

Estimating Firm-Specific Inefficiencies

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JLMS estimator

After estimating the expected inefficiency using methods of moment (MOM) or quasi-likelihood estimation (QLE), We can use JLMS estimator proposed by Jondrow et al. (1982) to estimate the firm-specific inefficiencies (Johnson & Kuosmanen, 2015). Under the assumption of a normally distributed error term and a half-normally distributed inefficiency term, they formulate the conditional distribution of inefficiency u_i , given ε_i , and propose the inefficiency estimator as the conditional mean $E(u_i|\varepsilon_i)$.

1. $E(u_i | \varepsilon_i)$: Following Kumbhakar & Lovell (2000), the conditional expected value of inefficiency are

- Production function

$$\begin{aligned} E(u_i | \varepsilon_i) &= \mu_{*i} + \sigma_* \left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[\frac{\phi(\varepsilon_i \lambda / \sigma)}{1 - \Phi(\varepsilon_i \lambda / \sigma)} - \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

where $\mu_* = -\varepsilon \sigma_u^2 / \sigma^2$ and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$.

- Cost function

$$\begin{aligned} E(u_i | \varepsilon_i) &= \mu_{*i} + \sigma_* \left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[\frac{\phi(\varepsilon_i \lambda / \sigma)}{1 - \Phi(-\varepsilon_i \lambda / \sigma)} + \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

where $\mu_* = \varepsilon \sigma_u^2 / \sigma^2$ and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$.

2. Technical inefficiency (TE):

- Production function
 - Logged Dependent Variable: $TE = \exp(-E(u_i | \varepsilon_i))$
 - Otherwise, $TE = \frac{\hat{y} - E(u_i | \varepsilon_i)}{\hat{y}}$
- Cost function
 - Logged Dependent Variable: $TE = \exp(E(u_i | \varepsilon_i))$
 - Otherwise, $TE = \frac{\hat{y} + E(u_i | \varepsilon_i)}{\hat{y}}$

References

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- Jondrow, J., Lovell, C. A. K., Materov, I. S. & Schmidt, P. (1982), On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics* 19, 233–238.
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