Exercise 3.1 MLE for the Bernoulli/ binomial model

$$\frac{d}{d\theta}p(D|\theta) = \frac{d}{d\theta}(\theta^{N_1}(1-\theta)^{N_0}) \tag{1}$$

$$= N_1\theta^{N_1-1}(1-\theta)^{N_0} - N_0\theta^{N_1}(1-\theta)^{N_0 1} \tag{2}$$

$$= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1(1-\theta) - N_0\theta) \tag{3}$$

$$= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1 - N\theta) \tag{4}$$

$$\therefore \theta_{\text{MLE}} = \frac{N_1}{N} \tag{5}$$

Exercise 3.2 Marginal likelihood for the Beta-Bernoulli model

$$p(D) = \frac{[(\alpha_1)\cdots(\alpha_1 + N_1 - 1)][(\alpha_0)\cdots(\alpha_0 + N_0 - 1)]}{(\alpha)\cdots(\alpha + N - 1)} \tag{6}$$

$$= \frac{[(\alpha_1)\cdots(\alpha_1 + N_1 - 1)][(\alpha_0)\cdots(\alpha_0 + N_0 - 1)]}{(\alpha_1 + \alpha_0)\cdots(\alpha_1 + \alpha_0 + N - 1)} \tag{7}$$

$$= \frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1 + \alpha_0 + N)}[(\alpha_1)\cdots(\alpha_1 + N_1 - 1)][(\alpha_0)\cdots(\alpha_0 + N_0 - 1)] \tag{8}$$

$$= \frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1 + \alpha_0 + N)}\frac{\Gamma(\alpha_1 + N_1)}{\Gamma(\alpha_1)}\frac{\Gamma(\alpha_0 + N_0)}{\Gamma(\alpha_0)} \tag{9}$$

$$= \frac{\Gamma(\alpha_1 + N_1)\Gamma(\alpha_0 + N_0)}{\Gamma(\alpha_1 + \alpha_0 + N)}\frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1)\Gamma(\alpha_0)} \tag{10}$$