

Exercise 3.1 MLE for the Bernoulli/ binomial model

$$\frac{d}{d\theta}p(D|\theta) = \frac{d}{d\theta}(\theta^{N_1}(1-\theta)^{N_0}) \quad (1)$$

$$= N_1\theta^{N_1-1}(1-\theta)^{N_0} - N_0\theta^{N_1}(1-\theta)^{N_0-1} \quad (2)$$

$$= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1(1-\theta) - N_0\theta) \quad (3)$$

$$= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1 - N\theta) \quad (4)$$

$$\therefore \theta_{\text{MLE}} = \frac{N_1}{N} \quad (5)$$

Exercise 3.2 Marginal likelihood for the Beta-Bernoulli model

$$p(D) = \frac{[(\alpha_1) \cdots (\alpha_1 + N_1 - 1)][(\alpha_0) \cdots (\alpha_0 + N_0 - 1)]}{(\alpha) \cdots (\alpha + N - 1)} \quad (6)$$

$$= \frac{[(\alpha_1) \cdots (\alpha_1 + N_1 - 1)][(\alpha_0) \cdots (\alpha_0 + N_0 - 1)]}{(\alpha_1 + \alpha_0) \cdots (\alpha_1 + \alpha_0 + N - 1)} \quad (7)$$

$$= \frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1 + \alpha_0 + N)} [(\alpha_1) \cdots (\alpha_1 + N_1 - 1)][(\alpha_0) \cdots (\alpha_0 + N_0 - 1)] \quad (8)$$

$$= \frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1 + \alpha_0 + N)} \frac{\Gamma(\alpha_1 + N_1)}{\Gamma(\alpha_1)} \frac{\Gamma(\alpha_0 + N_0)}{\Gamma(\alpha_0)} \quad (9)$$

$$= \frac{\Gamma(\alpha_1 + N_1)\Gamma(\alpha_0 + N_0)}{\Gamma(\alpha_1 + \alpha_0 + N)} \frac{\Gamma(\alpha_1 + \alpha_0)}{\Gamma(\alpha_1)\Gamma(\alpha_0)} \quad (10)$$

Exercise 3.3 Posterior predictive for Beta-Binomial model

$$Bb(1|\alpha'_1, \alpha'_0, 1) = \frac{B(1 + \alpha'_1, 1 - 1 + \alpha'_0)}{B(\alpha'_1, \alpha'_0)} \binom{1}{1} \quad (11)$$

$$= \frac{B(1 + \alpha'_1, \alpha'_0)}{B(\alpha'_1, \alpha'_0)} \quad (12)$$

$$= \frac{\Gamma(\alpha'_1 + \alpha'_0)}{\Gamma(\alpha'_1)\Gamma(\alpha'_0)} \frac{\Gamma(1 + \alpha'_1)\Gamma(\alpha'_0)}{\Gamma(1 + \alpha'_1 + \alpha'_0)} \quad (13)$$

$$= \frac{\Gamma(\alpha'_1 + \alpha'_0)}{\Gamma(\alpha'_1)\Gamma(\alpha'_0)} \frac{\alpha'_1\Gamma(\alpha'_1)\Gamma(\alpha'_0)}{(\alpha'_1 + \alpha'_0)\Gamma(\alpha'_1 + \alpha'_0)} \quad (14)$$

$$= \frac{\alpha'_1}{\alpha'_1 + \alpha'_0} \quad (15)$$

Exercise 3.4 Beta updating from censored likelihood

$$p(\theta, X < 3) = p(\theta)p(X < 3|\theta) \quad (16)$$

$$= p(\theta)\left(\sum_{k=0}^2 p(X = k|\theta)\right) \quad (17)$$

$$= p(\theta)\left(\sum_{k=0}^2 \theta^k (1 - \theta)^{(5-k)}\right) \quad (18)$$

$$= \text{Beta}(\theta|1, 1)\left(\sum_{k=0}^2 \theta^k (1 - \theta)^{(5-k)}\right) \quad (19)$$

$$= \sum_{k=0}^2 \theta^k (1 - \theta)^{(5-k)} \quad (20)$$

Exercise 3.5 Uninformative prior for log-odds ratio

$$p(\theta) = p(\phi) \left| \frac{d\phi}{d\theta} \right| \quad (21)$$

$$= p(\phi) \theta^{-1} (1 - \theta)^{-1} \quad (22)$$

$$\propto \text{Beta}(\theta|0, 0) \quad (\because p(\phi) \propto 1) \quad (23)$$