

Input data

- nurses $n \in \{1, \dots, N\}$, days $d \in \{1, \dots, D\}$, shifts $s \in \{1, \dots, S\}$
- week numbers $w \in \{0, \dots, W\}$, $W = \lfloor D/7 \rfloor$
- nurses demand per day and shift - $demand_{d,s} \in \mathbb{N}$
- workhours lower and upper limit per nurse - $maxhours_n \in \mathbb{N}$
- vacation requests - $VR \subset N \times D$
- preferred companions - $PC \subset N \times N$
- unpreferred companions - $UC \subset N \times N$
- preferred slots - $PS \subset N \times D \times S$
- unpreferred slots - $US \subset N \times D \times S$

Optimization variables: schedule for each nurse, day and shift

$$schedule_{n,d,s} \in \{0, 1\}$$

and interactions of nurses (weekends 2)

$$interaction_{n,n',d,s} \in \{0, 1\}$$

and "worked in given weekend" indicators

$$weekend_{n,w} \in \{0, 1\}$$

and "fraction of contract fulfilled" proportions, as well as lower and upper bound for those

$$\alpha_{min}, \alpha_{max}$$

Reward function

$$\begin{aligned} & \lambda_{PC} \cdot \sum_{(n,n') \in PC} \sum_{d,s} interaction_{n,n',d,s} \\ & - \lambda_{UC} \cdot \sum_{(n,n') \in UC} \sum_{d,s} interaction_{n,n',d,s} \\ & + \lambda_{PS} \cdot \sum_{(n,d,s) \in PS} schedule_{n,d,s} \\ & - \lambda_{US} \cdot \sum_{(n,d,s) \in US} schedule_{n,d,s} \\ & - \lambda_{WCS}(\alpha_{max} - \alpha_{min}) \quad (\text{tight alpha gap iff work distributed proportionately to workhours specified}) \end{aligned}$$

Constraints

$$\begin{aligned}
& \forall_{d,s} \sum_n schedule_{n,d,s} = demand_{d,s} \quad (\text{demand is met}) \\
& \forall_n \frac{24}{S} \sum_{d,s} schedule_{n,d,s} \leq maxhours_n \quad (\text{workhours limits are not exceeded}) \\
& \forall_{n,d} \sum_s schedule_{n,d,s} \leq 1 \quad (\text{max 1 shift per day}) \\
& \forall_{n,w} \sum_{d=7w+1}^{\min(7(w+1),D)} schedule_{n,d,S} \leq 6 \quad (\text{max 6 night shifts per week}) \\
& \forall_{n,d} schedule_{n,d,S} + schedule_{n,d+1,1} \leq 1 \quad (\text{can't continue past midnight}) \\
& \forall_{(n,d) \in VR} \forall_s schedule_{n,d,s} = 0 \quad (\text{vacations are respected}) \\
& \forall_{n,n',d,s} interaction_{n,n',d,s} \leq schedule_{n,d,s} \quad (\text{interactions 1}) \\
& \forall_{n,n',d,s} interaction_{n,n',d,s} \leq schedule_{n',d,s} \quad (\text{interactions 2}) \\
& \forall_{n,n',d,s} interaction_{n,n',d,s} \geq schedule_{n,d,s} + schedule_{n',d,s} - 1 \quad (\text{interactions 3}) \\
& \forall_{n,w} weekend_{n,w} \geq \frac{1}{S} \sum_s (schedule_{n,w+6,s} + schedule_{n,w+7,s}) \quad (\text{weekends 1}) \\
& \forall_{n,w} weekend_{n,w} \leq \sum_s (schedule_{n,w+6,s} + schedule_{n,w+7,s}) \quad (\text{weekends 2}) \\
& \forall_n \alpha_{min} \leq \frac{\frac{24}{S} \sum_{d,s} schedule_{n,d,s}}{workhours_n} \leq \alpha_{max} \quad (\text{alphas with bounds})
\end{aligned}$$