Exercise 3.1 MLE for the Bernoulli/ binomial model

$$
\begin{aligned}
\frac{d}{d\theta}p(D|\theta) &= \frac{d}{d\theta}(\theta^{N_1}(1-\theta)^{N_0}) && (1)\\
&= N_1\theta^{N_1-1}(1-\theta)^{N_0} - N_0\theta^{N_1}(1-\theta)^{N_0 1} && (2)\\
&= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1(1-\theta) - N_0\theta) && (3)\\
&= \theta^{N_1-1}(1-\theta)^{N_0-1}(N_1 - N\theta) && (4)\\
\therefore \theta_{\text{MLE}} &= \frac{N_1}{N} && (5)
\end{aligned}
$$

Exercise 3.2 Marginal likelihood for the Beta-Bernoulli model

$$
\begin{aligned}
p(D) &= \frac{[(\alpha_1)\cdots(\alpha_1+N_1-1)][(\alpha_0)\cdots(\alpha_0+N_0-1)]}{(\alpha)\cdots(\alpha+N-1)} && (6)\\
&= \frac{[(\alpha_1)\cdots(\alpha_1+N_1-1)][(\alpha_0)\cdots(\alpha_0+N_0-1)]}{(\alpha_1+\alpha_0)\cdots(\alpha_1+\alpha_0+N-1)} && (7)\\
&= \frac{\Gamma(\alpha_1+\alpha_0)}{\Gamma(\alpha_1+\alpha_0+N)}[(\alpha_1)\cdots(\alpha_1+N_1-1)][(\alpha_0)\cdots(\alpha_0+N_0-1)] && (8)\\
&= \frac{\Gamma(\alpha_1+\alpha_0)}{\Gamma(\alpha_1+\alpha_0+N)}\frac{\Gamma(\alpha_1+N_1)}{\Gamma(\alpha_1)}\frac{\Gamma(\alpha_0+N_0)}{\Gamma(\alpha_0)} && (9)\\
&= \frac{\Gamma(\alpha_1+N_1)\Gamma(\alpha_0+N_0)}{\Gamma(\alpha_1+\alpha_0+N)}\frac{\Gamma(\alpha_1+\alpha_0)}{\Gamma(\alpha_1)\Gamma(\alpha_0)} && (10)
\end{aligned}
$$

Exercise 3.3 Posterior prdictive for Beta-Binomial model

$$
\begin{aligned}
Bb(1|\alpha_1',\alpha_0',1) &= \frac{B(1+\alpha_1', 1-1+\alpha_0')}{B(\alpha_1',\alpha_0')}\binom{1}{1} && (11)\\
&= \frac{B(1+\alpha_1',\alpha_0')}{B(\alpha_1',\alpha_0')} && (12)\\
&= \frac{\Gamma(\alpha_1'+\alpha_0')}{\Gamma(\alpha_1')\Gamma(\alpha_0')}\frac{\Gamma(1+\alpha_1')\Gamma(\alpha_0')}{\Gamma(1+\alpha_1'+\alpha_0')} && (13)\\
&= \frac{\Gamma(\alpha_1'+\alpha_0')}{\Gamma(\alpha_1')\Gamma(\alpha_0')}\frac{\alpha_1'\Gamma(\alpha_1')\Gamma(\alpha_0')}{(\alpha_1'+\alpha_0')\Gamma(\alpha_1'+\alpha_0')} && (14)\\
&= \frac{\alpha_1'}{\alpha_1'+\alpha_0'} && (15)
\end{aligned}
$$