数学软件——短学期课程
Matlab 第二次作业

姓名：汪利军
学号：3140105707
班级：统计 1401

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目 录

1 问题重述

$$\begin{cases} -\nabla \cdot \left(\frac{\nabla u}{|\nabla u|}\right) & = f \\ u\Big|_{x=0} = 0, & u\Big|_{y=0} = 0 \\ \frac{\partial u}{\partial x}\Big|_{x=1} = y, & \frac{\partial u}{\partial y}\Big|_{y=1} = x \\ \Omega = [0,1] \times [0,1] \end{cases} \tag{1}$$

已知精确解为

$$u(x,y) = xy \tag{2}$$

于是

$$\begin{aligned} f &= -\nabla \cdot \left(\frac{\nabla u}{|\nabla u|}\right) && (3) \\ &= -\nabla \cdot \left(\frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}}\right) && (4) \\ &= -\left(-\frac{2xy}{(x^2+y^2)^{\frac{3}{2}}}\right) && (5) \\ &= \frac{2xy}{(x^2+y^2)^{\frac{3}{2}}} && (6) \end{aligned}$$

2 有限差分推导

将 $[0,1]N$ 等分，网格节点 $0 = x_0 < x_1 < \cdots < x_N = 1$

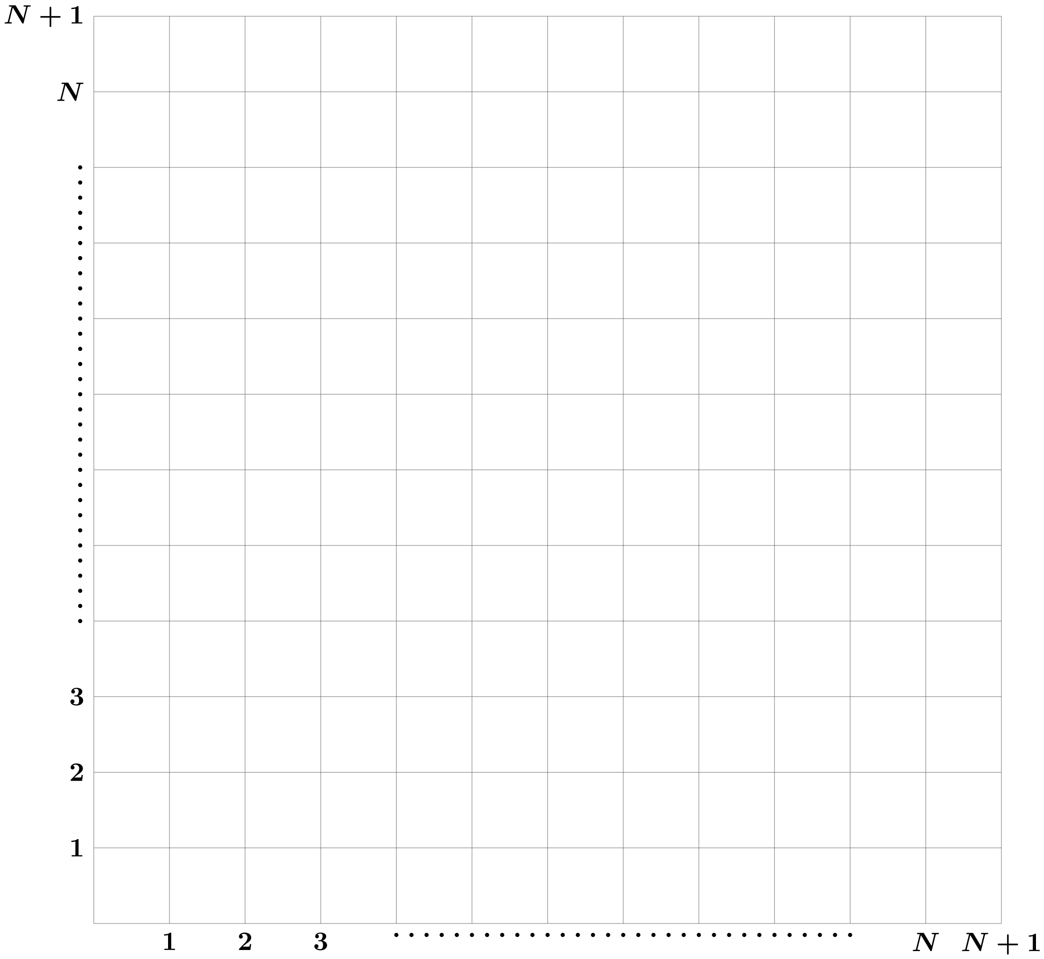


Figure 1: 网格剖分

$$x_i = ih \qquad h = \frac{1}{N} \tag{7}$$

$$(\frac{\partial u}{\partial x})_{ij} = \frac{u_{i+1,j} - u_{i-1,j}}{2h} + O(h^2) \tag{8}$$

$$(\frac{\partial u}{\partial y})_{ij} = \frac{u_{i,j+1} - u_{i,j-1}}{2h} + O(h^2) \tag{9}$$

$$\begin{aligned}
&\frac{\partial}{\partial x}\Big(\frac{1}{|\nabla u|}\frac{\partial u}{\partial x}\Big)_{ij} && (10)\\
=\ & \frac{1}{h}\Big[\frac{1}{|\nabla u|_{i+\frac{1}{2},j}}(\frac{u_{i+1,j}-u_{i,j}}{h}) - \frac{1}{|\nabla u|_{i-\frac{1}{2},j}}(\frac{u_{i,j}-u_{i-1,j}}{h})\Big] + O(h^2) && (11)\\
=\ & \frac{1}{h^2}\Big[\frac{1}{|\nabla u|_{i-\frac{1}{2},j}}u_{i-1,j} - (\frac{1}{|\nabla u|_{i-\frac{1}{2},j}} + \frac{1}{|\nabla u|_{i+\frac{1}{2},j}})u_{ij} + \frac{1}{|\nabla u|_{i+\frac{1}{2},j}}u_{i+1,j}\Big] && (12)\\
&\frac{\partial}{\partial y}\Big(\frac{1}{|\nabla u|}\frac{\partial u}{\partial y}\Big)_{ij} && (13)\\
=\ & \frac{1}{h}\Big[\frac{1}{|\nabla u|_{i,j+\frac{1}{2}}}(\frac{u_{i,j+1}-u_{i,j}}{h}) - \frac{1}{|\nabla u|_{i,j-\frac{1}{2}}}(\frac{u_{i,j}-u_{i,j-1}}{h})\Big] + O(h^2) && (14)\\
=\ & \frac{1}{h^2}\Big[\frac{1}{|\nabla u|_{i,j-\frac{1}{2}}}u_{i,j-1} - (\frac{1}{|\nabla u|_{i,j-\frac{1}{2}}} + \frac{1}{|\nabla u|_{i,j+\frac{1}{2}}})u_{ij} + \frac{1}{|\nabla u|_{i,j+\frac{1}{2}}}u_{i,j+1}\Big] && (15)\\
& && (16)
\end{aligned}$$

则

$$\begin{aligned}
&\nabla\cdot\Big(\frac{1}{|\nabla u|}\nabla u\Big)_{ij} && (17)\\
=\ & \frac{\partial}{\partial x}\Big(\frac{1}{|\nabla u|}\frac{\partial u}{\partial x}\Big)_{ij} + \frac{\partial}{\partial y}\Big(\frac{1}{|\nabla u|}\frac{\partial u}{\partial y}\Big)_{ij} && (18)\\
=\ & \frac{1}{h^2}\Big[\frac{1}{|\nabla u|_{i-\frac{1}{2},j}}u_{i-1,j} + \frac{1}{|\nabla u|_{i+\frac{1}{2},j}}u_{i+1,j} + \frac{1}{|\nabla u|_{i,j-\frac{1}{2}}}u_{i,j-1} + \frac{1}{|\nabla u|_{i,j+\frac{1}{2}}}u_{i,j+1} \\
& -(\frac{1}{|\nabla u|_{i-\frac{1}{2},j}} + \frac{1}{|\nabla u|_{i+\frac{1}{2},j}} + \frac{1}{|\nabla u|_{i,j-\frac{1}{2}}} + \frac{1}{|\nabla u|_{i,j+\frac{1}{2}}})u_{ij}\Big] + O(h^2) && (19)
\end{aligned}$$

$$\begin{aligned}
|\nabla u|_{i-\frac{1}{2},j} &= \Big|\Big((\frac{\partial u}{\partial x})_{i-\frac{1}{2},j}, (\frac{\partial u}{\partial y})_{i-\frac{1}{2},j}\Big)\Big| && (20)\\
&= \Big|\Big(\frac{u_{ij}-u_{i-1,j}}{h}, \frac{u_{i-\frac{1}{2},j+1}-u_{i-\frac{1}{2},j-1}}{2h}\Big)\Big| && (21)
\end{aligned}$$

又

$$\begin{cases} u_{i-\frac{1}{2},j\pm1} &= u_{i-1,j\pm1} + \frac{1}{2}h(\frac{\partial u}{\partial x})_{i-1,j\pm1} + O(h^2) \\ &= u_{i-1,j\pm1} + \frac{1}{2}h\cdot\frac{u_{i,j\pm1}-u_{i-2,j\pm1}}{2h} + O(h^2) \\ &= u_{i-1,j\pm1} + \frac{1}{4}(u_{i,j\pm1} - u_{i-2,j\pm1}) + O(h^2) \quad 当 i \neq 1 \\ u_{\frac{1}{2},j\pm1} &= u_{1,j\pm1} - \frac{1}{2}h\cdot\frac{u_{2,j\pm1}-u_{0,j\pm1}}{2h} + O(h^2) \quad 当 i = 1 \\ &= u_{1,j\pm1} - \frac{1}{4}u_{2,j\pm1} + O(h^2) \\ u_{i-\frac{1}{2},0} &= 0 \quad 当 j = 1 \end{cases} \tag{22}$$

同理可求得

$$|\nabla u|_{i+\frac{1}{2},j} = \left|\left((\frac{\partial u}{\partial x})_{i+\frac{1}{2},j}, (\frac{\partial u}{\partial y})_{i+\frac{1}{2},j}\right)\right| \tag{23}$$

$$= \left|\left(\frac{u_{i+1,j} - u_{i,j}}{h}, \frac{u_{i+\frac{1}{2},j+1} - u_{i+\frac{1}{2},j-1}}{2h}\right)\right| \tag{24}$$

$$|\nabla u|_{i,j-\frac{1}{2}} = \left|\left((\frac{\partial u}{\partial x})_{i,j-\frac{1}{2}}, (\frac{\partial u}{\partial y})_{i,j-\frac{1}{2}}\right)\right| \tag{25}$$

$$= \left|\left(\frac{u_{i+1,j-\frac{1}{2}} - u_{i-1,j-\frac{1}{2}}}{2h}, \frac{u_{i,j} - u_{i,j-1}}{h}\right)\right| \tag{26}$$

$$|\nabla u|_{i,j+\frac{1}{2}} = \left|\left((\frac{\partial u}{\partial x})_{i,j+\frac{1}{2}}, (\frac{\partial u}{\partial y})_{i,j+\frac{1}{2}}\right)\right| \tag{27}$$

$$= \left|\left(\frac{u_{i+1,j+\frac{1}{2}} - u_{i-1,j+\frac{1}{2}}}{2h}, \frac{u_{i,j+1} - u_{i,j}}{h}\right)\right| \tag{28}$$

以及

$$\begin{cases} u_{i+\frac{1}{2},j\pm1} &= u_{i,j\pm1} + \frac{1}{2}h(\frac{\partial u}{\partial x})_{i,j\pm1} + O(h^2) \\ &= u_{i,j\pm1} + \frac{1}{2}h\cdot\frac{u_{i+1,j\pm1}-u_{i-1,j\pm1}}{2h} + O(h^2) \\ &= u_{i,j\pm1} + \frac{1}{4}(u_{i+1,j\pm1} - u_{i-1,j\pm1}) + O(h^2) \\ u_{i\pm1,0} &= 0 \quad 当 j = 1 \\ u_{N+\frac{1}{2},j\pm1} &= u_{N,n\pm1} + \frac{1}{2}hy_{i+1} \end{cases}$$

类似地，

$$\begin{cases} u_{i\pm1,j-\frac{1}{2}} &= u_{i\pm1,j} - \frac{1}{2}h\left(\frac{\partial u}{\partial x}\right)_{i\pm1,j} + O(h^2) \\ &= u_{i\pm1,j} - \frac{1}{2}h \cdot \frac{u_{i\pm1,j+1}-u_{i\pm1,j-1}}{2h} + O(h^2) \\ &= u_{i\pm1,j} - \frac{1}{4}(u_{i\pm1,j+1} - u_{i\pm1,j-1}) + O(h^2) \\ u_{0,j-\frac{1}{2}} &= 0 \quad \text{当} i = 1 \\ u_{i\pm1,\frac{1}{2}} &= u_{i\pm1,1} - \frac{1}{2}h \cdot \frac{u_{i\pm1,2}-u_{i\pm1,0}}{2h} + O(h^2) \\ &= u_{i\pm1,1} - \frac{1}{4}u_{i\pm1,2} + O(h^2) \quad \text{当} j = 1 \end{cases}$$

$$\begin{cases} u_{i\pm1,j+\frac{1}{2}} &= u_{i\pm1,j} + \frac{1}{2}h\left(\frac{\partial u}{\partial x}\right)_{i\pm1,j} + O(h^2) \\ &= u_{i\pm1,j} + \frac{1}{2}h \cdot \frac{u_{i\pm1,j+1}-u_{i\pm1,j-1}}{2h} + O(h^2) \\ &= u_{i\pm1,j} + \frac{1}{4}(u_{i\pm1,j+1} - u_{i\pm1,j-1}) + O(h^2) \\ u_{0,j+\frac{1}{2}} &= 0 \end{cases}$$

为了表述方便. 记

$$p_1 = \frac{1}{h\left|\nabla u\right|_{i-\frac{1}{2},j}} \tag{29}$$

$$p_2 = \frac{1}{h\left|\nabla u\right|_{i+\frac{1}{2},j}} \tag{30}$$

$$p_3 = \frac{1}{h\left|\nabla u\right|_{i,j-\frac{1}{2}}} \tag{31}$$

$$p_4 = \frac{1}{h\left|\nabla u\right|_{i,j+\frac{1}{2}}} \tag{32}$$

则有

$$-\frac{1}{h^2}[p_1hu_{i-1,j} + p_2hu_{i+1,j} + p_3hu_{i,j-1} + p_4hu_{j,j+1} - (p_1+p_2+p_3+p_4)hu_{ij}] = f \tag{33}$$

从而有

$$(p_1+p_2+p_3+p_4)u_ij - p_1u_{i-1,j} - p_2u_{i+1,j} - p_3u_{j,j-1} - p_4u_{j,j+1} = hf \tag{34}$$

3 边界条件

3.1 左边界

$i = 1, 2 \leq j \leq N-1$ 时，有

$$u_{i-1,j} = 0 \tag{35}$$

于是

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{1j} - p_2u_{2j} - p_3u_{1,j-1} - p_4u_{1,j+1} &= hf_{i,j} + p_1u_{0j} \qquad (36)\\ &= hf_{i,j} \qquad (37)\end{aligned}$$

3.2 右边界

$i = N, 2 \leq j \leq N-1$ 时，有

$$\frac{\partial u}{\partial x}\Big|_{i+1,j} = y_j \tag{38}$$

于是

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{ij} - p_1u_{i-1,j} - p_3u_{i,j-1} - p_4u_{i,j+1} &= hf_{i,j} + p_2u_{N+1,j} \qquad (39)\\ &= hf_{i,j} + p_2u_{N+1,j} \qquad (40)\end{aligned}$$

对 $u_{N,j}$ 进行泰勒展开

$$\begin{aligned}u_{N,j} &= u_{N+1,j} - \frac{1}{h}\frac{\partial u}{\partial x} + O(h^2) \qquad (41)\\ &= u_{N+1,j} - \frac{1}{h}y_j \qquad (42)\end{aligned}$$

于是

$$u_{N+1,j} = u_{N,j} + \frac{1}{h}y_j \tag{43}$$

从而，式 (39) 为

$$
\begin{aligned}
& (p_1+p_2+p_3+p_4)u_{ij} - p_1 u_{i-1,j} - p_3 u_{i,j-1} - p_4 u_{i,j+1} && (44)\\
= \;& h f_{i,j} + p_2 u_{N+1,j} && (45)\\
= \;& h f_{i,j} + p_2 (u_{N,j} + \frac{1}{h} y_j) && (46)
\end{aligned}
$$

3.3 上边界

$2 \le i \le N-1, j = N$ 时，有

$$\frac{\partial u}{\partial x}|_{i,N+1} = x_i \tag{47}$$

于是有

$$
\begin{aligned}
& (p_1+p_2+p_3+p_4)u_{iN} - p_1 u_{i-1,N} - p_2 u_{i+1,N} - p_3 u_{i,N-1} && (48)\\
= \;& h f_{i,N} + p_4 u_{i,N+1} && (49)
\end{aligned}
$$

对 $u_{i,N}$ 进行泰勒展开

$$
\begin{aligned}
u_{i,N} &= u_{i,N+1} - \frac{1}{h}\frac{\partial u}{\partial y} + O(h^2) && (50)\\
&= u_{i,N+1} - \frac{1}{h} x_i && (51)
\end{aligned}
$$

于是

$$u_{i,N+1} = u_{i,N} + \frac{1}{h} x_i \tag{52}$$

从而，式 (49) 为

$$
\begin{aligned}
& (p_1+p_2+p_3+p_4)u_{iN} - p_1 u_{i-1,N} - p_2 u_{i+1,N} - p_3 u_{i+1,N} && (53)\\
= \;& h f_{i,j} + p_4 u_{i,N+1} && (54)\\
= \;& h f_{i,j} + p_4 (u_{iN} + \frac{1}{h} x_i) && (55)
\end{aligned}
$$

3.4 下边界

$2 \le i \le N-1, j=1$ 时，有

$$u_{i,j-1} = 0 \tag{56}$$

于是

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{i1} - p_1 u_{i-1,1} - p_2 u_{i+1,j} - p_4 u_{i,2} &= hf_{i,j} + p_3 u_{i,0} && (57)\\ &= hf_{i,j} && (58)\end{aligned}$$

3.5 节点

3.5.1 节点 (1,1)

$i=1, j=1$ 时有

$$\begin{aligned}u_{i-1,j} &= 0 && (59)\\ u_{i,j-1} &= 0 && (60)\end{aligned}$$

于是

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{11} - p_2 u_{21} - p_4 u_{12} &= hf_{x_1,y_1} + p_1 u_{01} + p_3 u_{10} && (61)\\ &= hf_{1,1} && (62)\end{aligned}$$

3.5.2 节点 (1,N)

$i=1, j=N$ 时，有

$$\begin{aligned}u_{i-1,j} &= 0 && (63)\\ \frac{\partial u}{\partial y}\Big|_{i,j+1} &= x_i && (64)\end{aligned}$$

于是

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{1N}-p_2u_{2N}-p_3u_{1,N-1} &= hf_{i,j}+p_1u_{0N}+p_4u_{1,N+1} &(65)\\ &= hf_{i,j}u_{0N}+p_4u_{1,N+1} &(66)\end{aligned}$$

对 $u_{1,N}$ 进行泰勒展开有

$$\begin{aligned}u_{1,N} &= u_{1,N+1}-\frac{1}{h}\frac{\partial u}{\partial y}+O(h^2) &(67)\\ &= u_{1,N+1}-\frac{1}{h}x_1 &(68)\end{aligned}$$

于是

$$u_{1,N+1}=u_{1,N}+\frac{1}{h}x_1 \tag{69}$$

于是式 (65) 为

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{1N}-p_2u_{2N}-p_3u_{1,N-1} &= hf_{i,j}+p_4u_{1,N+1} &(70)\\ &= hf_{i,j}+p_4(u_{1,N}+\frac{x_1}{h}) &(71)\end{aligned}$$

3.5.3 节点 (N,1)

$i=N,j=1$ 时，有

$$u_{N,j-1}=0 \tag{72}$$

$$\frac{\partial u}{\partial x}\Big|_{i+1,j}=y_j \tag{73}$$

于是有

$$\begin{aligned}(p_1+p_2+p_3+p_4)u_{N1}-p_1u_{N-1,1}-p_4u_{N2} &= hf_{i,j}+p_2u_{N+1,1}+p_3u_{N,0} &(74)\\ &= hf_{i,j}+p_2u_{N+1,1} &(75)\end{aligned}$$

对 u_{N1} 进行泰勒展开有

$$u_{N1} = u_{N+1,1} - \frac{1}{h}\frac{\partial u}{\partial x} + O(h^2) \tag{76}$$
$$= u_{N+1,1} - \frac{1}{h}y_1 \tag{77}$$

于是

$$u_{N+1,1} = u_{N,1} + \frac{1}{h}y_1 \tag{78}$$

于是式 (74) 为

$$(p_1+p_2+p_3+p_4)u_{N1} - p_1u_{N-1,1} - p_4u_{N2} \tag{79}$$
$$= hf_{i,j} + p_2u_{N+1,1} \tag{80}$$
$$= hf_{i,j} + p_2(u_{N,1} + \frac{y_1}{h}) \tag{81}$$

3.5.4 节点 (N,N)

$i = N, j = N$ 时，有

$$\frac{\partial u}{\partial x}\Big|_{i+1,j} = y_j \tag{82}$$
$$\frac{\partial u}{\partial y}\Big|_{i,j+1} = x_i \tag{83}$$

于是

$$(p_1+p_2+p_3+p_4)u_{NN} - p_1u_{N-1,N} - p_3u_{N,N-1} \tag{84}$$
$$= hf_{i,j} + p_2u_{N+1,N} + p_4u_{N,N+1} \tag{85}$$

由式 (43) 及式 (52)，有

$$u_{N+1,N} = u_{N,N} + \frac{1}{h}y_N \tag{86}$$
$$u_{N,N+1} = u_{N,N} + \frac{1}{h}x_N \tag{87}$$

于是式 (88) 可写成

$$(p_1+p_2+p_3+p_4)u_{NN}-p_1u_{N-1,N}-p_3u_{N,N-1} \tag{88}$$
$$= hf_{ij}+p_2(u_{NN}+\frac{1}{h}y_N)+p_4(u_{NN}+\frac{1}{h}x_N) \tag{89}$$

4 系数矩阵

设 $U=(u_{11},u_{21},\cdots,u_{N1},u_{12},\cdots,u_{N2},u_{13},\cdots,u_{N-1,N},u_{NN})'$,$A$ 为 N^2*N^2 的五对角矩阵.

$$A=\begin{bmatrix}
\sum_{i=1}^{4}p_i & -p_2 & \cdots & -p_4 & \cdot & \cdots & \cdots \\
-p_1 & \sum_{i=1}^{4}p_i & -p_2 & \cdots & -p_4 & \cdots & \cdots \\
\vdots & -p_1 & \sum_{i=1}^{4}p_i & \cdots & \cdots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-p_3 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdot & -p_3 & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \sum_{i=1}^{4}p_i
\end{bmatrix} \tag{90}$$

根据上述边界条件的讨论，我们有

$$F_{ij}=\begin{cases}
hf_{ij}+p_2(u_{Nj}+\frac{1}{h}y_j) & i=N,2\leq j\leq N-1 \\
hf_{ij}+p_4(u_{iN}+\frac{1}{h}x_i) & j=N,2\leq i\leq N-1 \\
hf_{ij}+p_2(u_{NN}+\frac{1}{h}y_N)+p_4(u_{NN}+\frac{1}{h}x_N) & i=j=N \\
hf_{ij} & others
\end{cases} \tag{91}$$

于是，我们的目标是求解线性方程

$$AU=F \tag{92}$$

其中，$F=(F_{11},F_{21},\cdots,F_{N1},F_{12},\cdots,F_{N2},F_{13},\cdots,F_{N-1,N},F_{NN})'$。$A$ 中的 p_i 是根据上一步的值确定出来的，每次算出的 u 又作为下一步 p_i 的输入

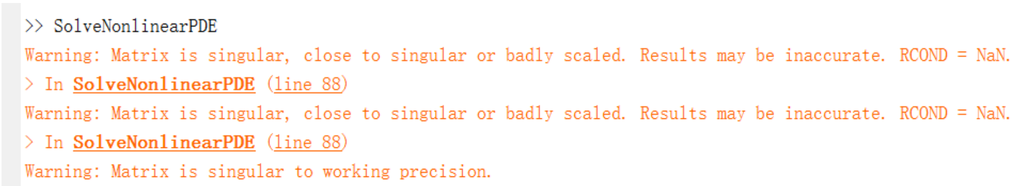
5 算法分析

由上述推导我们得出目标即是要求解方程 $AU = F$, 但 A 中含有与 u 有关的参数, 于是想到先给 u 一个初值，不断迭代直至收敛到解。所以代码思路基本上是这样

```
给定初值 u0;
根据 u0 组装出系数矩阵 A 及 F;
for lp = 1:maxIters
    u = A\F;
    err = norm(u-exu);
    if err < errStop
        break;
    end
    u0 = u;
end
plot;
```

6 运行结果

失败原因:



```
>> SolveNonlinearPDE
Warning: Matrix is singular, close to singular or badly scaled. Results may be inaccurate. RCOND = NaN.
> In SolveNonlinearPDE (line 88)
Warning: Matrix is singular, close to singular or badly scaled. Results may be inaccurate. RCOND = NaN.
> In SolveNonlinearPDE (line 88)
Warning: Matrix is singular to working precision.
```

Figure 2: 错误信息

时间有限，没有调试成功。

附录

代码如下:

```
function [err, step] = SolveNonlinearPDE
global N;
N = 10;
global h;
global x;
global y;
h = 1/(N+1);
maxLoop = 100;
errStop = 1.0e-6;

u = ones(N*N, 1);
exu = zeros(N*N, 1);

x = h * (1:N);
y = h * (1:N);

rowIdx = zeros(1,N*N+1);
F = zeros(N*N, 1);
for lp = 1:maxLoop
%% Assign value
%% Matrix A
%% the first row
i = 1;
j = 1;
rowIdx(1) = 1;
colIdx(1) = 1;
entries(1) = p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) +p4func
    (u,i,j);
F(1,1) = h * RightF(x(1),y(1));
for i = 2:N-1
    % (i,j) (i-1,j) (i+1,j) (i,j+1)
    rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
    colIdx = [colIdx, i+(j-1)*N, i-1+(j-1)*N, i+1+(j-1)*N, i + j*N
        ];
    entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i
        ,j) +p4func(u,i,j), -1*p1func(u, i, j) ,-1*p2func(u,i,j)
        ,-1*p4func(u, i,j) ];
    F(i+(j-1)*N) = h*RightF(x(i),y(j));
end
% (i,j) (i-1,j) (i,j+1)
rowIdx(N) = size(colIdx, 2) + 1;
colIdx = [colIdx, i+(j-1)*N, i-1+(j-1)*N, i+j*N];
entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j)
    + p4func(u,i,j), -1*p1func(u, i, j), -1*p4func(u, i,j)];
F(i+(j-1)*N) = h*RightF(x(i), y(j));
```

```matlab
for j = 2:N-1
    for i = 1:N
        if i == 1
            % (i,j) (i,j-1) (i+1,j) (i,j+1)
            rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
            colIdx = [colIdx, i+(j-1)*N, i+(j-2)*N, i+1+(j-1)*N, i+j*N ];
            entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) + p4func(u,i,j),-1*p3func(u,i,j), -1*p2func(u,i,j),-1*p4func(u,i,j)];
            F(i+(j-1)*N) = h*RightF(x(i),y(j));
        elseif i == N
            % (i,j) (i,j-1), (i-1, j) (i, j+1)
            rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
            colIdx = [colIdx, i+(j-1)*N, i+(j-2)*N, i-1+(j-1)*N,i+j*N ];
            entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) + p4func(u,i,j),-1*p3func(u,i,j), -1*p1func(u,i,j), -1*p4func(u,i,j)];
            F(i+(j-1)*N) = h*RightF(x(i),y(j)) + p2func(u,i,j)*(u(N+(j-1)*N)+1/h*y(j));
        else
            % (i,j) (i,j-1) (i-1,j) (i+1,j) (i,j+1)
            rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
            colIdx = [colIdx, i+(j-1)*N, i+(j-2)*N, i-1+(j-1)*N, i+1+(j-1)*N ,i+j*N];
            entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) + p4func(u,i,j),-1*p3func(u,i,j), -1*p1func(u,i,j), -1*p2func(u,i,j),-1*p4func(u,i,j)];
            F(i+(j-1)*N) = h*RightF(x(i),y(j));
        end
    end
end
% (i,j) (i,j-1) (i+1,j)
j = N;
i = 1;
rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
colIdx = [colIdx, i+(j-1)*N, i+(j-2)*N, i+1+(j-1)*N];
entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) + p4func(u,i,j), -1*p2func(u,i,j), -1*p4func(u,i,j)];
F(i+(j-1)*N) = h*RightF(x(i),y(j)) + p4func(u,i,j)*(u(i+(N-1)*j)+1/h*x(i));
for i = 2:N-1
    rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
    colIdx = [colIdx, i+(j-1)*N,i+(j-2)*N,i-1+(j-1)*N,i+1+(j-1)*N];
    entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j) + p4func(u,i,j),-1*p3func(u,i,j), -1*p1func(u,i,j), -1*p2func(u,i,j)];
```

```
    F(i+(j-1)*N) = h*RightF(x(i),y(j)) + p4func(u,i,j)*(u(i+(N-1)*
        j)+1/h*x(i));
end
i = N;
rowIdx(i+(j-1)*N) = size(colIdx, 2) + 1;
colIdx = [colIdx, i + (j-1)*N];
entries = [entries, p1func(u,i,j) + p2func(u,i,j) + p3func(u,i,j)
    + p4func(u,i,j)];
F(i+(j-1)*N) = h*RightF(x(i),y(j)) + p4func(u,i,j)*(u(i+(N-1)*j)
    +1/h*x(i)) + p2func(u,i,j)*(u(i+(j-1)*N)+1/h*y(j));
rowIdx(N*N+1) = N*N;
%%
u0 = ones(N*N,1);

    %%matspA = mysp2matsp(rowIdx, colIdx, entries);
    myfull = mysparse2full(rowIdx,colIdx,entries);
    u = myfull \ F;
    err = norm(abs(u - exu));
    if err < errStop
        break;
    end
    u0 = u;
end

%p1 = p1func(u, i, j);%p_{i-1/2,j}
%p2 = p2func(u, i, j);%p_{i+1/2,j}
%p3 = p3func(u, i, j);%p_{i,j-1/2}
%p4 = p4func(u, i, j);%p_{i,j+1/2}


end
function vp1 = p1func(u, m, n)
    global N;
    global h;
    global x;
    global y;
    if m == 1||n == N
        vpx = (u(m+(n-1)*N))^2;
        vpy1 = u(1+(n-1)*N) - 0.25*u(2+(n-1)*N);
        %vpx = (u(m+(n-1)*N))^2;
        %vpy1 = u(1+n*N) - 0.25*u(2+n*N);
    else
        vpx = (u(m+(n-1)*N) - u(m-1 + (n-1)*N))^2;
        vpy1 = u(m-1+n*N) + 0.25*(u(m+n*N)-u(m-2+n*N));
    end
    if n == 1 || m==1
        vpy2 = 0;
    elseif m==1 && n==2
        vpy2 = u(m+(n-1)*N) + 0.25*(u(m+(n-1)*N)-u(m+(n-1)*N));
```

```
    elseif m == 2 && n ==2
        vpy2 = u(m-1+(n-2)*N) + 0.25*(u(m+(n-2)*N)-u(m-1+(n-2)*N));
    else
        vpy2 = u(m-1+(n-2)*N) + 0.25*(u(m+(n-2)*N)-u(m-2+(n-2)*N));
    end
    vpy = 0.25*(vpy1 - vpy2)^2;
    vp1 = 1.0/(vpx + vpy);
end
function vp2 = p2func(u, m, n)
    global N;
    global h;
    global x;
    global y;
    if m < N
        vpx = (u(m+1+(n-1)*N) - u(m + (n-1)*N))^2;
    else
        vpx = (u(m+(n-1)*N)+1/h*y(n) - u(m + (n-1)*N))^2;
    end
    if m < N && n < N
        vpy1 = u(m+n*N) + 0.25*(u(m+1+n*N)-u(m-1+n*N));
    elseif m < N && n == N
        vpy1 = u(m+(n-1)*N)+1/h*x(m) + 0.25*(u(m+1+(n-1)*N)+1/h*x(m) - u(m-1+(n-1)*N)-1/h*x(m));
    elseif m == N && n < N
        vpy1 = u(m+n*N) + 0.25*(u(m+n*N)+1/h*y(n) - u(m-1+n*N));
    else
        vpy1 = u(m+(n-1)*N)+1/h*x(m) + 0.25*(u(m+(n-1)*N)+1/h*y(n) + u(m-1+(n-1)*N));
    end
    if n == 1
        vpy2 = 0;
    elseif m == N
        vpy2 = u(m+(n-2)*N) + 0.25*(u(m+(n-2)*N)+1/h*y(n)-u(m-1+(n-2)*N));
    else
        if m ==1
            vpy2 = 0;
        else
            vpy2 = u(m+(n-2)*N) + 0.25*(u(m+1+(n-2)*N)-u(m-1+(n-2)*N));
        end
    end
    vpy = 0.25*(vpy1 - vpy2)^2;
    vp2 = 1.0/(vpx + vpy);
end
function vp3 = p3func(u, m, n)
    global N;
```

```
    global x;
    global y;
    global h;
    if n == 1
        vpx1 = u(m+1+(n-1)*N) - 0.25*u(m+1+(2-1)*N);
    elseif n == N
        vpx1 = u(m+(n-1)*N) - 0.25*(u(m + (n-1)*N)+1/h*x(m) +1/h*y
            (n)- u(m+1 + (n-2)*N));
    elseif m == N
        vpx1 = u(m+(n-1)*N)+1/h*y(n) - 0.25*(u(m + (n-1)*N)+1/h*x(
            m)+1/h*y(n) - u(m + (n-2)*N)+1/h*y(n));
    else
        vpx1 = u(m+1+(n-1)*N) - 0.25*(u(m+1 + n*N) - u(m+1 + (n-2)
            *N));
    end
    if m == 1||n == 1
        vpx2 = 0;
    elseif n == N
        vpx2 = u(m-1+(n-1)*N) - 0.25*(u(m-1 + (n-1)*N) + 1/h*x(m)
            - u(m-1 + (n-2)*N));
    else
        vpx2 = u(m-1+(n-1)*N) - 0.25*(u(m-1 + n*N) - u(m-1 + (n-2)
            *N));
    end

    vpx = 0.25*(vpx1 - vpx2)^2;
    if n == 1
        vpy = 0;
    else
        vpy = (u(m+(n-1)*N) - u(m+(n-2)*N))^2;
    end
    vp3 = 1.0/(vpx + vpy);
end
function vp4 = p4func(u, m, n)
    global h;
    global N;
    global x;
    global y;
    if n == 1
        vpx1 = u(m+1+(n-1)*N) + 0.25*(u(m+1+n*N));
        vpx2 = 0;
    elseif n == N || m == N
        vpx1 = u(m+(n-1)*N)+1/h*y(n) + 0.25*(u(m + (n-1)*N)+1/h*x(
            m) - u(m + (n-2)*N));
        vpx2 = u(m-1+(n-1)*N) + 0.25*(u(m-1 + (n-1)*N) - u(m-1 + (
            n-2)*N));
    else
        vpx1 = u(m+1+(n-1)*N) + 0.25*(u(m + n*N)+1/h*y(n) - u(m+1
            + (n-2)*N));
```

```
        if m ==1 && n==1
            vpx2 = 0;
        else
            if m == 1 && n==2
                vpx2 = 0;
            else
                vpx2 = u(m-1+(n-1)*N) + 0.25*(u(m-1 + n*N) - u(m-1
                    + (n-2)*N));
            end
        end
    end
    vpx = 0.25*(vpx1 - vpx2)^2;
    if n == N
        vpy = (u(m+(n-1)*N)+1/h*x(m) - u(m+(n-1)*N))^2;
    else
        vpy = (u(m+n*N) - u(m+(n-1)*N))^2;
    end
    vp4 = 1.0/(vpx + vpy);
end
function vboundary = boundaryfunc(m, n)
    if m == 1 || n == 1
        vboundary = 0;
    elseif m == N
        vboundary = 1;
    elseif n == N
        vboundary = 1;
    else
        vboundary = 0;
    end
end
function v = RightF(x, y)
v = 2*x*y*(x^2+y^2)^(-3/2);
end
```