Introduction to Algebra

Algebra is a branch of mathematics that uses symbols, variables, and numbers to express relationships between quantities. It helps in solving problems where values are unknown and need to be found. In algebra, letters like xxx, yyy, and zzz represent unknown numbers.

Why Learn Algebra?

- **Problem-Solving Skills**: Algebra is used in a wide range of real-world applications, from finance to science and engineering.

- **Foundation for Higher Mathematics**: Algebra forms the basis for advanced topics in mathematics like calculus, statistics, and trigonometry.

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1. Basic Concepts of Algebra

1.1 Variables and Constants

- **Variable**: A symbol (often a letter) that represents an unknown or changeable number.

    o Example: xxx, yyy, or zzz

- **Constant**: A fixed number.

    o Example: 2, 7, -5

Example:

In the equation x+2=5x + 2 = 5x+2=5, xxx is the variable, and 2 and 5 are constants.

1.2 Expressions

An **algebraic expression** is a combination of variables, constants, and mathematical operations (like addition, subtraction, multiplication, and division).

- Example: 3x+43x + 43x+4, 5a2−2a+75a^2 - 2a + 75a2−2a+7

Key Terms:

- **Term**: A single part of an expression, which can be a number, a variable, or the product of both.

    o Example: In 3x+43x + 43x+4, the terms are 3x3x3x and 4.

- **Coefficient**: The numerical factor of a term that includes a variable.

    o Example: In 3x3x3x, the coefficient is 3.

1.3 Equations

An **equation** is a mathematical statement that asserts the equality of two expressions, containing an equal sign $=$.

- Example: $2x + 3 = 7$

In equations, you solve for the unknown variable.

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2. Operations in Algebra

2.1 Addition and Subtraction of Like Terms

Like terms are terms that have the same variable raised to the same power. Only like terms can be added or subtracted.

- Example: $3x + 5x = 8x$ (like terms can be combined)
- Example: $3x^2 + 5x$ (cannot be combined because the powers of x are different)

2.2 Multiplication of Terms

When multiplying terms, you multiply the coefficients and add the exponents of like variables.

- Example: $2x \times 3x = 6x^2$
- Example: $4a^2 \times 2a = 8a^3$

2.3 Division of Terms

To divide terms, divide the coefficients and subtract the exponents of like variables.

- Example: $\frac{6x^3}{2x} = 3x^2$

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3. Solving Linear Equations

3.1 What is a Linear Equation?

A **linear equation** is an equation where the highest power of the variable is 1. It graphs a straight line.

- Example: $2x + 3 = 7$

3.2 Steps to Solve a Linear Equation

1. **Simplify both sides** of the equation (combine like terms).

2. **Move variable terms** to one side and constant terms to the other side.

3. **Isolate the variable** by performing inverse operations (addition <-> subtraction, multiplication <-> division).

Example:

Solve $2x+3=7$:

1. Subtract 3 from both sides:
   $2x=4$

2. Divide by 2:
   $x=2$

3.3 Solving Equations with Variables on Both Sides

When variables appear on both sides of the equation, move them to one side before solving.

Example:

Solve $3x-4=2x+5$:

1. Subtract $2x$ from both sides:
   $x-4=5$

2. Add 4 to both sides:
   $x=9$

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4. Word Problems in Algebra

4.1 Translating Words into Algebraic Equations

When solving word problems, the key is to convert the written information into an algebraic equation.

Example 1:

A number is 3 more than twice another number. If their sum is 9, find the numbers.

1. Let the first number be x and the second number be $2x+3$.

2. Their sum is: $x+(2x+3)=9$

3. Simplify and solve:
   $x+2x+3=9$
   $3x+3=9$

$3x = 6$

$x = 2$

The first number is 2, and the second number is $2(2) + 3 = 7$.

4.2 Steps to Solve Word Problems

1. **Read the problem carefully** and identify what you need to find.

2. **Assign variables** to the unknown quantities.

3. **Translate the words into an equation** using algebra.

4. **Solve the equation**.

5. **Check your solution** by substituting it back into the original problem.

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5. Special Types of Equations

5.1 Quadratic Equations

A **quadratic equation** is an equation where the highest power of the variable is 2.

- Standard form: $ax^2 + bx + c = 0$

- Example: $2x^2 + 3x - 5 = 0$

5.2 Factoring Quadratics

Factoring is one method to solve quadratic equations.

Example:

Solve $x^2 + 5x + 6 = 0$ by factoring:

1. Factor the quadratic:
   $(x + 2)(x + 3) = 0$

2. Set each factor to zero:
   $x + 2 = 0$ or $x + 3 = 0$

3. Solve:
   $x = -2$ or $x = -3$

5.3 Using the Quadratic Formula

If the quadratic cannot be factored, use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- For $ax^2 + bx + c = 0$

Example:

Solve $2x^2 - 4x - 3 = 0$ using the quadratic formula:

1. Identify $a = 2$, $b = -4$, $c = -3$.

2. Substitute into the formula: $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$ $x = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4}$

3. Simplify:
   $x = \frac{4 \pm 6.32}{4}$

4. Solve for both values of x: $x = 2.58$ or $x = -0.58$

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6. Graphing Linear Equations

6.1 The Cartesian Plane

The Cartesian plane consists of two axes:

- The **x-axis** (horizontal)

- The **y-axis** (vertical)

6.2 Graphing Linear Equations

To graph a linear equation, use the slope-intercept form $y = mx + b$, where:

- m is the slope (rise over run)

- b is the y-intercept (where the line crosses the y-axis)

Example:

Graph $y = 2x + 3$:

1. The slope $m = 2$ means the line rises 2 units for every 1 unit it moves to the right.

2. The y-intercept $b = 3$ means the line crosses the y-axis at $y = 3$.

3. Plot the y-intercept and use the slope to plot another point, then draw the line through these points.

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7. Key Takeaways

- **Expressions vs. Equations**: Expressions are mathematical phrases, while equations show equality.

- **Solving Linear Equations**: Simplify, move terms, and isolate the variable.

- **Factoring and Quadratic Formula**: Both are methods for solving quadratic equations.

- **Graphing**: Understand how the slope and intercept define a line on the Cartesian plane.

Practice Questions

1. Simplify the expression: $4x+5x-2+34x + 5x - 2 + 34x+5x-2+3$.

2. Solve the equation: $3x-7=2x+53x - 7 = 2x + 53x-7=2x+5$.

3. Factor the quadratic: $x2-4x-12=0x^2 - 4x - 12 = 0x2-4x-12=0$.

4. Use the quadratic formula to solve $2x2+3x-2=02x^2 + 3x - 2 = 02x2+3x-2=0$.

5. Graph the equation $y=-3x+1y = -3x + 1y=-3x+1$.

Additional Resources

- **Khan Academy**: [Algebra Resources]

- **Purplemath**: [Algebra Lessons]

- **IXL**: [Algebra Practice]