \log\left[1 - u\left(1-\exp((1-a)y)\right)\right]$$