An Investigation into the Relationship Between the Length of Coffee Stirrers and their Maximum Deflection Distance when Subjected to a Fixed Load

Introduction

As engineers, the strength of materials and how much load they can hold play a fundamental role in building stable and sustainable structures. Despite the inherent strength of the materials, their dimensions also greatly impact their strength and elasticity. For instance, engineered spider silk can be several times stronger than a steel wire of similar size, but steel wire is usually stronger because it is thicker (Agnarsson et al., 2010). Hence, understanding material properties is essential when designing and building a structure. I first saw some experiments about Young's modulus of spaghetti noodles and their different diameters. I then got curious and wondered if similar experiments can be performed on other daily products such as coffee stirrers and straws. Although coffee stirrers are not commonly considered a building material, an investigation into its material properties can provide valuable insights to optimize efficiency during manuracturing, transportation, and usage. Since the coffee stirrer is a food-grade commodity made of wood and shaped to standardized shapes, its ambient humidity and wood quality can also affect the product. Deflection distances from a bending test, for instance, helps to predict whether the coffee stirrer is likely to break in usage or transit.

Background Information

Young's modulus, also known as the modulus of elasticity, is a fundamental property in physics that characterizes a material's stiffness and ability to resist deformation when subjected to an applied force. It quantifies the ratio of stress to strain within a material and plays a critical role in understanding the behavior of solids under mechanical loads (Jones & Ashby, 2019). In engineering and physics, Young's modulus serves as a key parameter for predicting and analyzing the response of materials to external forces, guiding design decisions and ensuring structural integrity. In such a graph (Figure 1), stress is plotted against strain, depicting the material's response to applied forces. For sapphire, characterized by its crystalline structure and high stiffness, the graph typically exhibits a steep linear region at low strains, indicating a high Young's modulus (Zherebtsov et al., 2019). This steep slope reflects sapphire's resistance to deformation under stress, illustrating its stiffness and brittleness. Conversely, rubber, with its amorphous molecular structure and low stiffness, displays a much gentler slope on the Young's modulus graph, signifying a lower modulus of elasticity. By analyzing the Young's modulus graph, engineers can discern distinct material behaviors, enabling informed decisions in material selection and design. In this investigation, it is expected that the coffee stirrers made out of birch wood to have a medium Young's modulus with mimimal malleability.

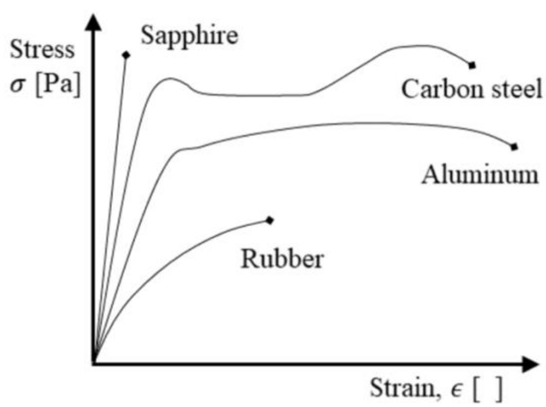


Figure 1. General Stress-Strain Curves for Various Materials

The lever system chosen for this experiment is a cantilever beam. Cantilever beam is a structural component that is supported at only one end and stretches horizontally as shown in Figure 2. The cantilever is the unsupported end that protrudes past the support point. In building, cantilever beams are frequently used to support overhanging structures like roofs and balconies (*A Complete Guide to Cantilever Beam | Deflections and Moments*, 2019). The support needs to be fixed to be able to support all forces and moments in all directions in order to guarantee that the structure is static. Typically, a cantilever beam is shown as follows, with the cantilevered end on the right and the support on the left.

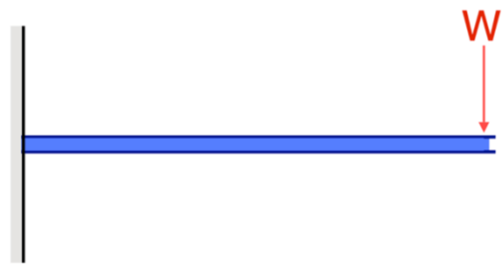


Figure 2. Diagram representation of cantilever beam (A Complete Guide to Cantilever Beam | Deflections and Moments, 2019)

The weight of the load imposed on the endpoint of the cantilever beam can be calculated using the following equation, assuming that gravity $g = 9.81\ m/s^2$ where W is the weight, m is the mass of the weight, and g is the acceleration due to gravity. In this scenario, W will be referred to as P in the modulus of elasticity equations.

$$W = m \cdot g$$

$$W = 0.500 \cdot 9.81$$

$$W = P = 4.91N$$

The modulus of elasticity equation that relates the length and cross sectional area of stirrer to the measured modulus of elasticity E is as follows, where P is the pressure on the endpoint of the stirrer in newtons, L is the length of the stirrer extended over the counter, I is the area moment of inertia (m^4) which quantifies the materials geometric resistance to bending, and δ_{max} is the maximum change of distance in the endpoint of the stirrer due to load (MechaniCalc, 2011).

$$E = \frac{PL^3}{3I\delta_{max}}$$

Since coffee stirrer sticks used in this experiment have a rectangular cross-sectional area, the area moment of inertia can be listed as the following where b is the base length of the stirrer sample, and h is the thickness of the stirrer.

$$I = \frac{bh^3}{12}$$

Substituting I into the original equation for the modulus of elasticity, one can get:

$$E = \frac{4PL^3}{bh^3\delta_{max}}$$

or

$$\delta_{max} = \frac{4PL^3}{bh^3E}$$

There are many variables in this equation that would remain constant in this experiment. E, the Young's modulus of elasticity will remain constant as it is specific to materials, and we will be using the same material across all trials. P, the load of the system will also remain constant, and so will the cross-sectional area of the coffee stirrers. Hence, the equation can be summarized in the following relationship.

$$\delta_{max} \propto L^3$$

Methodology

Research Question: How much does the length of a cantilever beam effect its maximum deflection when subjected to 0.500 kg of load measured using coffee stirrers?

Hypothesis: As the length of coffee stirrers increases, the distance of maximum deflection should increase as the deflection distance is directly proportional to the cubic of the lever length. That said, the system can also be explained that: when a fixed weight is applied to the end of a beam, energy is transferred from the weight to the beam. This energy transfer results in deflections of the beam, with the potential energy of the weight being converted into elastic potential energy stored within the beam. A longer stirrer would allow for more material and volume to absorb and distribute the force from the weights, reducing the stress levels within the beam, hence deflecting a longer distance while withstanding the force.

$$\delta_{max} = \frac{PL^3}{3EI}$$

Variables

Independent Variable: Length of extension (30 mm, 50 mm, 70 mm, 90 mm, 110 mm)

Dependent Variable: Distance of deflection (mm)

Controlled Variables:

1. Dimension of coffee stick (1mm thick, 5mm wide)
    a. Reason: coffee stirrers with distinct dimensions and masses will have different area moments of inertia, therefore affecting the deflection distance as the area moment of inertia changes
    b. Method: throughout all trials, all coffee stirrers used will be from the same pack to ensure they are identical to each other. Also, they will only be used once for each trial, avoiding any cracks and modifications to its sizing and strength after a trial.
2. Force applied onto the end point of the stirrer
    a. Reason: since this experiment assumes that the weight will be applied only at the end point of the system, we cannot place the weights anywhere on the stirrer, as it would then cause our calculations to deviate from the equations currently used. The
    b. Method: the string attached to the weight will always be secured 3mm away from the endpoint of the stirrer, tape may be used to secure the string and weight
3. Fixation on the fixed end of the cantilever

    a. Reason: the degree of fixation onto the counter may cause small shifts and slips due to the upward normal force imposed by the table and weight. If the c-clamp does not hold down the stirrer firmly, they will deflect more as the length of the lever increases.
    b. Method: whenever securing down the stirrer between trials, attempt to turn the clamp half of rotation further. Using a woodblock will help to distribute the downward force throughout the stirrer.
4. Ambient temperature of the experiment room and material
    a. Reason: at different temperature, materials exhibit difference in its flexibility and Young's modulus. The lower the temperature, the less flexible the material is, causing lower deflection lengths if the trials were conducted in a cold environment.
    b. Method: all trials were conducted in room temperature (22°C) to ensure uniformed flexibility of the material.

Materials

- C-Clamp (1)
- Coffee stirrers (15)
- 0.500kg weights (1)
- Fishing strings (~0.5m)
- Measurement stand or a ruler attached to a retort stand (1)
- Wood block (1)
- Ruler (1)
- Pencil (1)
- Scissors (1)
- Tape

Experimental Procedure

1) A 0.500kg weight is tightly secured onto a small piece of string.
2) The measurement stand is calibrated and leveled until the 0cm mark is at par with the height of the surface of the lab bench. Set up as illustrated in 'Apparatus'.
3) Mark 5 coffee stirrers **30 mm** away from the round endpoint of the stirrer using a ruler and a pencil.
4) One coffee stirrer is placed under the c-clamp and woodblock, tightened to the maximum.
    i) Ensure the edge of the length pencil marking is aligned with the edge of the lab bench.
    ii) Ensure the stirrer is perpendicular to the bench.
5) The weight is secured onto the coffee stirrer, some tape may be used to further ensure that it is secured.
6) Wait for the system to stabilize and turn motionless.
7) Using a ruler to guide, the end point of the coffee stirrer is measured using the measurement stand.
8) The data read is recorded.
9) Repeat steps 3-9 four more times to complete 5 trials for the scenarios.
10) Repeat steps 4-9 with each independent variable once (30 mm, 50 mm, 70 mm, 90 mm, 110 mm)

Apparatus

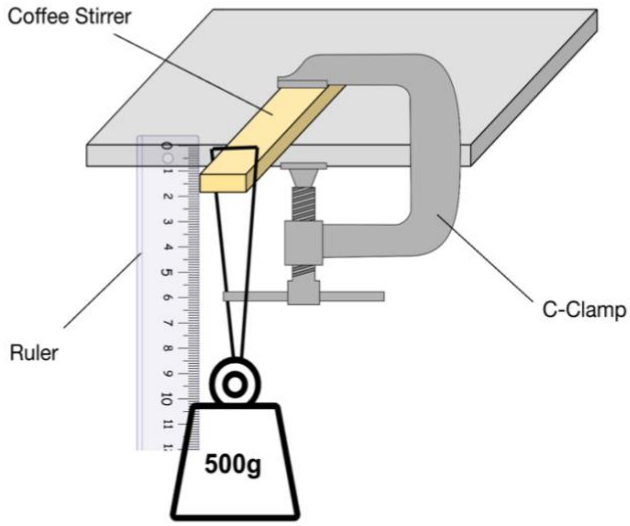


Figure 3. Simplified Experimental Apparatus

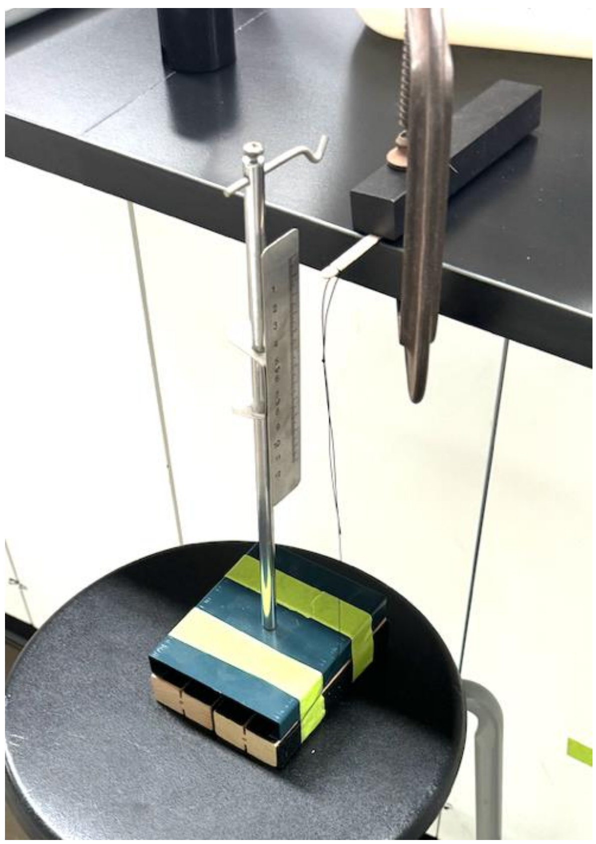


Figure 4. Actual experimental apparatus under testing conditions

Data Analysis

Table 1: Raw Data Table

Length (mm ± 0.5)	$\boldsymbol{\delta_{max}}$ (mm ± 0.5)				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
30.0	6.0	7.0	5.0	4.0	3.0
50.0	17.0	18.0	16.0	15.0	10.0
70.0	37.0	49.0	32.0	43.0	41.0
90.0	59.0	53.0	55.0	52.0	57.0
110.0	100.0	87.0	89.0	79.0	88.0

Sample Calculation

Average δ_{max} over 120 seconds changed - Sample calculation with Scenario 1

$$\delta_{max_{Average}}\ (\text{mm}) = \frac{trial\ 1 + trial\ 2 + trial\ 3 + trial\ 4 + trial\ 5}{5}$$

$$\delta_{max_{Average}}\ (mm) = \frac{6.0 + 7.0 + 5.0 + 4.0 + 3.0}{5}$$

$$\delta_{max_{Average}}\ (mm) = 5.0\ \pm\ 0.5\ mm$$

Range calculations for the Processed Data Table - Sample calculation with Scenario 1

$$\delta_{max_{Range}}(mm)\ = maximum\ \delta_{max}\ (mm)\ -\ minimum\ \delta_{max}\ (mm)$$

$$\delta_{max_{Range}}(mm) = 7.0 - 3.0$$

$$\delta_{max_{Range}}(mm)\ = 4.0$$

Margin of Error and Uncertainty calculations for the Processed Data Table - Sample calculation with Scenario 1

$$Margin\ of\ Error = \frac{Range}{2} = \frac{maximum\ \delta_{max}\ (mm)\ -\ minimum\ \delta_{max}\ (mm)}{2}$$

$$Margin\ of\ Error = \frac{4.0}{2}$$

$$Margin\ of\ Error = 2.0$$

Length cubed calculations for Table 3 – Sample calculation with Scenario 1

$$L^3 = 30^3 = 27000\ mm^3$$

Table 2: Processed Data Table

Length (mm ± 0.5)	$\delta_{max_{Average}}$(mm)	$\delta_{max_{Range}}$(mm)	Margin of Error (mm)
30.0	5.0	4	2
50.0	15.2	8	4
70.0	40.4	17	9
90.0	55.2	7	3
110.0	88.6	21	10

Figure 5. Graph of data graphed from Table 2

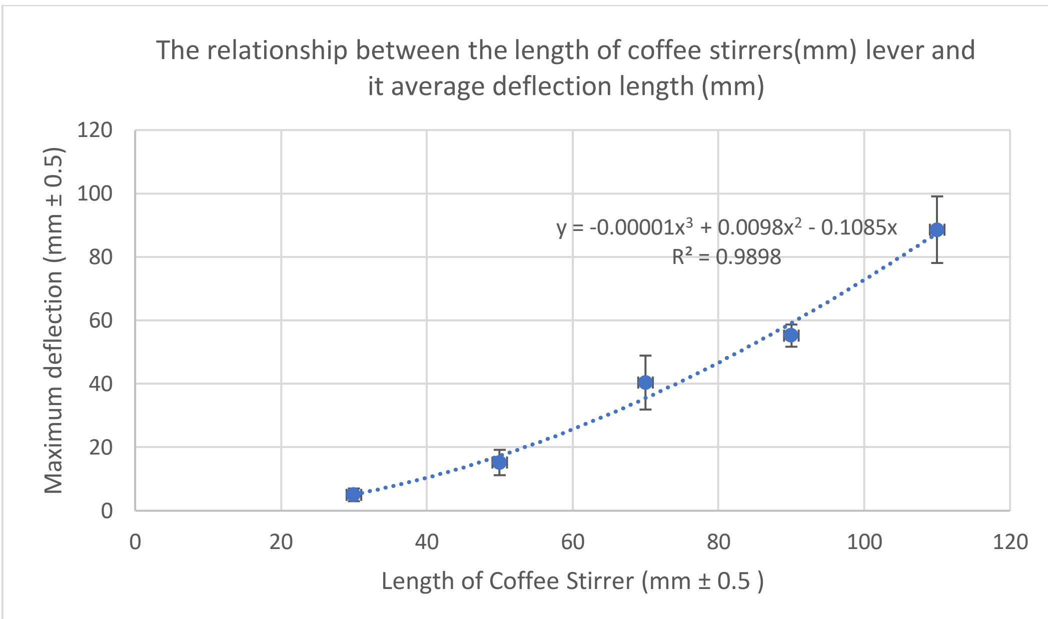
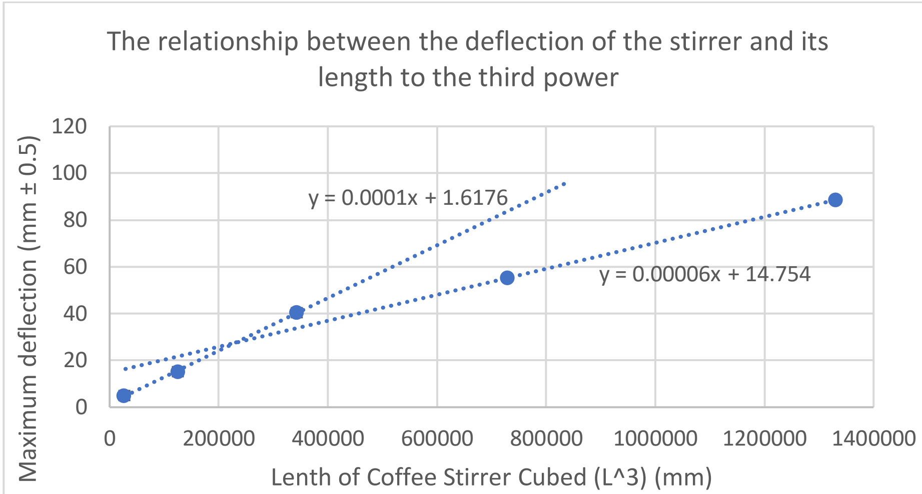


Table 3 Calculated Data with Length Cubed (L^3) as Independent Variable To Illustrate Direct Linear Relationship between the Two.

Length Cubed (mm^3)	Uncertainty in Length Cubed (mm^3)	$\delta_{max_{Average}}$ (mm ± 0.5)
27000	100	5.00
125000	400	15.2
343000	700	40.4
729000	1000	55.2
1331000	2000	88.6

Figure 6. Graph of Table 3 plotting average deflection distance $\boldsymbol{\delta_{max_{Average}}}$ directly to length of extension L^3.



Conclusion

Bending is a fundamental mechanical phenomenon governed by the interplay of applied forces and material properties, specifically the Young's modulus. When an external force is applied to a beam, it induces bending by generating internal stresses within the material. Young's modulus E then quantifies a material's resistance to deformation under tensile or compressive loads. Higher values of Young's modulus signify stiffer materials that exhibit less deformation for a given stress, while lower values indicate more flexible materials with greater deformation.

From the data collected and processed from experimentation, it can be concluded that the results support the initial hypothesis to a partial extent. One can already observe in Table 2 that the average maximum deflection increases as the length of the stirrer increased. By plotting a cubic graph of the force needed to bend the pasta sample against its length, one should expect a positive relationship between the two variables. Thus, as the length increases, the maximum deflection distance also increases exponentially.

In Figure 3, the average maximum deflection exhibited by the stirrer was plotted against the length of the stirrer. The line of best fit of this graph indicated a positive cubic relationship between the volume and average change in temperature, which was stated in the initial prediction as $\delta_{max} \propto L^3$. Hence, the hypothesis is correct with respect to the general trend as the constant modulus of elasticity and that it would result in a larger change in deflection when the length of the stirrer has increased.

In addition, one can notice that there was a relatively high precision in the data obtained, indicated by the small margins of error and the high correlation coefficient (0.9898) between the line of best fit and data points. However, if one were to extrapolate the line of best fit equation on a Cartesian plane, one would find that the trendline crosses the x-axis of at 1.125cm, meaning that at any length below that, the stirrer would exhibit a negative deflection distance. Since these behaviours are physically impossible given our experimental set-up, the data must have been skewed, lowering the precision of the measured data. Unless given that this relationship is true within error bars.

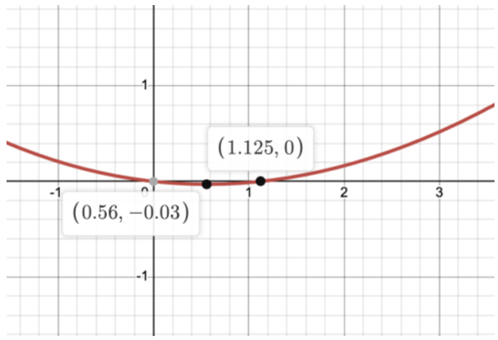


Figure 7. Graph of the trendline of data produced in Figure 5

The independent variable, length of the stirrer, is then cubed and plotted again the average deflection distance in order to gain a clearer understanding of their relationship, shown in figure 6. It was found that the first data points from scenarios 1-3 create a steeper linear trendline in comparison to the later two. The gradient of the two trendlines is equivalent to: $\frac{P}{3IE}$, illustrating the constant factor found in the proportional relationship of $\delta_{max} \propto L^3$. Through calculations below, one can deduce that the experimental Young's modulus is 34804 MPa, compared to researched Young's modulus of 11860 (Silver Birch (Gray Birch) Wood, n.d.). Two trendlines were used to illustrate the differences in the k gradient throughout different scenarios. The difference in experimental and theoretical Young's modulus is mostly due to systematic and random errors during data collection.

$$\delta_{max} \propto L^3$$

$$\delta_{max} = kL^3$$

$$k = \frac{P}{3IE}$$

$$0.00011262 = \frac{4.9}{\frac{5}{12} \cdot 3 \cdot E}$$

$$E = 34804\ MPa$$

There were a total of 5 trials conducted per volume and Table 2 demonstrates that the variations between each trial conducted. For instance, at the length of 7 cm, the average change in deflection was 4.04cm, but the range was 1.7cm, 42% of the measured change in deflection. This large margin of error is then reflected in Figure 3, with a larger vertical error bar when compared to other trials. This may create doubts from having high confidence and certainty in the trend described. That said, the horizontal error bars for the volume measurements are rather small, showing a high validity in the independent variables chosen. Additionally, all of the vertical error bars cross the trendline at some point, suggesting some random error in the data collection. The line of best fit does not pass through all 5 points but does pass through their respective error bars. In conclusion, the experiment supports the initial hypothesis regarding the relationship between stirrer length and the maximum deflection length, although data's precision and reliability could be improved.

Evaluation

Calculations for δ_{max} using actual value of Young's modulus (E) and length of extensions (L) of the coffee stirrers:

$$E = \frac{4PL^3}{bh^3\delta_{max}}$$

$$\delta_{max} = \frac{4P}{bh^3E} \cdot L^3$$

Given the Young's modulus of the coffee stirrers used in the trials was 11860 MPa for birch wood, and calculating the stirrer's area moment of inertia, we find the constant of the proportional relationship (Silver Birch (Gray Birch) Wood, n.d.):

$$\frac{12P}{3bh^3E} = \frac{4(4.90)}{(5.0)(1.0^3)(11860)} = 3.3 \times 10^{-4}$$

Using Scenario 5's stirrer extension length (110mm) to calculate the theoretical deflection distance:

$$\delta_{max} = \frac{4P}{bh^3E} \cdot L^3$$

$$\delta_{max} = 3.3 \times 10^{-4} \cdot 110^3$$

$$\delta_{max} = 439\ mm$$

Hence, the theoretical deflection length of the coffee stirrer is 439mm when it is extended 110mm over the counter. This is much larger than the experimental value of 88.6mm. In this case, this theoretical deflection length is completely unrealistic as the coffee stirrer cannot deflect a distance greater than the length itself, suggesting a maximum threshold length of which the bending can be measured. However, the higher theoretical deflection distance is also expected as the experimental apparatus had many systematic errors in its design, such as the use of weights and gravity rather than force sensors. At smaller angles of deflection, this is less significant, but at larger angles, the downward force of the weight is no longer perpendicular to the stirrer, causing almost half of the force to be diverged in the x-direction respective to the stirrer. Hence, reducing the force applied, and reducing the deflection length. To improve the accuracy of the experiment in finding the Young's modulus, the length of stirrer should be considerably smaller, and bending should be controlled at small angles to decrease the divergence in forces in the horizontal directions (or a force sensor could be used to directly apply downward force).

Source of Error	Significance and Evidence	Improvements
Systematic Errors Affecting Accuracy		
Placement of the weight: At larger lengths, specifically scenarios 4 and 5, the deflection distance of the coffee stirrers increased significantly, causing the stirrer to be close to normal to the ground. This posed significant challenges when trying to secure and balance the weights. Sometimes, tape was used to secure the weights onto the stirrers.	High significance, because: the slippage resulted in a large range of data collected, namely in scenario 5 where the stirrer was 11cm extended, resulting in a large margin of error. It is also evident in Graph 2, that the scenarios 4 and 5 are less accurate than scenarios 1-3 given that scenarios 4 and 5 have a lower slope than the expected value, which is likely caused by the underestimation of deflection as the force applied is much less in scenarios 4 and 5.	A force sensor attached to LoggerPro software could be used to push downwards on the coffee stirrers until the stirrers reached a certain deflection distance at various lengths rather than manually trying to secure weights solely by a string. Then, the force applied could be recorded as the dependent variable of the experiment, applying a force constantly perpendicular to the stirrer.
Location of the weight placement: Given the curvature at the end of the coffee stirrers, the weight was never placed exactly at the endpoint of the	Low significance, because: since the length of extension of stirrers were measured from the tip, this results in lower values of deflection distance as the weight is not exactly the	Can be eliminated by using a force sensor like above, and trimming the stirrers before experiment to ensure that the

stirrers, may have affected the weight distribution and internal stress of the material.	measured length away from the fixed end. Did not cause major outliers in the data but did contribute to the overall uncertainty in each datapoint.	weight is placed as close to the endpoint as possible.
Random Errors Affecting Precision		
Reusing coffee stirrers or using defected coffee stirrers: At lower extension lengths, the coffee stirrers were reused because there was no visible damage to the stirrers after each deflection at the shorter lengths. In rare cases, some coffee stirrers that were already bent or had cracks were used in the experiment.	Low significance, because: although it may have caused a greater deflection distance than expected given that the stirrers have been damaged and is now more flexible to bending, none of the stirrers used in the experiment had any visible alterations or cracks, and no distorted/damaged stirrers were used for the trials. May have caused the larger average deflection distance in Scenario 3, because at that point of data collection, the stirrers would have been used twice already for Scenarios 1&2.	Utilize new stirrers for every trial. Avoid using defected or bent stirrers in order to be consistent with the Young's modulus of the wood used in the stirrers.
Inherent uncertainties of the measurement stand: Given that the experiment's data is directly measured by humans, there can be great uncertainties to the data collected. Even the angle at which the stirrer is viewed at can impact the result greatly.	Low significance: The smallest increment on the stand was 0.1cm, giving us 0.05cm of uncertainty.	Use an electronic measurement tool such as a distance sensor with a lower uncertainty value.
Time of data collection: At larger lengths (scenarios 4 & 5), the deflection was so large that the weight was slipping, giving little time to stabilize the stirrer, then observe and record the data.	Negligible significance, because: the experiment is not time based. Although some time may be needed to stabilize the bending of the stirrer to record a precise distance, this error would be obsolete when using alternate methods to apply force.	Being insignificant, there is no improvement needed.

As an extension to the lab, it would be interesting to see how the area moment of inertia can change the deflection distance using an improved experimental apparatus. One can change the cross sectional shape or its dimensions, such as examining the difference in strength between rectangular, circular, and L-shaped beams, and potentially optimize the cost of producing the strongest beam. This would help me gain a better understanding of the material properties and how they can be manipulated to optimize for strengths.

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