Estimating Firm-Specific Inefficiencies

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JLMS estimator

After estimating the expected inefficiency using methods of moment (MOM) or quasi-likelihood estimation (QLE), We can use JLMS estimator proposed by Jondrow et al. (1982) to estimate the firm-specific inefficiencies (Johnson & Kuosmanen, 2015). Under the assumption of a normally distributed error term and a half-normally distributed inefficiency term, they formulate the conditional distribution of inefficiency u_i, given ε_i, and propose the inefficiency estimator as the conditional mean $E(u_i|\varepsilon_i)$.

1. $E(u_i \mid \varepsilon_i)$: Following Kumbhakar & Lovell (2000), the conditional expected value of inefficiency are

   - Production function

     $$\begin{aligned} E(u_i \mid \varepsilon_i) &= \mu_{*i} + \sigma_* \left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[\frac{\phi(\varepsilon_i \lambda/\sigma)}{1 - \Phi(\varepsilon_i \lambda/\sigma)} - \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

     where $\mu_* = -\varepsilon\sigma_u^2/\sigma^2$ and $\sigma_*^2 = \sigma_u^2\sigma_v^2/\sigma^2$.

   - Cost function

     $$\begin{aligned} E(u_i \mid \varepsilon_i) &= \mu_{*i} + \sigma_* \left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[\frac{\phi(\varepsilon_i \lambda/\sigma)}{1 - \Phi(-\varepsilon_i \lambda/\sigma)} + \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

     where $\mu_* = \varepsilon\sigma_u^2/\sigma^2$ and $\sigma_*^2 = \sigma_u^2\sigma_v^2/\sigma^2$.

2. Technical inefficiency (TE):

   - Production function
     - Logged Dependent Variable: $\text{TE} = \exp(-E(u_i \mid \varepsilon_i))$
     - Otherwise, $\text{TE} = \frac{\text{fitt} - E(u_i|\varepsilon_i)}{\text{fitt}}$
   - Cost function
     - Logged Dependent Variable: $\text{TE} = \exp(-E(u_i \mid \varepsilon_i))$
     - Otherwise, $\text{TE} = \frac{E(u_i|\varepsilon_i) - \text{fitt}}{\text{fitt}}$

References

Johnson, A. L. & Kuosmanen, T. (2015), An Introduction to CNLS and StoNED Methods for Efficiency Analysis: Economic Insights and Computational Aspects, *in* S. C. Ray, S. C. Kumbhakar & P. Dua (eds), *Benchmarking for Performance Evaluation: A Production Frontier Approach*, Springer, chapter 3, pp. 117–186.

Jondrow, J., Lovell, C. A. K., Materov, I. S. & Schmidt, P. (1982), On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics* 19, 233–238.

Kumbhakar, S. C. & Lovell, C. A. K. (2000), *Stochastic Frontier Analysis*, Cambridge University Press.