Quantization Techniques for Visualization of High Dynamic Range Pictures

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Abstract : *This paper proposes several techniques that enable to display high dynamic range pictures (created by a global illumination rendering program, for instance) on a low dynamic range device. The methods described here are based on some basic knowledge about human vision and are intended to provide "realistic looking" images on the visualization device, even with critical lighting conditions in the rendered scene. The main features of the new techniques are speed (only a handful of floating point operations per pixel are needed) and simplicity (only one single parameter, which can be empirically evaluated has to be provided by the user). The goal of this paper is not to propose a psychovisual or neurological model for subjective perception, but only to described some experimental results and propose some possible research directions.*
Keywords : Dynamic Range, Subjective Brightness Perception, Tone Reproduction

1 Motivations

The last task of the rendering step in computer graphics is to display the computed picture on some visualization device. More precisely, it implies to quantize every floating point "intensity" value (expressed either as radiance or luminance) computed during the rendering process, in order to map to one of the N single integer values accepted by a typical visualization device (cathodic monitor, laser printer). This step is far from being trivial, the main difficulty is to find a good *quantization function* which bypasses the limitations of the device (limited color gamut, limited dynamic range) in order to display a picture that evokes the same visual sensation as the equivalent real scene [8]. Several comprehensive solutions, using work done in color science [13], have been proposed for color gamut limitations [9, 6, 10]. But, except two very recent papers [3, 12], dynamic range limitations have usually been ignored or solved only for specific cases.

This paper proposes some fast and simple quantization techniques to display high dynamic range pictures on low dynamic range devices. For simplicity, the algorithms are first explained on greyscale pictures (Section 3 and 4) before being extended to color pictures (Section 5). Notice that these techniques do not pretent to be a comprehensive solution based on a more or less complex psychovisual model for subjective perception, but only some better alternatives to the "gamma-corrected clamping technique" that is widely used in the global illumination community. Every quantization technique detailed here will be tested on the two pictures shown on Figure 1. For the first one, the ratio between the highest and the lowest intensity value (*ie* dynamic range) is 2 500, whereas for the second one the dynamic range is 30 000. Such dynamic ranges are common both in real life and in computer imagery generated by global illumination. Therefore finding a technique to display correctly these pictures on current visualization devices should be of great interest for the global illumination community.

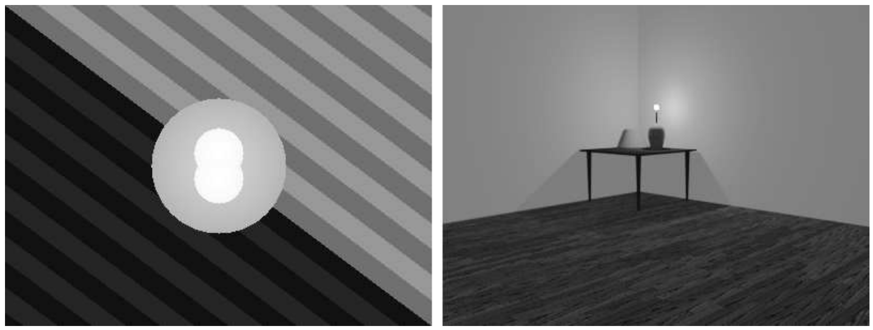

Figure 1 : High dynamic range test pictures
(a) Abstract picture showing stripes and circles of different intensities
(b) Room with a table and a wooden floor lit by an incandescent bulb[1]

2 Quantization of Greyscale Pictures

As recalled in the introduction, every intensity value *Val* of the computed picture has to be quantized in order to map one of the N single values in $[0, N\text{–}1]$ accepted by a typical visualization device. This process uses a so-called *tone reproduction function* (TRF, for short) F and can be formulated as :

$$Q\ (Val) = \lfloor N\ F(Val) \rfloor \quad \text{where} \quad F : [0, HiVal] \longrightarrow [0, 1] \tag{1}$$

With Equation 1, $F(Val) = 1$ maps to the out-of-range value N. This particular case can be easily detected, and *Val* be remapped to $N - 1$. Despite this annoying additional test, we find Equation 1 preferable to the more usual $Q(Val) = \lfloor (N-1)\ F(Val) + 0.5 \rfloor$ because it provides N equal quantization steps.

Finding a good TRF is a complex task which involves many different fields such as anatomy of the human eye, technology of the visualization device, color science, principles of subjective perception [8]. As said, until very recent papers [12, 3], most work on TRF functions in computer graphics has been done on color reproduction, usually ignoring brightness reproduction. The most widely used TRF in computer graphics is *gamma-corrected linear mapping* :

$$F_q\ (Val) = \left(\frac{Val}{HiVal} \right)^{1/q} \quad \text{where} \quad q \in [1, 3] \tag{2}$$

[1] This picture was provided by courtesy of Peter Shirley, University of Indiana.

In fact, two successive operators are applied in that function : first, a linear scaling (*ie* division by HiVal) which brings the intensity values into the range [0,1] and second, a gamma-correction which compensates for the non-linear response of the visualization device (usual values for q range from 1 to 3). The result of gamma-corrected linear mapping on our pictures (see Picture 1, Left Line) shows the inadequacy of this quantization for high dynamic range pictures: almost the whole scene (except the luminaires) is mapped to black or very dark gray.

A second popular TRF in computer graphics is *gamma-corrected clamping* :

$$F_{p,q}\left(Val\right) = \begin{cases} \left(Val/p\right)^{1/q} & \text{if } Val < p \\ 1 & \text{otherwise} \end{cases} \quad \text{where} \quad p \in [LoVal, HiVal] \quad q \in [1,3] \tag{3}$$

The main limitation of clamping is that a satisfying value for p can almost never be found. Either it is too high (see Picture 1, Middle Line) and then dark features (woodgrain of the floor, for instance) of the scene are all mapped to black, or it is too low (see Picture 1, Right Line) and then bright details (the bulb and its neighborhood) are all mapped to white. A “magic formula” that circulates in the global illumination community is to give to p the value of the brightest object which is not a luminaire (Middle Line of Picture 1 has been obtained so). But notice that such a formula implies that the quantization is done directly by the rendering process, because it is the only place where the luminaire/non-luminaire information is available.

Another TRF that has been often recommended but rarely implemented [2] is *logarithmic mapping* :

$$F_{p,q}\left(Val\right) = \left(\frac{\log\left(1 + p\ Val\right)}{\log\left(1 + p\ HiVal\right)}\right)^{1/q} \quad \text{where} \quad p \in [0, \infty) \quad q \in [1,3] \tag{4}$$

Logarithmic mapping has been proposed in the literature with or without gamma-correction. This fact can be explained when plotting the curves of logarithm mapping (see Figure 2a) and gamma-correction (see Figure 2b) : the shapes are almost identical. It means that gamma-corrected logarithmic scaling can be simulated with simple logarithmic scaling by giving a larger value to p. Applied on our test pictures, nice results are obtained (see Picture 2, Left Line) showing both the woodgrain on the floor and the details near the bulb.

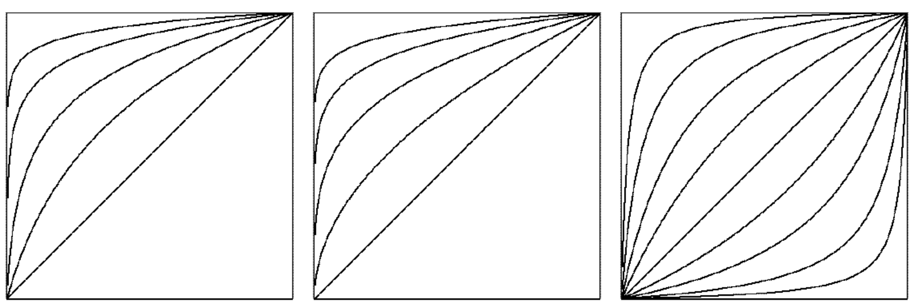

Figure 2 : Mapping functions for different values of p
(a) Logarithmic mapping **(b)** Exponentiation mapping **(c)** Rational mapping

Theoretical justifications of logarithm mapping are usually based on an old result in human vision, dating from the beginning of the century, known as Weber's law [7]. This law follows from experimentation on brightness perception that consists of exposing an observer to a uniform field of intensity Val in which a disk is gradually increased with a quantity ΔVal. The value ΔVal from which the observer perceives the disk is called brightness discrimination threshold, and Weber noticed that $\Delta Val / Val$ is constant for a wide range of intensity values.

More complete and more recent experimentation [11, 5] have shown that Weber's experimental results (valid for the discrimination threshold) cannot be integrated to derive a logarithmic law (valid throughout the range of brightnesses). In fact, the actual law looks more like an exponentiation law ($\Delta Val / Val^k$ is constant) where $k \in [0, 1]$ depends on the intensity of the surrounding environment and the dynamic range of the picture. Thus, a possible TRF including both brightness perception and gamma-correction could be *exponentiation mapping* :

$$F_{p,q}\ (Val) = \left(\frac{Val}{HiVal} \right)^{p/q} \quad \text{where} \quad p \in [0, 1] \quad q \in [1, 3] \tag{5}$$

Consequently, what we have here is a gamma-corrected linear scaling with a new gamma factor $q' = q/p$ where nice results may be obtained by a judicious value for p (see Picture 2, Middle Line). In other words, it means that the natural reflex (which everyone has succumbed to one day) of pushing the gamma factor over the value recommended by the manufacturer of the visualization device when displaying dark pictures, has got at least some theoretical justifications.

3 Uniform Rational Quantization

Though they can provide very good results, logarithmic and exponentiation mapping suffer from a serious weak point : there is no good rule to generate automatically a value for parameters p and q. Of course, q is theoretically given by the manufacturer but, as noted in [4], the gamma law is a first order approximation, usually only valid for a specific surrounding luminance and a specific setting of the device contrast and brightness. Moreover, as recalled in many technical manuals of printers or monitors, the gamma value is fluctuating under environment changes (heat, hygrometry, atmospheric pressure). Therefore finding some satisfying values implies a wasteful try-and-look process for almost every picture (more than 20 trials have been needed to get Left and Middle Line of Picture 2).

In order to correct this weak point, we propose a new quantization scheme using a so-called *rational mapping* function :

$$F_p\ (Val) = \frac{p\ Val}{p\ Val - Val + HiVal} \quad \text{where} \quad p \in [1, \infty) \tag{6}$$

This TRF is intended to account for the non-linear response of both the visualization device and subjective perception. When looking at the shape of the function for different values of p (see Figure 2c), one can see that very similar curves are obtained compared to logarithmic or exponentiation (the same figure also shows that symmetric curves according to the first diagonal can be obtained by replacing p by $1/p$; this property will be used in Section 4). Therefore when applying rational mapping on our test pictures, results are almost identical to the previous techniques (see Picture 2, Right Line), but the new function needs only 1 division, 1 multiplication, 1 subtraction and 1 addition, which is very economic compared to logarithm or exponentiation.

Automatic Generation of Parameter But the most interesting feature of the new TRF is that we propose a scheme to generate automatically the value of p. As this parameter controls how the non-linear response of the visualization device and subjective perception is accounted for, it should ideally depend on all the factors that are involved in a specific combination of observer, device and viewing conditions.

A possible solution is to use a scheme similar to the one proposed by Tumblin & Rushmeier [12] where 7 different parameters enable to control the shape of the TRF : gamma factor, maximum device contrast, maximum device luminance and the degree of adaptation in both the real world observer and display observer. While this technique is very comprehensive and does provide impressive results, we find it a bit too complex to become of general use for computer graphics. Indeed, the role of some parameters is hard to understand for a non-specialist user; and even for a specialist, giving a meaningful value to some of these parameters implies to use specific measuring instruments such as a photometer. Therefore, the user is more or less forced to work with the default values provided in [12], giving a sort of unflexible TRF hard to adapt to specific viewing conditions.

The automatic parameter generation process that we propose is based on the assumption that what really changes on a visualization device when several viewing parameters are modifed (average brightness of the surrounding environment, contrast and brightness setting for the device, observation distance...) is the value M of the darkest gray level that can be clearly distinguished from black. This value M could be used in the following manner : almost every rendering program includes an "epsilon" value (beneath which computed intensities are considered as negligible) that usually generates the smallest non-zero intensity value $LoVal$ of the picture. Therefore, it seems natural that a quantization process should map the smallest non-zero intensity of the picture to the darkest non-black gray of the device : $Q(LoVal) = M$

With our rational mapping function, it gives :

$$p = \frac{M\ HiVal - M\ LoVal}{N\ LoVal - M\ LoVal} \simeq \frac{M\ HiVal}{N\ LoVal} \tag{7}$$

The value of M can be easily provided by the user without any measuring instrument. For instance, we use the following process on our workstation : a black window is displayed on the screen, in which several squares of various intensities are drawn at random positions (see Figure 3) — randomness insures that no spatial regularity would bias the perception. The user simply clicks on the darkest square he is able to see; the intensity of the selected square gives the value of parameter M. In a similar way, by printing or photographing the resulting pattern, the same process may be used to find the value of M for a given printer or camera setting.

This automatic generation of p using Equation 7 has been tested on numerous images and/or viewing conditions, and it always enabled to display a picture where all the desired details were visible. Therefore, the main result illustrated on Picture 2 is not the similarity between the results provided by the three quantization techniques, but rather the fact that rational mapping enables to obtain these results, without any guesswork or hand fitting.

The reader may ask why the process $Q(LoVal) = M$ could not be used with logarithmic or exponentiation mapping in order to generate automatically the value of p. In fact, we have tried it, but in general, it does not provide very good results. We think that the reason lies on the symmetry of the rational mapping curves along the second diagonal (see Figure 2c), symmetry that does not exist with logarithmic or exponentiation mapping. This symmetry implies that low and high values are treated in a reciprocal way (when the dynamic range of low values is multiplied by a given factor, the dynamic range of high values is divided by the same factor) and therefore a somewhat smoother mapping is provided.

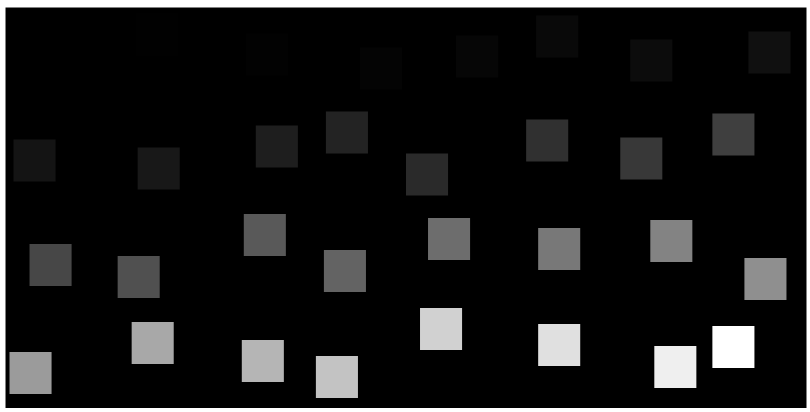

Figure 3 : Selection of the darkest non-black gray level

4 Non Uniform Rational Quantization

The quantization process presented in Section 3 can be called *uniform* in the sense that each pixel intensity is quantized with the same tone mapping function. Therefore two pixels intensities Val_1 and Val_2, with $Val_1 \leq Val_2$, will be mapped to quantized values Q_1 and Q_2, such as $Q_1 \leq Q_2$. In real life, this is not always true because the subjective brightness of an object is highly dependent on the average surrounding brightness — everyone knows the classical optical illusion in which two identical gray squares put on different backgrounds appear with different subjective brightnesses. Chiu et al. [3] were the first in computer graphics to consider this property and stated that a quantization technique should be spatially *non-uniform*.

This non-uniform subjective perception is usually explained in vision science by the fact that an observer does not view a scene as a whole [1]. In fact, his eyes are continuously moving from one point to another, and for each point on which the eye focuses, there exists a surrounding zone that creates some local visual adaptation and thus modifies brightness perception. If we want to display a picture that evokes the subjective perception of the scene, this eye behaviour has to be included during the visualization process.

We propose the following empirical scheme to account for non-uniform subjective perception. For each pixel, an intensity value $ZoVal$ is computed corresponding to the average intensity of a given zone surrounding the pixel. This value $ZoVal$ is then used to modify parameter p of Equation 6, in order to create a specific parameter p' (and thus a specific TRF) for each pixel. According to the rules governing subjective perception [7, 5] when $ZoVal$ is low (resp. high), the pixel should appears brighter (resp. darker) to the observer, which means that p' should be smaller (resp. larger) than p.

Because of the (quasi-)logarithmic response of subjective brightness perception, a fundamental quantity of our picture is the geometrical mean :

$$MiVal = \sqrt{LoVal\ HiVal} \tag{8}$$

which divides the dynamic range into two equal sub-ranges :

$$\frac{HiVal}{MiVal} = \frac{MiVal}{LoVal} = \sqrt{\frac{HiVal}{LoVal}} \tag{9}$$

Therefore the ratio $ZoVal/MiVal$ characterizes the brightness of a zone : if it is smaller (resp. larger) than 1, the zone will be considered as dark (resp. bright). In our scheme, this ratio is used to modify parameter p in the following manner :

$$p' = p\ \left(1 - k + k\frac{ZoVal}{MiVal}\right) \quad \text{with} \quad k \in [0, 1] \tag{10}$$

where k represents the weight of non-uniformity that is included in the TRF (*ie* when $k = 0$, the mapping is uniform). Giving a meaningful value to k is difficult because, to our knowledge, the relative importance of global vs. local brightness perception has never been studied (we have used $k = 0.5$ in our experimentations).

So, to get a complete non-uniform quantization technique, the only point that remains to solve is to find a scheme to compute $ZoVal$ for each pixel. In fact, we have tested three different schemes (low pass filtering, micro-zone, segmentation) which are detailed in the next paragraphs.

Low-Pass Filtering Because we want to compute the average intensity value $ZoVal$ of a zone surrounding a pixel, a natural idea is to apply a low-pass filter on the picture. Thus, $ZoVal$ will be computed as a weighted sum of neighboring intensities; the weights as well as the extent of the neighbourhood are given by the convolution matrix of the filter. Such a process has been used by Chiu et al. [3] in their non-uniform mapping scheme. We have tried a similar idea, and though we have used a different convolution matrix and a different mapping function, we came to the same conclusion : non-uniform quantization driven by a low-pass filtered picture creates some unacceptable artifacts.

A low-pass filtered version of our test pictures can be seen on Picture 2, Left Line. Using the intensity values $ZoVal$ provided by that filtered image to control the non-uniform mapping with Equation 10 yields bad results (see Picture 3, Middle Line) where unnatural dark bandings are visible. Nevertheless, it should be noted that, if low-pass filtering is inadequate for our quantization scheme, we have found it very useful to simulate another specific phenomenon involved in human vision : dazzling effects (see Picture 3, Right Line) can be obtained by simply reversing the transformation of parameter p in the non-uniform mapping (*ie* replacing $ZoVal/MiVal$ by $MiVal/ZoVal$ in Equation 10).

Micro-Zones When reducing the size of the convolution matrix, dark artifacts progressively vanish. Finally best results are obtained when the size of the matrix is one, which means that the surrounding zone of a pixel is reduced to a micro-zone composed of the pixel itself, and thus $ZoVal = Val$. This result is somewhat surprising because reducing to surrounding zone to one pixel provides in fact a uniform mapping.

Comparing the result of such a mapping (see Picture 4, Left Line) with the previous rational mapping on our test pictures (see Picture 2, Right Line) shows much nice results for the former (the woodgrain appears much better and the light bulb is better antialiased). We think that this result comes from the fact that the new uniform mapping creates larger dynamic ranges (which may eventually overlap) for specific parts of the picture. Notice that by precomputing $1-k$ and $k/MiVal$, this new rational mapping function involves only an additional cost of 2 multiplications and 1 addition per pixel, compared to the rational mapping function presented in Section 3.

Segmentation The last scheme that we have tried is to segment the picture into zones of similar intensity values. For each zone, the average intensity $ZoVal$ is computed and each pixel of the zone will get the same $ZoVal$. Image segmentation is a complex problem and an algorithm dealing with every configuration has not yet been found. But we have here a simplified case (greyscale picture where only a few number of zones are wanted) and therefore classical techniques such as gradient or histogram thresholding should work for most pictures.

A segmented version of our test pictures id shown on Picture 4, Middle Line. Using the value $ZoVal$ provided by that segmented image to control the non-uniform mapping with Equation 10 yields nice results (see Picture 4, Right Line). But compared to the micro-zone technique, differences can hardly be seen (a bit less saturation around the light bulb) and therefore, we think that the improvement is not worth the overhead involved by the segmentation step.

5 Extension to Color Pictures

In this section, we propose to extend our quantization scheme to color pictures. Only images created with a trichromatic model during the rendering step are considered here, but the technique can be straightforwardly adapted to spectral models [6, 9]. The only imperative is to be able to compute the Y coordinate of the CIE XYZ color system.

Let's suppose that we have created a color picture using three floating point values per pixel (Val_r, Val_g, Val_b), one for each primary color R,G,B. A naive extension of the quantization scheme could be to compute 3 dynamic ranges, 3 parameters (p_r, p_g, p_b) and 3 rational mapping functions $F_{p_r}(Val_r)$, $F_{p_g}(Val_g)$ and $F_{p_b}(Val_b)$. Because dynamic ranges may be very different for each primary color, the resulting mapping functions may have very different shapes and then many color shifts are likely to be created by such a process.

One has to remember that our tone mapping function is intended to compensate for non-linear response of both the visualization device and subjective brightness perception. When looking at technical characteristics of trichromatic devices, one can see that the response is nearly identical for the three primary colors. When looking at the conclusions of experimentation on subjective perception, one can see that the brightness discrimination law yields, more or less, throughout the visible spectrum, depending only on the relative sensitivity of the eye to the tested color. These two results tend to prove that, in a first order approximation, the tone mapping operator should probably be achromatic.

Starting from that presumption, we propose the following extension of our TRF for color pictures. For each pixel (Val_r, Val_g, Val_b), the corresponding achromatic intensity Val is computed. Val is in fact the Y coordinate of the CIE XYZ color system (*ie* luminous efficiency of human eye across the visible spectrum). Therefore, if the XYZ coordinates of the primary colors used by the visualization device are provided by the manufacturer, they could be used. If these coordinates are not available, Val could be computed according to some television color standard. For instance, with NTSC[2] is gives [6] :

$$Val = 0.299\ Val_r + 0.587\ Val_g + 0.114\ Val_b \tag{11}$$

This operation creates a greyscale picture for which the tone mapping function $F_p(Val)$ is defined by Equations 6, 7 and 10. This TRF insures that $Val' = F_p(Val)$ will be the new (normalized) intensity of the pixel. Therefore, what we want is to find (Val'_r, Val'_g, Val'_b) such as

$$Val' = 0.299\ Val'_r + 0.587\ Val'_g + 0.114\ Val'_b \tag{12}$$

There is an infinite number of solutions that fulfill Equation 12. Work done on a similar problem for color gamut [6, 10] has shown that minimal color shifts occur

[2] Equation 11 is valid for an additive color device (e.g. CRT) with R,G,B as primary colors. For a subtractive color device (e.g. printer) with C,M,Y as primary colors, Val is given by : $1 - 0.402\, Val_c + 0.174\, Val_m - 0.772\, Val_y$.

when the ratios between primary colors are preserved during the tranformation :

$$\frac{Val'_r}{Val_r} = \frac{Val'_g}{Val_g} = \frac{Val'_b}{Val_b} \tag{13}$$

Let's suppose that $max\ (Val_r,\ Val_g,\ Val_b) = Val_r$. It means that

$$\exists\ (u,v) \in [0,1]^2 \quad / \quad Val_g = u\ Val_r \quad \text{and} \quad Val_b = v\ Val_r \tag{14}$$

We have

$$Val = Val_r\ (0.299 + 0.587\ u + 0.114\ v) \tag{15}$$

and according to Equation 13, we want

$$Val' = Val'_r\ (0.299 + 0.587\ u + 0.114\ v) \tag{16}$$

thus

$$Val'_r = \frac{Val'}{0.299 + 0.587\ u + 0.114\ v} \qquad Val'_g = u\ Val'_r \qquad Val'_b = v\ Val'_r \tag{17}$$

which defines the TRF for color pictures.

6 Conclusion

This paper has reported some experimentations about quantization techniques for visualization of computer generated pictures, leading to the following conclusions :

- Arbitrary high dynamic ranges (greyscale or color) pictures can be displayed on arbitrary low range devices by the use of an original rational tone mapping function F_p.
- A possible technique has been proposed (called *micro-zone rational mapping*) which uses a specific function F_p for which parameter p is changed according to the intensity of each pixel.
- One main feature of the quantization is *speed* : for a color picture, only 4 divisions, 12 multiplications and 5 additions per pixel are required.
- Another main feature is *simplicity* : the algorithm is controlled by only 3 parameters, the highest intensity *HiVal* of the picture, the lowest non-zero intensity *LoVal* (which can either be provided by the user or computed by a first pass on the picture) and the value M of the darkest non-black gray visible on the device under the current viewing conditions.

Pictures

Each picture (see picture plates, below) is composed of three columns where one column represents one quantization technique. For each technique, four images are shown : an abstract picture composed of lines and circles, an room with a table and a wooden floor lit by an incandescent bulb, as well as two close-ups of specific parts of this latter image (the light bulb and the rightmost table foot).

Picture 1 : LEFT : Linear quantization
MIDDLE : Clamping with a large value RIGHT : Clamping with a small value

Picture 2 : LEFT : Logarithmic quantization
MIDDLE : Exponentiation quantization RIGHT : Rational quantization

Picture 3 : LEFT : Low-pass filtered image
MIDDLE : Low-pass quantization RIGHT : Inverse low-pass quantization

Picture 4 : LEFT : Micro-zone quantization
MIDDLE : Segmented image RIGHT : Segmentation quantization

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