Input data

- nurses $n \in \{1, \ldots, N\}$, days $d \in \{1, \ldots, D\}$, shifts $s \in \{1, \ldots, S\}$
- week numbers $w \in \{0, \ldots, W\}, W = \lfloor D/7 \rfloor - 1$, weekdays $\delta \in \{0, ..., 6\}$
- nurses demand per day and shift - $demand_{d,s} \in \mathbb{N}$
- workhours lower and upper limit per nurse - $maxhours_n \in \mathbb{N}$
- vacation requests - $VR \subset N \times D$
- preferred companions - $PC \subset N \times N$
- unpreferred companions - $UC \subset N \times N$
- preferred slots - $PS \subset N \times D \times S$
- unpreferred slots - $US \subset N \times D \times S$

Optimization variables: schedule for each nurse, day and shift

$$schedule_{n,d,s} \in \{0, 1\}$$

and interactions of nurses (weekends 2)

$$interaction_{n,n',d,s} \in \{0, 1\}$$

and "worked in given weekend" indicators

$$\texttt{weekend}_{n,s} \in \{0, 1\}, \texttt{min\_weekends\_worked}, \texttt{max\_weekends\_worked}$$

and "fraction of contract fulfilled" proportions, as well as lower and upper bound for those

$$\alpha_{min},\ \alpha_{max}$$

Reward function

$$\lambda_{PC} \cdot \sum_{(n,n') \in PC} \sum_{d,s} interaction_{n,n',d,s}$$

$$-\lambda_{UC} \cdot \sum_{(n,n') \in UC} \sum_{d,s} interaction_{n,n',d,s}$$

$$+\lambda_{PS} \cdot \sum_{(n,d,s) \in PS} schedule_{n,d,s}$$

$$-\lambda_{US} \cdot \sum_{(n,d,s) \in US} schedule_{n,d,s}$$

$-\lambda_{\text{WHS}}(\alpha_{max} - \alpha_{min})$ (prefer equal work to max work hours ratio)

$-\lambda_{\text{W}}(\texttt{max\_weekends\_worked} - \texttt{min\_weekends\_worked})$ (prefer equal busy weekends distribution)

Constraints

$\forall_{d,s} \sum_{n} schedule_{n,d,s} = demand_{d,s}$ (demand is met)

$\forall_{n} \frac{24}{S} \sum_{d,s} schedule_{n,d,s} \leq maxhours_{n}$ (workhours limits are not exceeded)

$\forall_{n,d} \sum_{s} schedule_{n,d,s} \leq 1$ (max 1 shift per day)

$\forall_{n,w} \sum_{d=7w+1}^{\min(7(w+1),D)} schedule_{n,d,S} \leq 6$ (max 6 night shifts per week)

$\forall_{n,d}\ schedule_{n,d,S} + schedule_{n,d+1,1} \leq 1$ (can't continue past midnight)

$\forall_{(n,d)\in VR}\ \forall_{s}\ schedule_{n,d,s} = 0$ (vacations are respected)

$\forall_{n,n',d,s}\ interaction_{n,n',d,s} \leq schedule_{n,d,s}$ (interactions 1)

$\forall_{n,n',d,s}\ interaction_{n,n',d,s} \leq schedule_{n',d,s}$ (interactions 2)

$\forall_{n,n',d,s}\ interaction_{n,n',d,s} \geq schedule_{n,d,s} + schedule_{n',d,s} - 1$ (interactions 3)

$\forall_{n,w}\ weekend_{n,w} \geq \frac{1}{S} \sum_{s} (schedule_{n,7w+6,s} + schedule_{n,7w+7,s})$ (weekends 1)

$\forall_{n,w}\ weekend_{n,w} \leq \sum_{s} (schedule_{n,7w+6,s} + schedule_{n,7w+7,s})$ (weekends 2)

$\forall_{n}\ \texttt{min\_weekends\_worked} \leq \sum_{n} \texttt{weekend}_{n,s} \leq \texttt{max\_weekends\_worked}$ (min/max weekends worked are computed)

$\forall_{n}\ \alpha_{min} \leq \frac{\frac{24}{S} \sum_{d,s} schedule_{n,d,s}}{workhours_{n}} \leq \alpha_{max}$ (alphas with bounds)

$\forall_{n,w,\delta,s}\ \texttt{rest\_24h\_indicator}_{n,w,\delta,s} \geq \frac{\sum_{0\leq i\leq S-1} schedule_{n,1+7w+\delta+\left\lfloor \frac{s+i}{s} \right\rfloor,1+(s-1+i) \bmod S}}{S}$

$\forall_{n,w}\ \sum_{\delta,s} \texttt{rest\_24h\_indicator}_{n,w,\delta,s} \leq 6S$