

# Lineare Algebra II (LA) Übungsblatt 12

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## Aufgabe 1

a)

*Beweis.* Für alle  $v, w, z \in \mathbb{R}^3$  gilt:

$$\begin{aligned} v \times (w \times z) &= v \times \begin{pmatrix} w_2 z_3 - w_3 z_2 \\ w_3 z_1 - w_1 z_3 \\ w_1 z_2 - w_2 z_1 \end{pmatrix} \\ &= \begin{pmatrix} v_2(w_1 z_2 - w_2 z_1) - v_3(w_3 z_1 - w_1 z_3) \\ v_3(w_2 z_3 - w_3 z_2) - v_1(w_1 z_2 - w_2 z_1) \\ v_1(w_3 z_1 - w_1 z_3) - v_2(w_2 z_3 - w_3 z_2) \end{pmatrix} \\ &= \begin{pmatrix} v_2 w_1 z_2 - v_2 w_2 z_1 - v_3 w_3 z_1 + v_3 w_1 z_3 \\ v_3 w_2 z_3 - v_3 w_3 z_2 - v_1 w_1 z_2 + v_1 w_2 z_1 \\ v_1 w_3 z_1 - v_1 w_1 z_3 - v_2 w_2 z_3 + v_2 w_3 z_2 \end{pmatrix} \\ &= \begin{pmatrix} w_1(v_2 z_2 + v_3 z_3) - z_1(v_2 w_2 + v_3 w_3) \\ w_2(v_3 z_3 + v_1 z_1) - z_2(v_3 w_3 + v_1 w_1) \\ w_3(v_1 z_1 + v_2 z_2) - z_3(v_1 w_1 + v_2 w_2) \end{pmatrix} \\ &= \begin{pmatrix} w_1(v_2 z_2 + v_3 z_3 + v_1 z_1 - v_1 z_1) - z_1(v_2 w_2 + v_3 w_3 + v_1 w_1 - v_1 w_1) \\ w_2(v_3 z_3 + v_1 z_1 + v_2 z_2 - v_2 z_2) - z_2(v_3 w_3 + v_1 w_1 + v_2 w_2 - v_2 w_2) \\ w_3(v_1 z_1 + v_2 z_2 + v_3 z_3 - v_3 z_3) - z_3(v_1 w_1 + v_2 w_2 + v_3 w_3 - v_3 w_3) \end{pmatrix} \\ &= \begin{pmatrix} (w_1(v_2 z_2 + v_3 z_3 + v_1 z_1) - w_1 v_1 z_1) - (z_1(v_2 w_2 + v_3 w_3 + v_1 w_1) - z_1 v_1 w_1) \\ (w_2(v_3 z_3 + v_1 z_1 + v_2 z_2) - w_2 v_2 z_2) - (z_2(v_3 w_3 + v_1 w_1 + v_2 w_2) - z_2 v_2 w_2) \\ (w_3(v_1 z_1 + v_2 z_2 + v_3 z_3) - w_3 v_3 z_3) - (z_3(v_1 w_1 + v_2 w_2 + v_3 w_3) - z_3 v_3 w_3) \end{pmatrix} \\ &= \begin{pmatrix} w_1(v_2 z_2 + v_3 z_3 + v_1 z_1) - z_1(v_2 w_2 + v_3 w_3 + v_1 w_1) \\ w_2(v_3 z_3 + v_1 z_1 + v_2 z_2) - z_2(v_3 w_3 + v_1 w_1 + v_2 w_2) \\ w_3(v_1 z_1 + v_2 z_2 + v_3 z_3) - z_3(v_1 w_1 + v_2 w_2 + v_3 w_3) \end{pmatrix} \\ &= \langle v, z \rangle w - \langle v, w \rangle z \end{aligned}$$

□