Input data

- nurses $n \in \{1, \ldots, pNumerOfNurses\} = sNurses$
- days $d \in \{1, \ldots, pNumberOfDays\} = sDays$
- shifts $s \in \{1, \ldots, pNumberOfShifts\} = sShifts$ and $pShiftLength = 24/pNumberOfShifts$
- week numbers $w \in \{0, \ldots, pNumberOfWeeks\} = sWeeks, pNumberOfWeeks = \lceil pNumberOfDays/7 \rceil - 1$, weekdays $\delta \in \{0, ..., 6\}$
- $pMaxNightShifts$
- nurses demand per day and shift - $pDemand_{d,s} \in \mathbb{N}$
- workhours upper limit per nurse - $pWorkhoursLimit_n \in \mathbb{N}$
- vacation requests - $sVacations \subset sNurses \times sDays$
- preferred companions - $sPreferredCompanions \subset sNurses \times sNurses$
- unpreferred companions - $sUnpreferredCompanions \subset sNurses \times sNurses$
- preferred slots - $sPreferredSlots \subset sNurses \times sDays \times sShifts$
- unpreferred slots - $sUnpreferredSlots \subset sNurses \times sDays \times sShifts$

Optimization variables: schedule for each nurse, day and shift

$$vSchedule_{n,d,s} \in \{0, 1\}$$

and interactions of nurses

$$vInteraction_{n,n',d,s} \in \{0, 1\}$$

and minimal and maximal number of worked weekends

$$vWeekend_{n,w} \in \{0, 1\}$$

and lower and upper bound of "fraction of contract fulfilled" proportions ($\frac{pShiftLength \cdot \sum_{d,s} vSchedule_{n,d,s}}{pWorkhoursLimit_n}$)

$$vAlphaMin,\ vAlphaMax$$

Reward function

$$\begin{aligned}
&\lambda_{PC} \cdot \sum_{(n,n') \in PC} \sum_{d,s} vInteraction_{n,n',d,s} \\
-&\lambda_{UC} \cdot \sum_{(n,n') \in UC} \sum_{d,s} vInteraction_{n,n',d,s} \\
+&\lambda_{PS} \cdot \sum_{(n,d,s) \in PS} vSchedule_{n,d,s} \\
-&\lambda_{US} \cdot \sum_{(n,d,s) \in US} vSchedule_{n,d,s} \\
-&(\alpha_{max} - \alpha_{min}) \\
&\lambda_{\text{WHS}}(\alpha_{max} - \alpha_{min}) \quad \text{(prefer equal work to max work hours ratio)} \\
&\lambda_{\text{W}}(\texttt{vMaxWeekendsWorked} - \texttt{vMinWeekendsWorked}) \quad \text{(prefer equal busy weekends distribution)}
\end{aligned}$$

Constraints

$\forall_{d,s} \sum_{n} vSchedule_{n,d,s} = vDemand_{d,s}$ (demand is met)

$\forall_{n} pShiftLegth \cdot \sum_{d,s} vSchedule_{n,d,s} \leq pWorkhoursLimit_{n}$ (workhours limits are not exceeded)

$\forall_{n,d} \sum_{s} vSchedule_{n,d,s} \leq 1$ (max 1 shift per day)

$\forall_{n,w} \sum_{d=7w+1}^{\min(7(w+1),D)} vSchedule_{n,d,S} \leq pMaxNightShifts$ (respect night shifts weekly limit)

$\forall_{n,d} vSchedule_{n,d,S} + vSchedule_{n,d+1,1} \leq 1$ (can't continue past midnight)

$\forall_{(n,d)\in sVacations} \forall_{s} vSchedule_{n,d,s} = 0$ (vacations are respected)

$\forall_{n,n',d,s} vInteraction_{n,n',d,s} \leq vSchedule_{n,d,s}$ (interactions 1)

$\forall_{n,n',d,s} vInteraction_{n,n',d,s} \leq vSchedule_{n',d,s}$ (interactions 2)

$\forall_{n,n',d,s} vInteraction_{n,n',d,s} \geq vSchedule_{n,d,s} + vSchedule_{n',d,s} - 1$ (interactions 3)

$\forall_{n} vAlphaMin \leq \frac{pShiftLength \cdot \sum_{d,s} vSchedule_{n,d,s}}{pWorkhoursLimit_{n}} \leq vAlphaMax$ (alphas with bounds)

$\forall_{n,w} vWeekend_{n,w} \geq \frac{1}{S} \sum_{s} (vSchedule_{n,7w+6,s} + vSchedule_{n,7w+7,s})$ (weekends 1)

$\forall_{n,w} vWeekend_{n,w} \leq \sum_{s} (vSchedule_{n,7w+6,s} + vSchedule_{n,7w+7,s})$ (weekends 2)

$\forall_{n}$ $\texttt{vMinWeekendsWorked} \leq \sum_{n} \texttt{vWeekend}_{n,s} \leq \texttt{vMaxWeekendsWorked}$ (min/max weekends worked are computed)

$\forall_{n,w,\delta,s}$ $\texttt{vRest24hIndicator}_{n,w,\delta,s} \geq \frac{\sum_{0\leq i\leq pNumberOfShifts-1} vSchedule_{n,d',s'}}{pNumberOfShifts}$,

where $d' := 1 + 7w + \delta + \left\lfloor \frac{s+i}{pNumberOfShifts} \right\rfloor, s' := 1 + (s - 1 + i) \bmod pNumberOfShifts$

$\forall_{n,w} \sum_{\delta,s} \texttt{vRest24hIndicator}_{n,w,\delta,s} \leq 6S$