

# PROGRAMMING IN HASKELL



## Chapter 6 - Recursive Functions

# Introduction

As we have seen, many functions can naturally be defined in terms of other functions.

```
fac :: Int → Int  
fac n = product [1..n]
```

fac maps any integer  $n$  to the product of the integers between 1 and  $n$ .

Expressions are evaluated by a stepwise process of applying functions to their arguments.

For example:

```
    fac 4  
=  
    product [1..4]  
=  
    product [1,2,3,4]  
=  
    1*2*3*4  
=  
    24
```

# Recursive Functions

In Haskell, functions can also be defined in terms of themselves. Such functions are called recursive.

```
fac 0 = 1  
fac n = n * fac (n-1)
```

fac maps 0 to 1, and any other integer to the product of itself and the factorial of its predecessor.

For example:

```
= fac 3
= 3 * fac 2
= 3 * (2 * fac 1)
= 3 * (2 * (1 * fac 0))
= 3 * (2 * (1 * 1))
= 3 * (2 * 1)
= 3 * 2
= 6
```

## Note:

- $\text{fac } 0 = 1$  is appropriate because 1 is the identity for multiplication:  $1 * x = x = x * 1$ .
- The recursive definition diverges on integers  $< 0$  because the base case is never reached:

```
> fac (-1)
```

```
*** Exception: stack overflow
```

# Why is Recursion Useful?

- Some functions, such as factorial, are simpler to define in terms of other functions.
- As we shall see, however, many functions can naturally be defined in terms of themselves.
- Properties of functions defined using recursion can be proved using the simple but powerful mathematical technique of induction.

# Recursion on Lists

Recursion is not restricted to numbers, but can also be used to define functions on lists.

```
product :: Num a => [a] -> a
product []      = 1
product (n:ns) = n * product ns
```

product maps the empty list to 1,  
and any non-empty list to its head  
multiplied by the product of its tail.



For example:

```
product [2,3,4]
=
2 * product [3,4]
=
2 * (3 * product [4])
=
2 * (3 * (4 * product []))
=
2 * (3 * (4 * 1))
=
24
```

Using the same pattern of recursion as in product we can define the length function on lists.

```
length :: [a] → Int
length []      = 0
length (_:xs) = 1 + length xs
```

length maps the empty list to 0,  
and any non-empty list to the  
successor of the length of its tail.

For example:

```
length [1,2,3]
=
1 + length [2,3]
=
1 + (1 + length [3])
=
1 + (1 + (1 + length []))
=
1 + (1 + (1 + 0))
=
3
```

Using a similar pattern of recursion we can define the reverse function on lists.

```
reverse :: [a] → [a]
reverse []      = []
reverse (x:xs) = reverse xs ++ [x]
```

reverse maps the empty list to the empty list, and any non-empty list to the reverse of its tail appended to its head.

For example:

```
reverse [1,2,3]
=
reverse [2,3] ++ [1]
=
(reverse [3] ++ [2]) ++ [1]
=
((reverse [] ++ [3]) ++ [2]) ++ [1]
=
(([] ++ [3]) ++ [2]) ++ [1]
=
[3,2,1]
```

# Multiple Arguments

Functions with more than one argument can also be defined using recursion. For example:

- Zipping the elements of two lists:

```
zip :: [a] → [b] → [(a,b)]  
zip []      _      = []  
zip _      []      = []  
zip (x:xs) (y:ys) = (x,y) : zip xs ys
```

## ■ Remove the first n elements from a list:

```
drop :: Int → [a] → [a]
drop 0 xs      = xs
drop _ []      = []
drop n (_:xs) = drop (n-1) xs
```

## ■ Appending two lists:

```
(++) :: [a] → [a] → [a]
[]      ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

# Quicksort

The quicksort algorithm for sorting a list of values can be specified by the following two rules:

- The empty list is already sorted;
- Non-empty lists can be sorted by sorting the tail values  $\leq$  the head, sorting the tail values  $>$  the head, and then appending the resulting lists on either side of the head value.



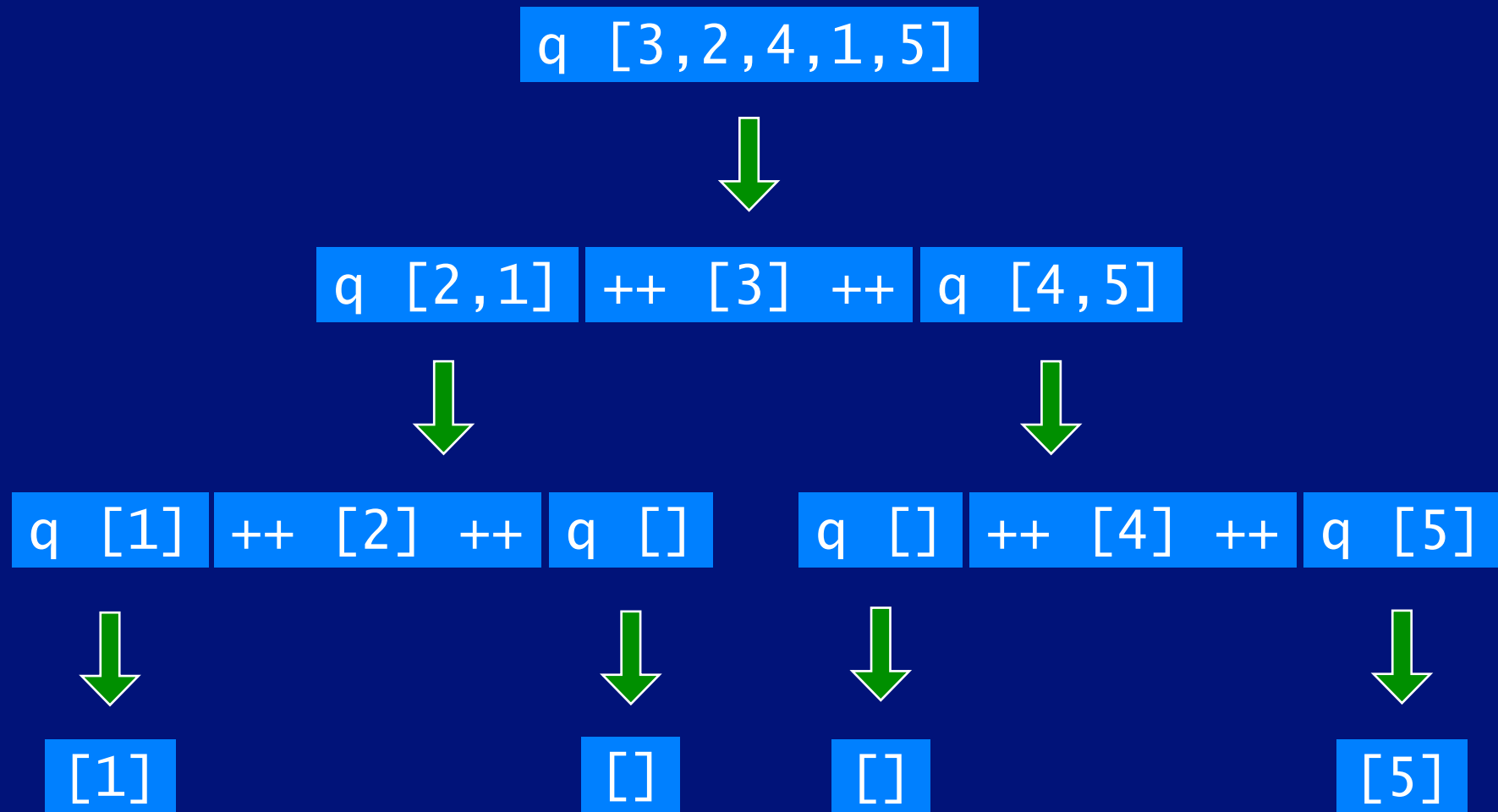
Using recursion, this specification can be translated directly into an implementation:

```
qsort :: Ord a => [a] -> [a]
qsort []      = []
qsort (x:xs) =
    qsort smaller ++ [x] ++ qsort larger
  where
    smaller = [a | a <- xs, a <= x]
    larger  = [b | b <- xs, b > x]
```

Note:

- This is probably the simplest implementation of quicksort in any programming language!

For example (abbreviating qsort as q):



# Exercises

(1) Without looking at the standard prelude, define the following library functions using recursion:

- Decide if all logical values in a list are true:

```
and :: [Bool] → Bool
```

- Concatenate a list of lists:

```
concat :: [[a]] → [a]
```

- Produce a list with n identical elements:

```
replicate :: Int → a → [a]
```

- Select the nth element of a list:

```
(!!) :: [a] → Int → a
```

- Decide if a value is an element of a list:

```
elem :: Eq a ⇒ a → [a] → Bool
```

## (2) Define a recursive function

```
merge :: Ord a => [a] -> [a] -> [a]
```

that merges two sorted lists of values to give a single sorted list. For example:

```
> merge [2,5,6] [1,3,4]
```

```
[1,2,3,4,5,6]
```

### (3) Define a recursive function

```
mSort :: Ord a => [a] -> [a]
```

that implements merge sort, which can be specified by the following two rules:

- Lists of length  $\leq 1$  are already sorted;
- Other lists can be sorted by sorting the two halves and merging the resulting lists.