Homework 1
PM569 Spatial Statistics, Fall 2019

Due September 20, 2019 (end of day)

Theoretical Semivariograms and Covariance

1. For the following theoretical semivariogram functions:
   - Linear: $\gamma(h) = \tau^2 + \sigma^2 h$ if $h > 0; 0$ otherwise
   - Power model: $\gamma(h) = \tau^2 + \sigma^2 h^\lambda$ if $h > 0; 0$ otherwise
   - Rational quadratic: $\gamma(h) = \tau^2 + \sigma^2 \frac{h^2}{(1+\phi h^2)}$ if $h > 0; 0$ otherwise
   - Powered exponential: $\gamma(h) = \tau^2 + \sigma^2(1 - exp(-|\phi h|^\lambda)$ if $h > 0; 0$ otherwise
   - Wave: $\gamma(h) = \tau^2 + \sigma^2(1 - \frac{sin(\phi h)}{\phi h})$ if $h > 0; 0$ otherwise

   a. Plot the semivariograms (on the same plot) fixing $\tau^2 = 0.5, \sigma^2 = 4, \phi = 6, \lambda = 0.5$ but choosing your own distances.
   b. Using the plot from a), provide a description of the differences in the four models despite having the same fixed parameter values. Also explain whether or not each semivariogram corresponds to a stationary process.
   c. For the two power models, vary the value λ and plot the different semivariograms. What does the power parameter λ represent?
   d. For the rational quadratic and wave models, vary the values of τ^2, σ^2, ϕ (one at a time) and describe how the semivariogram changes.

2. Find/calculate and plot the **covariogram** (covariance as a function of distance) for the exponential, spherical and Gaussian functions shown in class.

3. The Matern covariance function is widely used in the spatial statistical literature. The function is:

$$C(h) = \sigma^2 \frac{1}{\Gamma(\kappa)2^{\kappa-1}}(\sqrt{2\kappa}\frac{h}{\phi})^\kappa K_\kappa(\sqrt{2\kappa}\frac{h}{\phi})$$

   Where κ is a smoothness parameter, Γ is a gamma function, and K_κ is a modified Bessel function. Using the functions gamma() and besselK(), write the above Matern covariance function in R and test and plot it with your choice of parameter values. What do you see changing for small κ (i.e. $\kappa \to 0$) versus large κ (i.e. $\kappa \to \infty$)?

Empirical Semivariograms

For this question we will use meteorological data from weather stations in the Carolinas at the time of Hurricane Dorian, September 6, 2019 (averaged over hourly measurements 10am-3pm). We will focus on wind speed (m/s) and atmospheric pressure (millibars) to examine the strength and location of the hurricane.
The data were acquired from the National Oceanic and Atmospheric Administration's National Weather Service ftp (ftp.ncdc.noaa.gov/pub/data/noaa/)

4. Perform exploratory data analysis: examine the data distributions of wind speed and atmospheric pressure and provide summary statistics. Create two maps in leaflet showing the locations and color gradients for the values of the meteorological parameters. Please describe any spatial trends that can be visualized in the data.

5. Project the latitude and longitude to UTM coordinates. Use these x,y values in the subsequent questions.

6. Plot empirical semivariograms (robust) for wind speed and atmospheric pressure. Try two different semivariograms for each variable (e.g. change max distance, number of bins).

7. Provide an eyeball estimate of the nugget, sill and range and plot a curve of a theoretical semivariogram (your choice of which function to use) on top of your preferred binned semivariogram.

8. Discuss your findings by comparing the spatial aspects of each variable. Do you think one or the other is a better indicator of the hurricane's strength and scope?

9. Determine if there is a linear spatial trend in wind speed or pressure.

10. Determine if there is anisotropy in either variable.