

Recommendation Systems

Tool: Clustering, Collaborative and content filtering

The Analytics Edge:

Recommendation systems build models around users' preferences to help personalize the user experience. Examples include Netflix that provides recommendations on movies to users, Amazon that provides recommendation of items of almost any type to users, eHarmony that provides recommendations on compatible matches for single men and women. The large number of choices available, the massive amount of data available through online transactions help companies use tools of analytics such as clustering and collaborative and content filtering to find the "right" items to recommend to users.

The challenges in recommendation systems pertain to obtaining real-time recommendations, making good recommendations since poor recommendations can imply frustrated customers, and dealing with users for which very little information is available to users with high activity.

The Netflix Story

Netflix is a company that provides DVD by mail service and on-demand streaming of films and TV series. The company was founded in 1997 and has over 60 million subscribers globally with annual revenues of roughly \$5.5 billion in 2014. Netflix is now available in Singapore too. The company started with Hastings (one of the Co-founders of Netflix) who was forced to pay \$40 in fines when he returned the Apollo 13 DVD well past its due date. As of April 2014, Netflix has 81 million subscribers worldwide.

Netflix provides a monthly flat fee service rate for rental of DVDs. A subscriber creates an ordered list of films to rent. Films are delivered by US mail to subscribers who can keep the rented discs as long as they desire with a limit on how many discs that a subscriber can have on loan simultaneously. To rent a new film, the subscriber must mail back the current DVD upon the receipt of which Netflix will ship the next disc in the rental queue. (Flat rate DVD by mail service)

Now increasingly, Netflix is moving more towards the video on demand streaming services on the Internet.

Netflix prize

In 2006, Netflix held the Netflix prize competition to predict the user preferences and beat its existing Netflix movie recommendation engine known as Cinematch by at least 10%.

Netflix provided a training data set of 100 480 507 ratings that 480 189 users gave (over 100 million) to 1770 movies. Each movie was rated from 1 to 5 by the users expressing how much they liked or disliked it.

\approx 100 million ratings
\approx 480 thousand users
\approx 18 thousand movies

The data was collected from 1998 to 2005. In addition it withheld the ratings of 2817131 (≈ 3 million ratings) data points from the same subscribers over the same set of movies as a "qualifying" dataset. Any participating team had to predict the ratings on the entire qualifying data set but were informed of the accuracy only for roughly half the data (a "quiz" set of 1408342 ratings). This was used to calculate the leaderboard. The other half (a "test" set of 1408789) of ratings was used to determine the ultimate winners. Only the judges knew the quiz & test set partition in the qualifying dataset. Note that in the training dataset, the average user rated over 200 movies and the average movie was rated by over 5000 users. However some movies had only 3 ratings in the training set while one user had rated over 17000 movies (wide variance in the dataset).

The predictions (which could be any number, not necessarily in the set $\{1, 2, 3, 4, 5\}$) were scored against the true ratings in terms of RMSE (root mean squared error). The goal was to minimize the Root mean squared error. The trivial algorithm that used the average rating for each movie from the training set produced a RMSE of 1.0540 on the quiz set. The algorithm used by Netflix at that time called Cinematch scored on RMSE of 0.9514 on the quiz data using the training set to build the model (roughly 10% improvement). On the test set, Cinematch had a RMSE of 0.9525 (close to quiz dataset). To win the 1 million \$ prize, the team had to beat Cinematch by another 10%, to achieve 0.8572 on the test set. ~~achieve~~. Once the RMSE on the quiz set was 0.8572 or lesser, 30 days was provided to submit additional candidate predictions. As long as no team won the grand prize \$50,000 was awarded as long as for each year they improved on the previous winner by at least 1%. Teams could submit one attempt per day.

Netflix prize

Oct 2, 2006



Oct 8, 2006



June 2007



Nov 13, 2007



2008



June 26, 2009



Last Call (30 days)



July 25, 2009



July 26, 2009



Final standing on
leaderboard



Sep 18, 2009

Competition launched

WXY7 Consulting beats Cinematch

Over 20000 teams registered

Bell Kor (Team of 3 AT&T lab researchers) won 50000\$ Progress Prize with 8.43% improvement over Cinematch

Teams Bell Kor & Pragmatic Chaos joined together to win 50000\$ prize with 9.4% improvement

Team BellKor Pragmatic Chaos achieved 10.05% improvement (quiz test RMSE of 0.8558)
Netflix entered last call for Grand Prize.

Team Ensemble achieved 10.09%.

Netflix stopped gathering submissions

Ensemble 10.10% improvement
BellKor Pragmatic Chaos 10.09%
quiz set results RMSE = 0.8554

On the test set, both teams had a test RMSE of 0.8567. But BellKor submitted it 20 min earlier making them the winners of the 1M\$ Netflix prize

The team Bell Kori's Pragmatic Chaos consisted of two researchers from AT&T Labs, two researchers from Austrian at the Commodo Research & Consulting, one researcher from Yahoo and two researchers from Pragmatic Theory.

The team published a description of the algorithm as required by the competition.

A variety of methods was used in the final winning predictive algorithm. Successful analytics often needs a combination of a variety of approaches to be successful. These approaches included collaborative filtering, matrix factorization, regression models, LASSO technique all combined into ensemble methods.

Movie Lens

Movie Lens is a recommendation system for movie set up by GroupLens research, a research lab at the University of Minnesota.

GroupLens collects and makes data on movie ratings from MovieLens available for research.

Clustering

In supervised learning, typically one has access to a set of p features (or predictor variables) x_1, \dots, x_p and an output (or response variable) y . The goal is to predict y from x_1, \dots, x_p . We have seen several examples of this including linear regression, logistic regression, discrete choice models, classification and regression trees, random forests.

In unsupervised learning, one has access to a set of p features x_1, \dots, x_p but no associated response variable. Clustering is an example of such an approach that is used to discover subgroups within the data.

One of the main challenges with unsupervised learning is that it is far more subjective since there is no clear way to perform cross-validation or validate results on a test set.

However these techniques are important in several domains such as clustering groups of shoppers based on their browsing and purchasing histories on say Amazon. Then an individual shopper can be preferentially shown items based on the purchase histories of other shoppers in the cluster (Build a model for each cluster)

Cluster observations into distinct groups such that.

- 1) Data within a group are similar to each other
- 2) Data in different groups are different from each other

K-means clustering

Given n observations $\bar{x}_i = (x_{i1}, \dots, x_{ip})$ for $i = 1, \dots, n$, partition it into K clusters C_1, \dots, C_K where:

$$C_1 \cup C_2 \cup \dots \cup C_K = \{1, \dots, n\}$$

$$C_x \cap C_{x'} = \emptyset \quad \text{for } x \neq x'$$

The clusters are chosen so as to minimize the within cluster sum of squares:

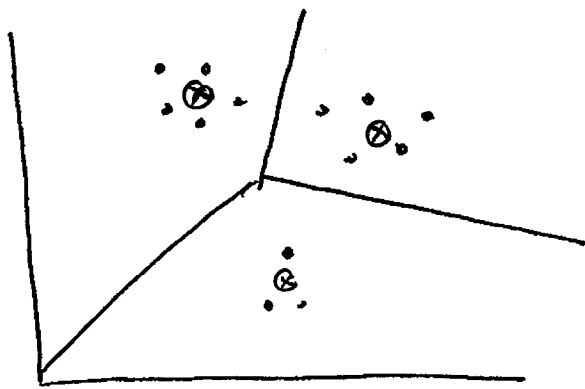
$$\min_{\substack{C_1, \dots, C_K \\ \mu_1, \dots, \mu_K}} \sum_{x=1}^K \sum_{i \in C_x} \|\bar{x}_i - \mu_x\|^2 \quad \left(\begin{array}{l} \text{where} \\ \|\bar{x}_i - \mu_x\| \\ = \sqrt{(x_{i1} - \mu_{x1})^2 + \dots + (x_{ip} - \mu_{xp})^2} \end{array} \right)$$

Here the optimal μ_1, \dots, μ_K are the mean of the observations in clusters C_1, \dots, C_K , namely

$$\mu_x = \frac{1}{|C_x|} \sum_{i \in C_x} x_i$$

The observations are partitioned into clusters in which each observation belongs to the cluster with the nearest mean serving as a prototype of the cluster.

$K = 3$



Partition the
Space of data
into Voronoi cells

(Region where all
the points in that
cell are closer to
the \otimes than any other

Given K clusters and n observations there are K^n ways to partition which grows rapidly as n increases. Solving the K -means problem exactly is challenging but efficient heuristic algorithms that quickly converge to the local optimum exist.

Algorithm (Lloyd's algorithm)

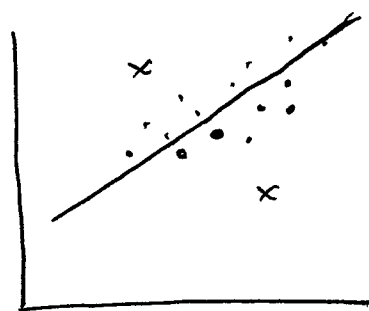
- 1) Start with a random set of K means.
For example this could be K observations from the data set
- 2)
 - a) Assignment step: Assign each observation to the cluster whose centroid (mean) is the closest.
 - b) Update step: Calculate the new centroid (mean) of the observations in the new cluster

Repeat steps a) and b) till the assignment no longer changes.

Example : $K = 2$



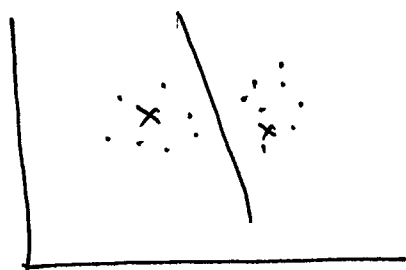
Assign
→



↓ Update



Assign
←



↓
Continue till convergence to local optimum

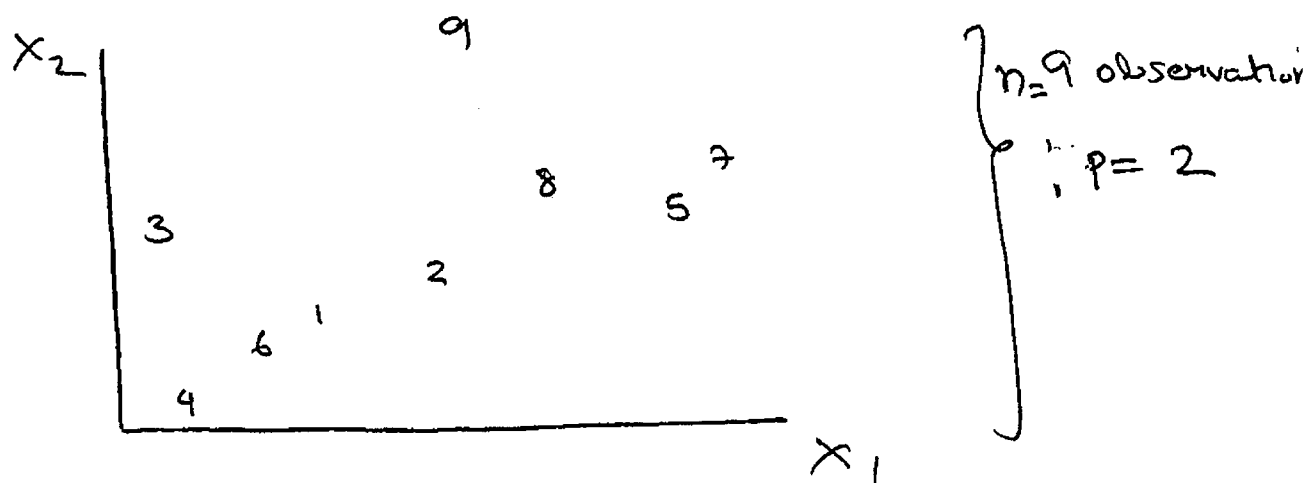
Alternate approach is start by randomly allocating each observation to a cluster and then using the update and assignment steps iteratively

The number of clusters (K) has to be decided before running the algorithm. The K-means algorithm works well for both small & large datasets. Since randomization is involved one can also try running the algorithm multiple times to try & choose the best clusters.

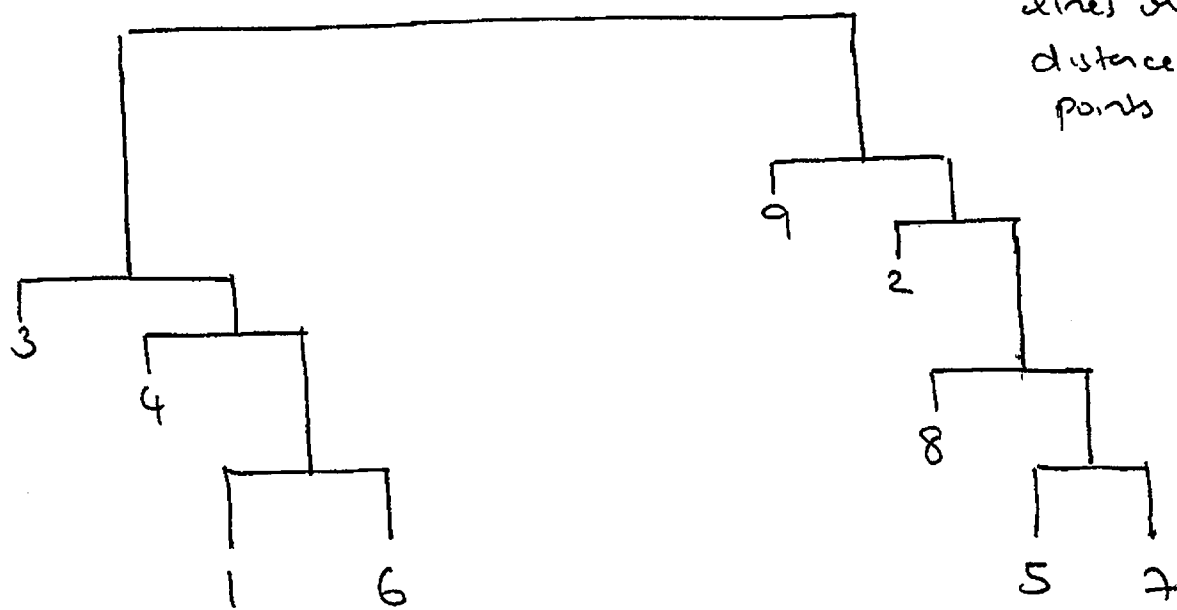
One however needs to check if the clusters are reasonable - looking at statistic of each cluster or looking at an outcome variable which is not used to cluster but clustering can help prediction then.

Hierarchical Clustering

This method does not need K to be prespecified and provides a tree based representation of the observations (called a dendrogram).



Dendrogram representation



Heights of vertical lines represents distance between points or clusters

Each leaf of dendrogram is an observation.

Fusion of two leaves into a branch corresponds to similar observations.

Observations that fuse at bottom of tree are very similar while fuse at top tends to be quite different. Height of fusion indicates how different observations are.

Note that similarity of observations should be based on the location on the vertical axis where branches containing two observations are fused first.

Similarity of two observations cannot be based on their proximity along horizontal axis.

For example 9 is not closer to 2 any more than it is to 8, 5 or 7.

To identify clusters from a dendrogram, make a horizontal cut across the dendrogram. The number of vertical lines that it crosses will be the number of clusters.

The furthest this horizontal line can move up and down without hitting one of the horizontal lines the better. Also depending on the application, you might have some sense on how many clusters you want.

The term hierarchical refers to the observation that the clusters obtained by cutting the dendrogram at a particular height is nested within clusters obtained by cutting it at a greater height.

Algorithm (Hierarchical clustering)

- 1) Begin with n observations and a measure of pairwise dissimilarities (a total of $\frac{n(n-1)}{2}$ measures). Treat each of the observation as its own cluster.
- 2) For $i = n, n-1, \dots, 2$
 - a) Examine all pairwise inter cluster dissimilarities among the i clusters and identify pairs of clusters that are most similar. Fuse these clusters. The dissimilarity between clusters helps indicate height of dendrogram where fusion should be placed.
 - b) Compute pairwise inter cluster dissimilarity among $i-1$ remaining clusters.

Note that measuring distances (dissimilarities) between clusters has several choices. Say a Euclidean distance metric is used. Then one can ~~define~~ define the linkage (dissimilarity between two groups of observations) for example as Complete (Maximal distance between two points in clusters A and B), Average (Average of distance between any two observations, one in A and the other in B)

Ward's criterion is another such method that minimizes the total within cluster variance. Here the pair of clusters that are merged such that it leads to minimum increase in total within cluster variance after merging.

$$\begin{aligned} \text{Within cluster variance} &= \frac{1}{|C_k|} \sum_{i, i' \in C_k} \sum_{d=1}^p (x_{ij} - x_{i'd})^2 \\ &= 2 \sum_{i \in C_k} \sum_{d=1}^p (x_{ij} - \mu_{kd})^2 \end{aligned}$$

$$\text{where } \mu_{kd} = \frac{\sum_{i \in C_k} x_{ij}}{|C_k|}$$

Note:

$$\begin{aligned} \sum_i \|x_i - y\|^2 &= \sum_i \|x_i - \mu + \mu - y\|^2 \quad \left(\text{where } \mu = \frac{\sum_i x_i}{n}\right) \\ &= \sum_i \|x_i - \mu\|^2 + \sum_i \|\mu - y\|^2 + 2 \sum_i (x_i - \mu)'(\mu - y) \\ &= \sum_i \|x_i - \mu\|^2 + n \|\mu - y\|^2 \end{aligned}$$

Hence

$$\begin{aligned} \sum_{i, i'} \|x_i - x_{i'}\|^2 &= n \sum_i \|x_i - \mu\|^2 + n \sum_d \|\mu - x_d\|^2 \\ &= 2n \sum_i \|x_i - \mu\|^2 \end{aligned}$$

K-means clustering typically runs much faster than hierarchical clustering

(linear versus quadratic computational time in number of observations)

K-means will provide different results in different runs due to randomization while Hclust will give repeatable solutions

Analysis on Movie Lens dataset

```
movies ← read.csv("movies.csv", stringsAsFactors  
= FALSE)
```

The movies dataframe consists of two variables with 8569 movies with movie ID and title

To read in movie genres, we use the following commands to check we are reading it in properly.

```
countfields ← count.fields("genres.csv", sep = "1")
```

This helps to count the number of fields as separated by "1" in each row of genres.csv

`countfields[1]` This gives value of 5 which means first movie has 5 genres listed

`min(countfields)` Each movie has between
`max(countfields)` 1 and 7 genres

```
genres ← read.csv("genres.csv", header = FALSE,  
sep = "1", col.names = c("X1", "X2", "X3",  
"X4", "X5", "X6", "X7"))
```

This reads in a dataframe with 8569 observations & 7 variables with column names "X1" to "X7".

`header = FALSE` helps validate variable names are not provided in top row.

Note that each variable has different number of factor levels. To obtain the overall set, we use:

```
fac <- union(union(union(union(union(union(
  levels(g$x1), levels(g$x2)), levels(g$x3)), levels(g$x4),
  levels(g$x5)), levels(g$x6)), levels(g$x7))
```

fac has a total of 20 categories from Action to IMAX where " " simply illustrates missing category. (or not provided)

To standardize across all variables:

```
g$x1 <- factor(g$x1, fac)
```

⋮

```
g$x7 <- factor(g$x7, fac)
```

```
levels(g$x1)
```

"Action" "Adventure" "Animation" "Children"

"Comedy" "Crime" "Documentary" "Drama"

"Fantasy" "Film-Noir" "Horror" "Musical"

"Mystery" "Romance" "Sci-Fi" "Thriller"

"War" "Western" " " "IMAX"

$M \leftarrow \text{matrix}(0, \text{nrow} = 8569, \text{ncol} = 20)$
(Creates a matrix with 8569 rows (movies)
and 20 columns (categories))

$\text{colnames}(M) \leftarrow \text{fac}$

Assign columnnames to matrix as genres

$\text{for}(i \text{ in } 1:8569) \{$

$M[i, \text{genres}[i, "x1"]] \leftarrow 1$

\vdots

$M[i, \text{genres}[i, "x2"]] \leftarrow 1 \}$

Creates a matrix with entry = 1 where a
movie is of a particular genre & 0
otherwise.

$\text{Data} \leftarrow \text{as.data.frame}(M)$

$\text{Data} \$ \text{title} \leftarrow \text{movies} \$ \text{title}$

Creates a data frame & introduces the
movie title into the data frame.

$\text{Data} \leftarrow \text{Data}[, -19]$

Drops the 19th column which corresponds
to "" category.

Hierarchical clustering in R

```
distances <- dist (Data[, 1:19], method =  
"euclidean")
```

The function `dist(.)` computes distances between movies using the first 19 columns (genres of movies) with Euclidean distances.

A total of $\frac{8568 \times 8569}{2} = \frac{36709596}{2}$

distances are computed.

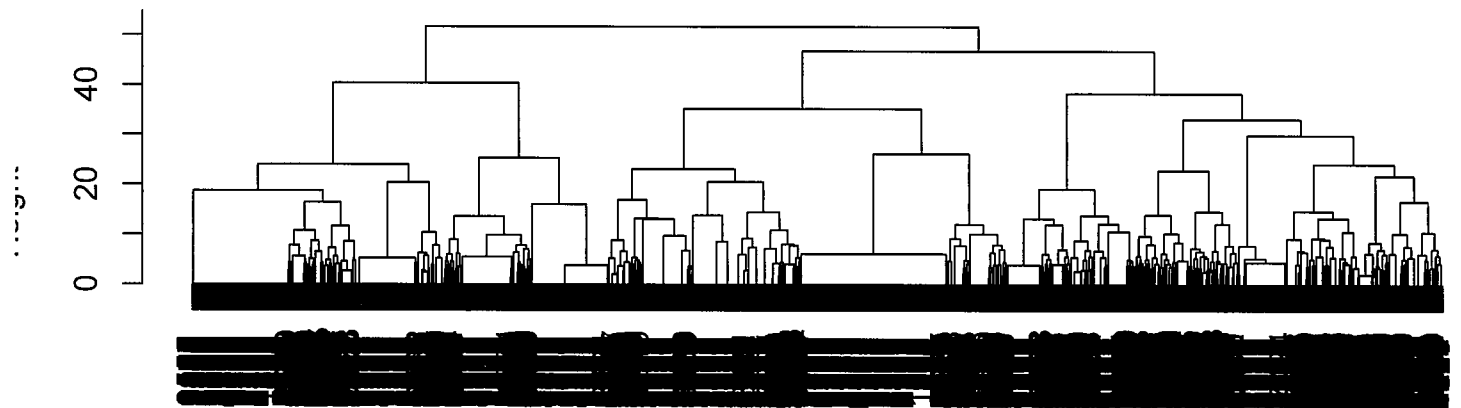
```
clusterMovies1 <- hclust (distances, method =  
"ward.D2")
```

Performs hierarchical cluster analysis using the distances. The Ward's method aims to find compact, spherical clusters.

```
plot (clusterMovies1)
```

Plots a dendrogram of clusters. This lists all points at the bottom making it hard to read in this example.

Cluster Dendrogram



distances
hclust (*, "ward.D2")

```
clusterGroups1 <- cutree (ClusterMovies1, k = 10)
```

Cuts a tree resulting from the `hclust(.)` into 10 groups.

```
tapply (Data[,1], clusterGroups1, mean)
```

Computes the average value across the cluster groups for the Action variable. Higher value indicates many movies in the cluster are action movies

```
Cat1 <- matrix (0, nrow = 19, ncol = 10)
```

```
for (i in 1:19) {
```

```
  Cat1[i,] <- tapply (Data[,i], clusterGroups1, mean) }
```

```
rownames (Cat1) <- colnames (Data)[1:19]
```

This helps create a matrix `Cat` where rows denote categories and columns indicate clusters

Cluster 1 : Fantasy, Comedy, Children, Adventure

Cluster 2 : Romance, Comedy, Drama

Cluster 3 : Comedy, Drama

Cluster 4 : Drama, Thriller, Crime

Cluster 5 : Sci-Fi, Adventure, Action

Cluster 6 : Horror, Thriller

Cluster 7 : Drama

Cluster 8 : Romance, Drama

Cluster 9 : Documentary

Cluster 10 : War, Drama, Action

Subset (Data & title, cluster Groups == 6)

disto out all movies in category 6
(horror, thriller)

Subset (Data, movies & title == "Grand Budapest
Hotel, The (2014)")

This is row number 8418 in the
data frame

cluster Groups [8418]

This is the comedy, drama cluster (3)

Subset (Data, movies & title == "Moneyball (2011)")

This is row number 7925 in the data
frame

cluster Groups [7925]

This is in the drama cluster (7)

Subset (Data, movies & title == "X-Men: First Class (2011)")

Row number 7849

cluster Groups [7849]

This is put in cluster (10).

k-means clustering in R

set.seed(1)

ClusterMovies2 <- kmeans (Data[,1:19],
Centers = 10, nstart = 20)

This performs a k-means clustering using
 $k=10$ and 20 random initial configurations

Here a random set of rows are chosen as
initial centers.

ClusterMovies2 \$ tot.within ss

Total within cluster sum of squares (we want
this number to be small)

set.seed(1)

ClusterMovies3 <- kmeans (Data[,1:19], centers = 10,
nstart = 1)

ClusterMovies3 \$ tot.within ss

Comparing ^{7601.8}~~7601.8~~ with ^{7324.7}~~7324.7~~ it is clear that
picking more starting points helps provides
better solutions

cluster(k),
fit ← 0

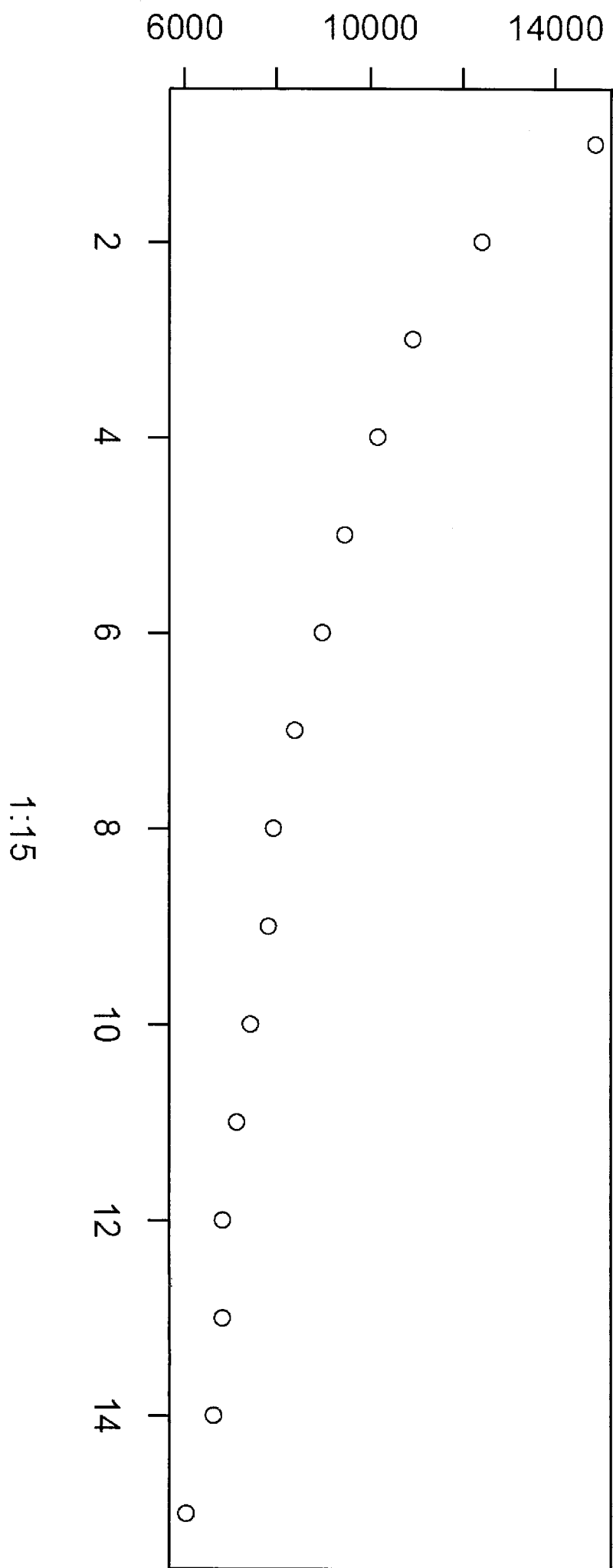
for (k in 1:15) {

clusterMovies4 ← kmeans (Data[, 1:19], Centers = k,
nstart = 20)

Fit[k] ← clusterMovies4\$tot.withinss

plot(1:15, fit)

This plots the error in the cluster fit as
the number of clusters increases



Cat 2 \leftarrow matrix (0, nrow = 19, ncol = 10)

for (i in 1:19) {

Cat2[i,] \leftarrow tapply(Date[,i], clusterMovies2 \$ cluster,
mean)

}

rownames(Cat2) \leftarrow colnames(Date)[1:19]

Cluster 1 : Horror

Cluster 2 : Crime, Drama, Thriller, Action

Cluster 3 : Drama

Cluster 4 : Documentary

Cluster 5 : Action, Adventure, Sci-Fi, Thriller

Cluster 6 : Comedy

Cluster 7 : Thriller, Horror

Cluster 8 : Children, Adventure, Animation, Fantasy, Comedy

Cluster 9 : War, Drama

Cluster 10 : Comedy, Drama

Recommendation Systems

There are two main types of recommendation systems:

1) Collaborative filtering : Recommendations are made based on attributes of users.

Each user is represented by a vector of items where the entry gives the customer's rating of the item. This vector will typically have many empty entries (only a small fraction is rated or purchased)

Last.fm creates a list of recommended songs for you by observing what the user listens to and likes and comparing against other users

2) Content filtering : Recommendations are made based on attributes of items.

Each item is represented by a set of attributes (such as genre of movie, keywords on webpage).

Pandora uses the attributes of a song such as style, artist to seed the station with other songs with similar attributes.

3) Hybrid recommendation Systems :

A combination of both collaborative and Content filtering might be useful.

Netflix is such an example. For example

Netflix makes recommendations by comparing the watching and searching habits of similar users (collaborative filtering) as well as recommending movies that share similar attributes (content filtering).

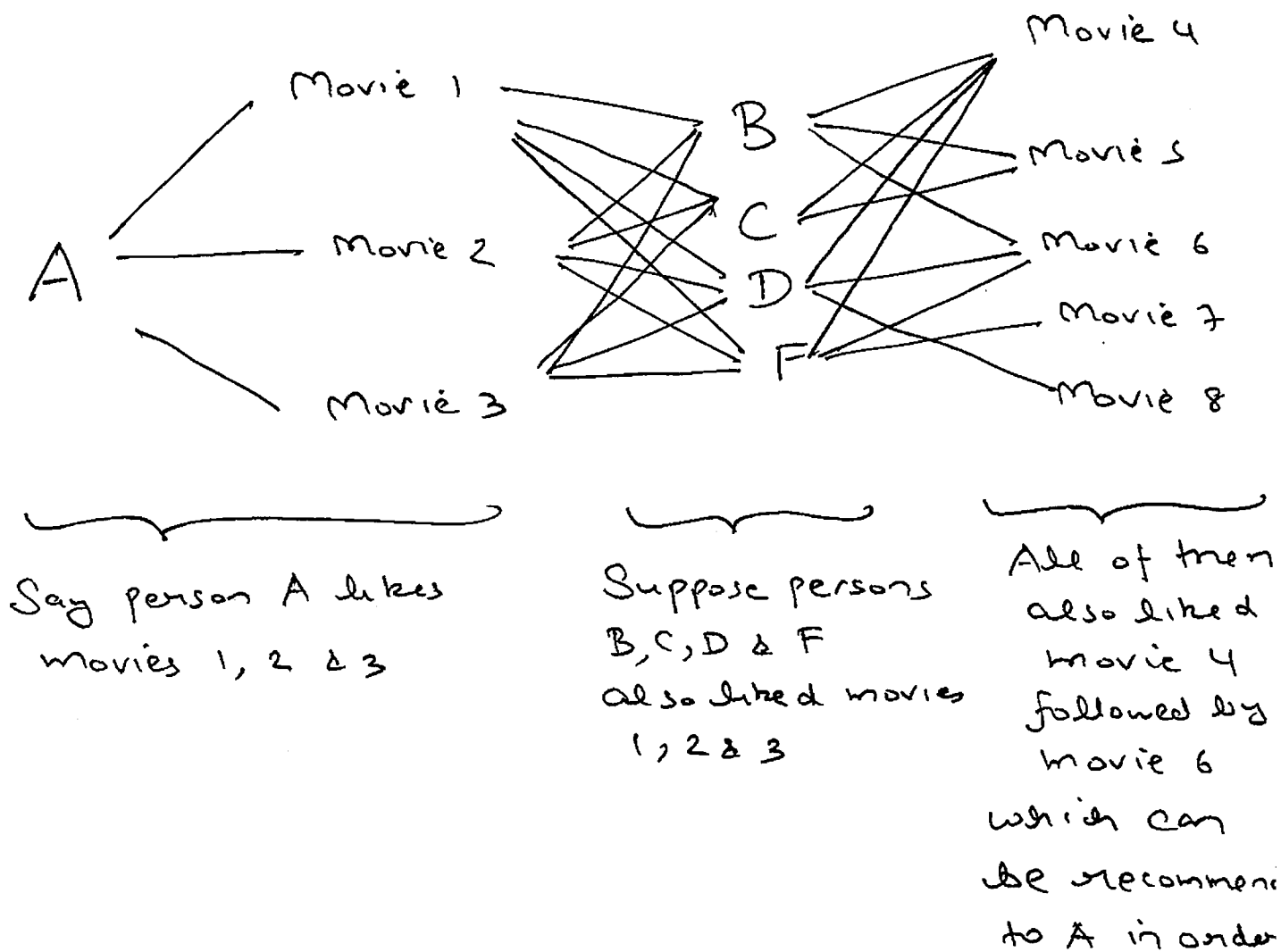
Content filtering often works better than Collaborative filtering when the user has not rated or purchased many items. As soon as one item is liked, content filtering can be used to make recommendations.

However if a user has rated many items, it is hard for content filtering to make recommendations since there might be many similar items to liked items. Furthermore in collaborative filtering it is possible to recommend items not previously considered in user's history. Thus it is possible to get them to purchase items they might not consider

Collaborative filtering

In this approach the data of ratings of users for items is used to predict missing ratings or create a top-N recommendation list for an active user.

Note that collaborative filtering will suffer from a cold start problem in comparison to content filtering since it will be unable to address new products and users. On the other hand it is domain free (does not depend on the item attributes).



Techniques for Collaborative filtering

Let $r_{u,i}$ = Rating of user u for item i

Baseline model

A simple baseline model is to predict the Average rating based on item popularity average.

$$\underbrace{\hat{b}_{u,i}}_{\text{Baseline prediction for user } u \text{ for item } i} = \bar{r}_i \quad \left(\begin{array}{l} \text{Average rating} \\ \text{for item } i \\ \text{across all users} \\ \text{who rated it} \end{array} \right)$$

Another approach is based on random recommendations or item popularity. User based collaborative filtering (no. of users who have rated it)

This is based on identifying other users whose ratings are similar to that of the active user and use their rating on other items to predict what the active (current) user will like.

To measure the similarity of user to user, the Pearson correlation can be used

$$\underbrace{S(u,v)}_{\text{Similarity between users } u \text{ and } v} = \frac{\sum_{i \in I_u \cap I_v} (r_{ui} - \bar{r}_u) (r_{vi} - \bar{r}_v)}{\sqrt{\sum_{i \in I_u \cap I_v} (r_{ui} - \bar{r}_u)^2 \sum_{i \in I_u \cap I_v} (r_{vi} - \bar{r}_v)^2}}$$

Here I_u, I_v are items rated by users u and v and \bar{r}_u, \bar{r}_v are average ratings of users u and v .

This similarity metric was used in Netflix.

Alternative similarity measures include the cosine similarity.

$$S(u, v) = \frac{\sum_{i \in I_u \cap I_v} r_{ui} r_{vi}}{\sqrt{\sum_{i \in I_u \cap I_v} r_{ui}^2} \sqrt{\sum_{i \in I_u \cap I_v} r_{vi}^2}}$$

To predict the rating the simplest method is to choose a set of neighbors of user u , denoted by N_u (say k nearest neighbors or at least a certain level of similarity),

$$\underbrace{p_{ui}}_{\text{Predicted rating for user } u \text{ for item } i} = \frac{\sum_{v \in N_u} r_{vi}}{|N_u|}$$

We can also use the observation that some users are more similar to u using the similarity metric.

$$p_{ui} = \frac{\sum_{v \in N_u} S(u, v) r_{vi}}{\sum_{v \in N_u} S(u, v)}$$

While all users could be used in the set N_u , it helps to restrict to a smaller number of neighbors (in the range of 20 to 250 typically)

Item based collaborative filtering

This is based on identifying items for users that are similar to the items they like. If two items have similar rating patterns across users the items are similar. Users will prefer items that are similar where the similarity metric is measured as say:

$$S(i, j) = \frac{\sum_{u \in U_i \cap U_j} r_{ui} r_{uj}}{\sqrt{\sum_{u \in U_i \cap U_j} r_{ui}^2} \sqrt{\sum_{u \in U_i \cap U_j} r_{uj}^2}}$$

To predict the rating, use $S(i, j)$ to identify a neighborhood N_i of items for item i :

$$P_{ui} = \frac{\sum_{j \in N_i} S_{ij} r_{uj}}{\sum_{j \in N_i} S_{ij}}$$

The performance of recommender algorithms can be improved by normalizing the ratings so as to compensate for user's differences on rating scales.

For example, there could be bias caused by users who consistently rate higher than (or lower than) other users.

Let $\hat{r}_{ui} = r_{ui} - \bar{r}_u$. This centers the scores. After applying collaborative filtering and normalizing back to original scale gives:

$$\underbrace{p_{ui}}_{\substack{\text{Predicted rating} \\ \text{for user } u \text{ for} \\ \text{item } i}} = \bar{r}_u + \frac{\sum_{v \in N_u} (s_{uv})(r_{vi} - \bar{r}_v)}{\sum_{v \in N_u} (s_{uv})}$$

Other transformations include taking the rating variance into account.

To measure the quality of predicted ratings

$$\text{RMSE (root mean squared error)} = \sqrt{\frac{\sum_{i,j} (p_{ui} - r_{ui})^2}{n}}$$

where p_{ui} = predicted rating for user u for item i , r_{ui} is real rating and n is the total number of ratings that are predicted.

Analytics on MovieLens data set

```
ratings <- read.csv("ratings.csv")
```

```
str(ratings)
```

The ratings data frame consists of 100023

Observations of 3 variables where

UserId (Identity of user), movieId (Identity of movie), rating (0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5)
706 users, 8532 movies

```
max(table(ratings$userId))
```

```
min(table(ratings$userId))
```

Users have rated from 20 movies

up to 2268 movies (User 16, User 516
17, ..., 684)
which (table(ratings\$userId) == 20)

```
max(table(ratings$movieId))
```

```
min(table(ratings$movieId))
```

Movies have from 1 up to 337 ratings

Data ← matrix (nrow = length(unique(ratings\$userId))
ncol = length(unique(ratings\$movieId)))

This creates an empty matrix with 706 rows and 8552 columns

rownames(Data) ← unique(ratings\$userId)

colnames(Data) ← unique(ratings\$movieId)

This names the rows and columns with the unique users and movies in the data set

for (i in 1:nrow(ratings)) {

Data[as.character(ratings\$userId[i]),
as.character(ratings\$movieId[i])] ← ratings\$rating[i]

}

This fills in the entries of the Data matrix with movie ratings

Data.norm ← Data - rowMeans(Data, na.rm=TRUE)

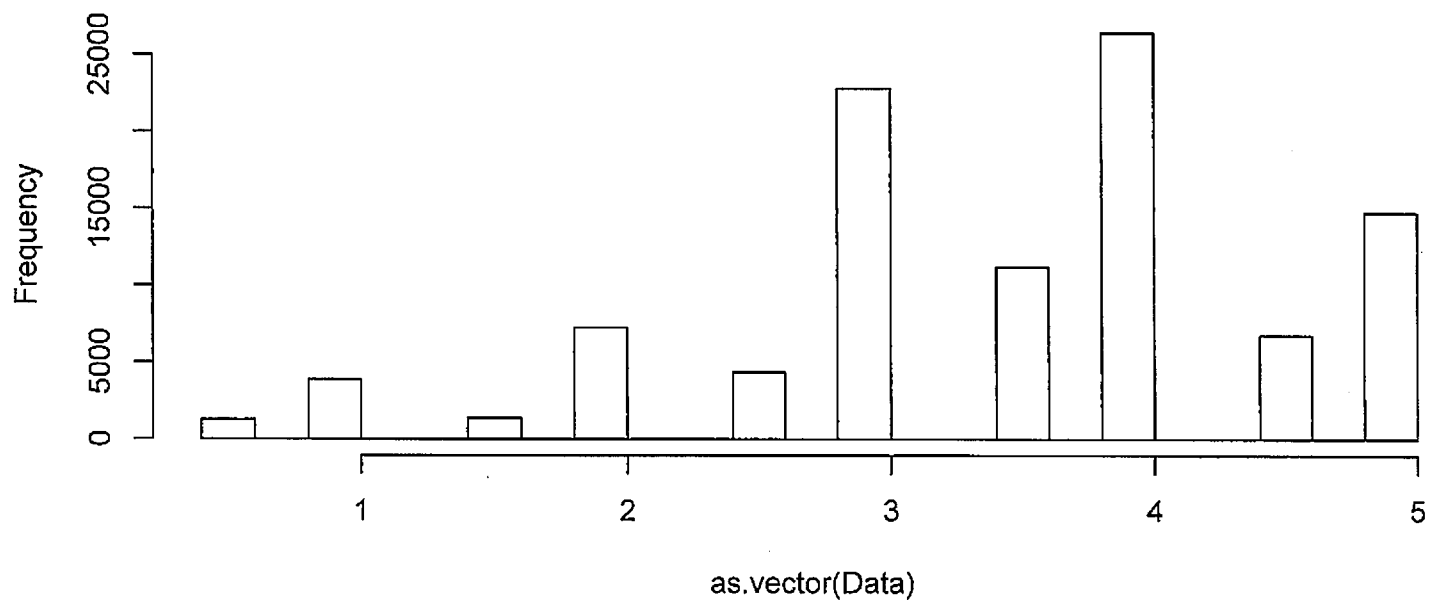
Normalize the ratings of each user so that bias is reduced.

hist(as.vector(Data))

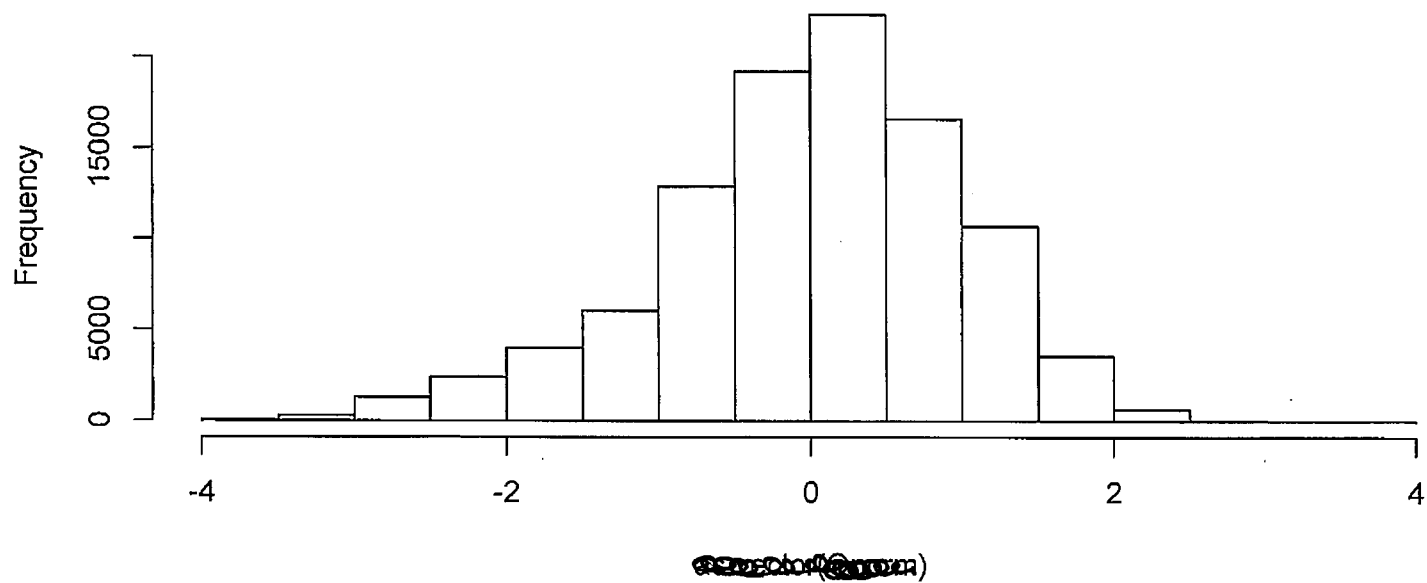
hist(as.vector(Data.norm))

Histograms of raw & normalized ratings indicate greater skew in raw ratings & more symmetric in normalized ratings

Histogram of as.vector(Data)



Data norm
 Histogram of as.vector(C.norm)



Set.seed(1)

Spl1 \leftarrow Sample(1:nrow(Data), 0.98 * nrow(Data))

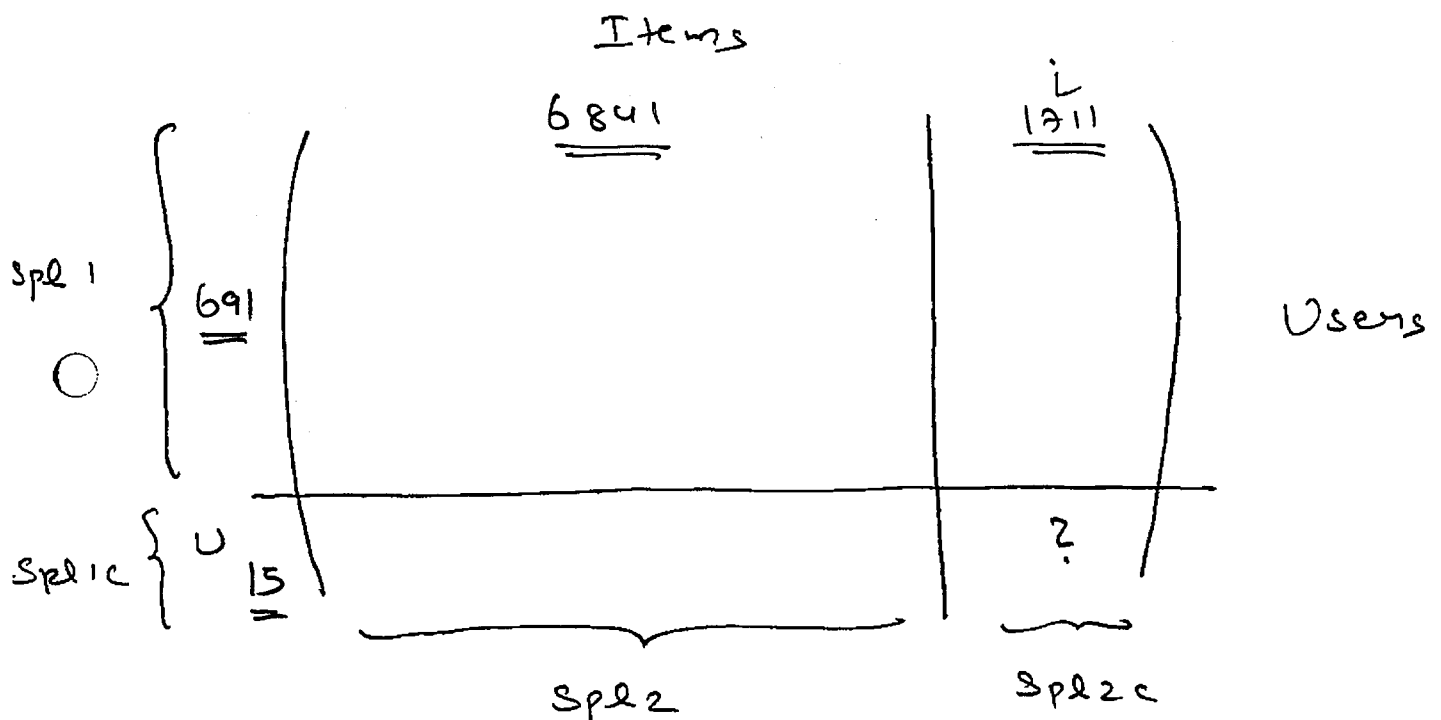
Set.seed(2)

Spl2 \leftarrow Sample(1:ncol(Data), 0.8 * ncol(Data))

Spl1c \leftarrow setdiff(1:nrow(Data), Spl1)

Spl2c \leftarrow setdiff(1:ncol(Data), Spl2)

- This splits the data into the following format so as to apply recommendation techniques by randomly selecting rows & columns



User Pred \leftarrow matrix (nrow = length(spl1c), ncol = length(spl2c))

Base 1 \leftarrow matrix (nrow = length(spl1c),
ncol = length(spl2c))

Base 2 \leftarrow matrix (nrow = length(spl1c),
ncol = length(spl2c))

Create three different prediction approaches
and corresponding predicted rating matrices)

for (i in 1:length(spl1c)) {

Base1[i,] \leftarrow colMeans (Data[spl1, spl2c],
na.rm = TRUE) }

This computes predicted ratings by averaging
out the item rating for all the users in
the training set (spl1)

○

for (j in 1:length(spl2c)) {

Base2[,j] \leftarrow rowMeans (Data[spl1c, spl2],) }
na.rm = TRUE

This computes predicted ratings by averaging
over the user rating for all the items in
the training set (spl1)

`Cor <- matrix (nrow = length(spl1), ncol = 1)`

`Order <- matrix (nrow = length(spl1c), ncol = length(spl1))`

The Cor object will keep track of correlation between users while Order will sort users in terms of decreasing correlations

```
for (i in 1:length(spl1c)) {
```

```
  for (j in 1:length(spl1)) {
```

```
    Cor[j] <- cor(Data[spl1c[i], spl2],  
                  Data[spl1[j], spl2],
```

```
                  use = "pairwise.complete.obs")
```

```
    V <- order(Cor, decreasing = TRUE, na.last = NA)
```

```
    Order[i,] <- c(V, rep(NA, times = length(spl1) - length(V)))  
  }
```

This creates an Order matrix of dimension 15×691 where each row corresponds to neighbors of the users in the spl1c in decreasing order of Pearson correlation

The NA's indicate that there are some users who have no common ratings of movies with the user.

for (i in 1:length(spl1c)) {

UserPred[i,] ← ColMeans(Data[spl1[Order[i,1:250]
spl2c], na.rm=TRUE))

This computes user predictions by looking at
the 250 nearest neighbors & averaging
equally over all these user ratings in the
items in spl2c

$$\text{RMSE UserPred} \leftarrow \text{Sqrt} \left(\text{mean} \left(\left(\text{Data}[\text{spl1c}, \text{spl2c}] - \text{UserPred} \right)^2, \text{na.rm} = \text{TRUE} \right) \right)$$

$$\text{RMSE} = 0.898$$

$$\text{RMSE Base 1} \leftarrow \text{Sqrt} \left(\text{mean} \left(\left(\text{Data}[\text{spl1c}, \text{spl2c}] - \text{Base 1} \right)^2, \text{na.rm} = \text{TRUE} \right) \right)$$

$$\text{RMSE Base 2} \leftarrow \text{Sqrt} \left(\text{mean} \left(\left(\text{Data}[\text{spl1c}, \text{spl2c}] - \text{Base 2} \right)^2, \text{na.rm} = \text{TRUE} \right) \right)$$

$$\text{RMSE} = 0.931 \text{ \& } 0.995 \text{ respectively}$$

We can also vary the neighborhood set to
see the effect & also try weighting.

```
RMSE <- rep(NA, times = 490)
```

```
for (k in 10:499) {
```

```
  for (i in 1:length(spl1c)) {
```

```
    UserPred[i,] <- colMeans( Data[spl1[order[i,1:k],  
                                   spl2c], na.rm=TRUE)
```

```
  }
```

```
RMSE(k-10) <- sqrt( mean( (Data[spl1c, spl2c] - UserPred)  
                        na.rm=TRUE )^2 )
```

```
plot(10:499, rm)
```

```
abline(h = 0.931)
```

....

