CS 320: Concepts of Programming Languages

Lecture 5: Lists, Lists, and Lists

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Today's Abstraction

- The most important data structure that OCaml provides is a *LIST*
- Today, we will learn all there's to know about lists
- As usual, we will study syntax, type system, and semantics of lists
- (you must be getting tired of this by now)
- (*but I will keep repeating this till my last lecture*)
- Note: for assignments, please create all files with dummy functions before running "`dune test`"

List Type

- Very much like the linked list data structure and not vector
- *Formal Syntax:*
  - `[]` for an empty list; also called "nil"
  - `x::l` for "cons"-ing element `x` to the front of list `l`
  - `[`x_1`; `x_2`; `x_3`; …; `x_n`]` can also be used to define a fixed list

Some List Examples

'a list

```
let l1 = []
```

int list

```
let l2 = 1::l1
```

int list

```
let l3 = 2::3::l2
```

int list

```
let l4 = [1; 2; 3]
```

```
(* Are l3 and l4 equal? *)
```

Some List Examples

'a list

```
let l1 = []
```

int list

```
let l2 = 1::l1
```

int list

```
let l3 = 2::3::l2
```

int list

```
let l4 = [1; 2; 3]
```

```
(* Are l3 and l4 equal? *)
```

:: is right associative

So, this would be equivalent to

```
2::(3::l2)
```

Example: Generating a List

- Generate a list of first n natural numbers

```
int -> int list
let rec generate n =
  if n = 0
    then []
    else n::(generate (n-1))

(* generate 5 = [5; 4; 3; 2; 1] *)
```

```
int -> int list
let generate n =
  let rec gen_helper n k =
    if n = 0
      then []
    else k::(gen_helper (n-1) (k+1))
  in
  gen_helper n 1

(* generate 5 = [1; 2; 3; 4; 5] *)
```

Typing Rule for Lists

$$\frac{}{\Gamma \vdash [\,] : \alpha \text{ list}}$$

- `[]` can have any type

$$\frac{\Gamma \vdash x : \tau \qquad \Gamma \vdash \ell : \tau \text{ list}}{\Gamma \vdash (x :: \ell) : \tau \text{ list}}$$

- If `x` has type τ, then ℓ must have type τ `list` and `x::`ℓ has type τ `list`

$$\frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau \qquad \ldots \qquad \Gamma \vdash e_n : \tau}{\Gamma \vdash [e_1; e_2; \ldots; e_n] : \tau \text{ list}}$$

- All elements of the list must have type τ, then `[e1; e2; ...; en]` has type τ `list`

Semantics Rule for Lists

$$\frac{}{[\,] \Downarrow [\,]}$$

- `[]` is a value

$$\frac{e \Downarrow v \qquad \ell \Downarrow [v_1;\ v_2;\ \ldots v_n]}{e :: \ell \Downarrow [v;\ v_1;\ v_2;\ \ldots v_n]}$$

- If `e` evaluates to `v`, and ℓ evaluates to `[v1; v2; …; vn]`, then `x::`ℓ evaluates to `[v; v1; v2; …; vn]`

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

- If each element `ei` evaluates to `vi`, then the list evaluates to `[v1; v2; …; vn]`

Example of Semantics

$$[2 + 5; 3 * 3; 4 - 10]$$

Example of Semantics

$$[2 + 5; 3 * 3; 4 - 10]$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\frac{}{[2+5; 3*3; 4-10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \dots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \dots e_n] \Downarrow [v_1;\ v_2;\ \dots v_n]}$$

Example of Semantics

$$\frac{2+5 \Downarrow 7}{[2+5; 3*3; 4-10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\frac{2 + 5 \Downarrow 7 \qquad\qquad 3 * 3 \Downarrow 9 \qquad\qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\frac{2 + 5 \Downarrow 7 \qquad 3 * 3 \Downarrow 9 \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\frac{2 + 5 \Downarrow 7 \qquad 3 * 3 \Downarrow 9 \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\dfrac{\dfrac{2 \Downarrow 2 \qquad 5 \Downarrow 5 \qquad \color{red}{7 = 2 + 5}}{2 + 5 \Downarrow 7} \qquad 3 * 3 \Downarrow 9 \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\dfrac{\dfrac{2 \Downarrow 2 \qquad 5 \Downarrow 5 \qquad \color{red}{7 = 2 + 5}}{2 + 5 \Downarrow 7} \qquad 3 * 3 \Downarrow 9 \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 * v_2}{e_1 * e_2 \Downarrow v} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]}$$

Example of Semantics

$$\dfrac{\dfrac{2 \Downarrow 2 \qquad 5 \Downarrow 5 \qquad 7 = 2 + 5}{2 + 5 \Downarrow 7} \qquad \dfrac{3 \Downarrow 3 \qquad 3 \Downarrow 3 \qquad 9 = 3 * 3}{3 * 3 \Downarrow 9} \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 * v_2}{e_1 * e_2 \Downarrow v} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\, e_2;\, \ldots e_n] \Downarrow [v_1;\, v_2;\, \ldots v_n]}$$

Example of Semantics

$$\dfrac{\dfrac{2 \Downarrow 2 \qquad 5 \Downarrow 5 \qquad \color{red}{7 = 2 + 5}}{2 + 5 \Downarrow 7} \qquad \dfrac{3 \Downarrow 3 \qquad 3 \Downarrow 3 \qquad \color{red}{9 = 3 * 3}}{3 * 3 \Downarrow 9} \qquad 4 - 10 \Downarrow -6}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 * v_2}{e_1 * e_2 \Downarrow v} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 - v_2}{e_1 - e_2 \Downarrow v}$$

Example of Semantics

$$\dfrac{\dfrac{2 \Downarrow 2 \qquad 5 \Downarrow 5 \qquad 7 = 2 + 5}{2 + 5 \Downarrow 7} \qquad \dfrac{3 \Downarrow 3 \qquad 3 \Downarrow 3 \qquad 9 = 3 * 3}{3 * 3 \Downarrow 9} \qquad \dfrac{4 \Downarrow 4 \qquad 10 \Downarrow 10 \qquad -6 = 4 - 10}{4 - 10 \Downarrow -6}}{[2 + 5; 3 * 3; 4 - 10] \Downarrow [7; 9; -6]}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 * v_2}{e_1 * e_2 \Downarrow v} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \Downarrow v}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad \ldots \qquad e_n \Downarrow v_n}{[e_1;\ e_2;\ \ldots e_n] \Downarrow [v_1;\ v_2;\ \ldots v_n]} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad v = v_1 - v_2}{e_1 - e_2 \Downarrow v}$$

How do we use Lists?

- So far, we have seen how to construct lists
- But, how do we use lists? (there's no point building them if we can't use them)
- To use lists, we go back to our new best friend: pattern matching
- Remember, lists have two possibilities: '`nil`' (or `[]`) and '`cons`' (or `h::t`) where `h` is the head of the list and `t` is the tail of the list
-
```
match <expr> with
| [] -> <expr>
| h::t -> <expr>
| ......
```

Example: Length of a List


```
'a list -> int
let rec length lst =
  match lst with
  | [] -> 0
  | h::t -> 1 + (length t)
```

- If the list is empty, i.e., `[]`, then return `0`
- Else, add `1` to the length of `t`
- Note the type: what is `'a list`?

Polymorphic Type of List

- What should be the type of `lst`?
- Should it be `int list`? Or `bool list`? Or something else?
- Technically, it can be `α list` for any type `α`
- In OCaml, this is written as `'a list`
- These functions are called polymorphic functions (similar to generics in Java, etc.)
- These functions are defined for all types and can be used at any type.

Polymorphic Functions

- This length function can be applied to any argument
- ```
  length [1; 2; 3] = 3
  length [4.; 1.; 2.; 5.] = 4
  length [“cs320”, “cs599”] = 2
  ```
- In fact, they only need to be defined once and can be called with different types without defining them individually for each type
- This makes them powerful! They are crucial for data structures like stacks, queues, trees that can store arbitrary data
  ```

Example: Sum of a List


```
int list -> int
let rec sum lst =
  match lst with
  | [] -> 0
  | h::t -> h + sum t
```

- If the list is empty, i.e., `[]`, then return `0`
- Else, add `h` to the sum of `t`
- *Challenge: Write a function that can sum up an arbitrary list type*

Lists are Immutable

- Like every type so far, lists are also immutable
- We create new lists from the older list
- For example:

```
int list -> int list
let rec inc lst =
  match lst with
  | [] -> []
  | h::t -> (h+1)::(inc t)
```

- Calling `inc` does not mutate the argument

Formal Typing Rule for Pattern Match

$$\frac{\Gamma \vdash e : \tau \ \mathsf{list} \qquad \Gamma \vdash e_1 : \tau' \qquad \Gamma, h : \tau, t : \tau \ \mathsf{list} \vdash e_2 : \tau'}{\Gamma \vdash \mathsf{match}\ e\ \mathsf{with}\ [\,] \to e_1 \mid h :: t \to e_2 : \tau'}$$

- `e` must have type τ `list`
- Each branch should have the same type: e_1 has type τ'
- For the second branch, add `h :` τ and `t :` τ `list` to the context
- In this context, e_2 must have type τ'

Formal Semantics Rule for Pattern Match

$$\frac{e \Downarrow [\,] \qquad e_1 \Downarrow v_1}{(\text{match } e \text{ with } [\,] \to e_1 \mid h :: t \to e_2) \Downarrow v_1}$$

$$\frac{e \Downarrow (v :: vs) \qquad [v/h]\,[vs/t]e_2 \Downarrow v_2}{(\text{match } e \text{ with } [\,] \to e_1 \mid h :: t \to e_2) \Downarrow v_2}$$

- **Since `e : `τ` list`, it can either evaluate to an empty list or a non-empty list (*a consequence of preservation*)**
- **First rule handles empty case, Second rule handles non-empty case**

Patterns can be Deep


```
match <expr> with
  | [] -> <expr>
  | [h1; h2] -> <expr>
  | h1::h2::t -> <expr>
  | h::t -> <expr>
  | ……
```

Patterns can be Deep

- `match <expr> with`

```
| [] -> <expr>
| [h1; h2] -> <expr>
| h1::h2::t -> <expr>
| h::t -> <expr>
| ......
```

[] for empty list

Patterns can be Deep

- `match <expr> with`

```
| [] -> <expr>

| [h1; h2] -> <expr>

| h1::h2::t -> <expr>

| h::t -> <expr>

| ......
```

`[]` for empty list

`[h1; h2]` for a list of size exactly 2

Patterns can be Deep

- `match <expr> with`

```
  | [] -> <expr>
  | [h1; h2] -> <expr>
  | h1::h2::t -> <expr>
  | h::t -> <expr>
  | ......
```

`[]` for empty list

`[h1; h2]` for a list of size exactly 2

`h1::h2::t` for a list of size at least 2

Patterns can be Deep

```
match <expr> with
  | [] -> <expr>
  | [h1; h2] -> <expr>
  | h1::h2::t -> <expr>
  | h::t -> <expr>
  | ……
```

`[]` for empty list

`[h1; h2]` for a list of size exactly 2

`h1::h2::t` for a list of size at least 2

`h::t` for a list of size at least 1

There's Nothing Special about Lists

- Recall that programmers can define their own types in OCaml
- List is just one such type; can be defined as follows:

```
type intlist =
  | Nil
  | Cons of int * intlist
```

- This is a data-carrying variant; a value of list type can either be `Nil` or `Cons` of an integer (head) and another list (tail)
- The built-in list type just uses `[]` for `Nil` and `::` for `Cons`

Creating Lists


```
type intlist =
  | Nil
  | Cons of int * intlist
```

- Empty list can be created using:

```
intlist
let x = Nil
```

- Bigger lists can be created using:

```
intlist
let x = Cons (1, (Cons (2, (Cons (3, Nil)))))
```

You Can Still Pattern Match


- ```
  match <expr> with
  | Nil -> <expr>
  | Cons(h, t) -> <expr>
  | ……
  ```

- **What should be the typing rule?**

$$\frac{\Gamma \vdash e : \text{int list} \qquad \Gamma \vdash e_1 : \tau' \qquad \Gamma, h : \text{int}, t : \text{int list} \vdash e_2 : \tau'}{\Gamma \vdash \text{match } e \text{ with Nil} \rightarrow e_1 \mid \text{Cons}(h, t) \rightarrow e_2 : \tau'}$$

- *Homework: Write down the semantics rules for this expression*
  ```

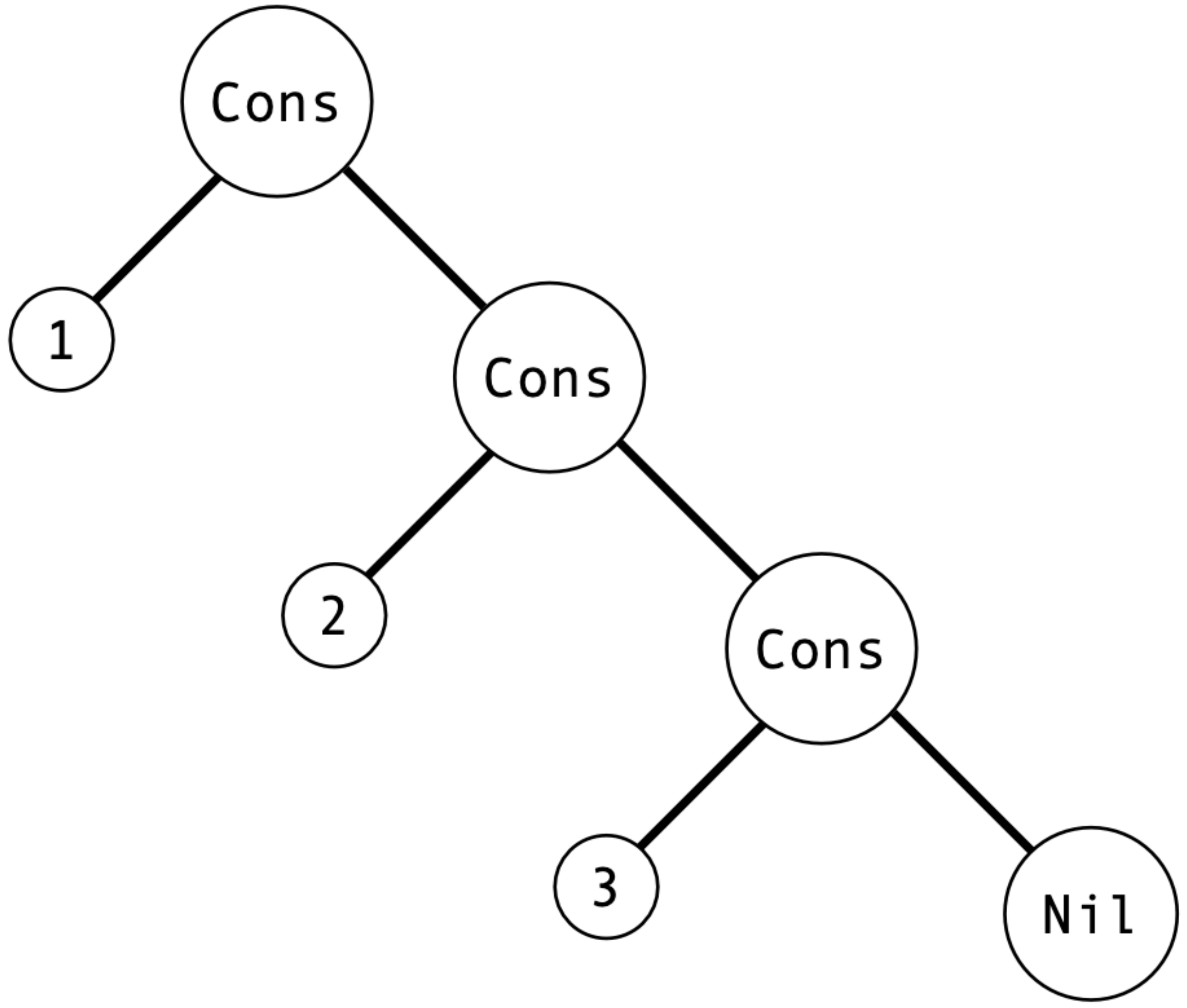
Patterns Can Still be Deeeeep


```
match <expr> with
  | Nil -> <expr>
  | Cons(h, Nil) -> <expr>
  | Cons(h1, Cons(h2, Nil)) -> <expr>
  | Cons(h1, Cons(h2, t)) -> <expr>
  | Cons(h, t) -> <expr>
  | ……
```

- *Homework: Write down the typing and semantics rules for this expression*

Pictorial Representation

intlist
```
let x = Cons (1, (Cons (2, (Cons (3, Nil)))))
```



We can also use Recursive Records


```
type intlist = { head : int; tail : intlist}
```

- But I don't like this representation. Tell me why?
- How do you represent an empty list?
- How about this instead? Types can also be mutually recursive

```
type intlist = { head : int; tail : intlist_option }

and intlist_option =
  | None
  | Some of intlist
```

How to Represent All Lists?

- The answer is *polymorphism*!
- Like functions, types can also be polymorphic
- Main idea: element inside a list can be of "*any*" type; "*any*" types are represented using `'a`, `'b`, `'c`, ……
- Let's do some examples!

```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

int list

```
let x = Cons (1, Cons (2, Nil))
```

Creating Specific Typed Lists


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

int list

```
let x = Cons (1, Cons (2, Nil))
```

string list

```
let y = Cons ("One", Cons ("Two", Nil))
```

float list

```
let z = Cons (1., Cons (2., Nil))
```

Defining Polymorphic Functions


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

'a list -> int

```
let rec length lst =
  match lst with
  | Nil -> 0
  | Cons(_, t) -> 1 + length t
```

Defining Polymorphic Functions


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

'a list -> int

```
let rec length lst =
  match lst with
  | Nil -> 0
  | Cons(_, t) -> 1 + length t
```

'a list -> 'a list

```
let rec swap2 lst =
  match lst with
  | Nil -> Nil
  | Cons(h1, Cons(h2, t)) -> Cons (h2, Cons (h1, swap2 t))
  | Cons(h, Nil) -> Cons(h, Nil)
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
let rec sum lst          =
  match lst with
  | Nil ->
  | Cons(h, t) ->
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
let rec sum lst           =
  match lst with
  | Nil -> zero
  | Cons(h, t) ->
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
let rec sum lst zero      =
  match lst with
  | Nil -> zero
  | Cons(h, t) ->
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
let rec sum lst zero add =
  match lst with
  | Nil -> zero
  | Cons(h, t) ->
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
let rec sum lst zero add =
  match lst with
  | Nil -> zero
  | Cons(h, t) -> add h (sum t zero add)
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

```
'a list -> 'b -> ('a -> 'b -> 'b) -> 'b
let rec sum lst zero add =
  match lst with
  | Nil -> zero
  | Cons(h, t) -> add h (sum t zero add)
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

'a list -> 'b -> ('a -> 'b -> 'b) -> 'b

```
let rec sum lst zero add =
  match lst with
  | Nil -> zero
  | Cons(h, t) -> add h (sum t zero add)
```

int -> int -> int

```
let add_int a b = a + b
```

int list -> int

```
let sum_int lst = sum lst 0 add_int
```

Can Size be Polymorphic?


```
type 'a list =
  | Nil
  | Cons of 'a * 'a list
```

'a list -> 'b -> ('a -> 'b -> 'b) -> 'b

```
let rec sum lst zero add =
  match lst with
  | Nil -> zero
  | Cons(h, t) -> add h (sum t zero add)
```

int -> int -> int

```
let add_int a b = a + b
```

int list -> int

```
let sum_int lst = sum lst 0 add_int
```

float -> float -> float

```
let add_float a b = a +. b
```

float list -> float

```
let sum_float lst = sum lst 0. add_float
```

More Polymorphic Types


```
type 'a option =
  | None
  | Some of 'a
```

```
'a list -> 'a option
let head lst =
  match lst with
  | Nil -> None
  | Cons(h, _) -> Some h
```

- This may or may not hold a value; useful when not all inputs have a valid output; like the head function cannot return a value when list is empty
- Some people use *null pointers* for this. Please don't EVER use null pointers
- Null pointers were called the "*billion-dollar mistake*" by Tony Hoare, the person who invented them

Generalization of Option Type


```
type ('a, 'e) result =
  | Ok of 'a
  | Err of 'e
```

```
'a list -> ('a, string) result
let head lst =
  match lst with
  | Nil -> Err "Empty list has no head"
  | Cons(h, _) -> Ok h
```

- Polymorphic types can have multiple parameters
- Allows users to return a result or an error message

Tail Recursion

- Some recursive functions can be made *tail recursive*
- What is tail recursive?
- A recursive function is tail recursive when you do not perform any computation on the result of the recursive call
- ```
  let rec factorial n =
  if n = 0 then 1 else n * factorial (n-1)
  ```
- *Not Tail Recursive:* The result of the recursive call to factorial is multiplied with "n"
- Can it be made tail recursive? Yes!
  ```

Tail Recursion


```
int -> int
let factorial n =
  let rec fact_helper n acc =
    if n = 0 then acc else fact_helper (n-1) (acc*n)
  in
  fact_helper n 1
```

- Make the result another argument
- `acc` (short for accumulator) stores the result of factorial; when `n` reaches `0`, we simply return `acc`
- In the else branch, the recursive call is directly returned

Tail Recursion with Lists


```
'a list -> 'a list
let rev lst =
  let rec rev_helper lst acc =
    match lst with
    | Nil -> acc
    | Cons(h, t) -> rev_helper t (Cons(h, acc))
  in
  rev_helper lst Nil
```

- **Can every recursive function be made tail recursive? Theoretically speaking, yes! There is an entire PL theory called "*Continuation Passing Style*" based on this. Much harder in practice though**
- ***Homework: Use reverse to implement list append using tail recursion***

Homework

- Read about records
- Practice as many typing derivations and semantics derivations as you can! Please!
- Write out the typing rules for other destructors for tuples
- Read OCP 3.1, 3.7