

# Estimating Firm-Specific Inefficiencies

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## JLMS estimator

After estimating the expected inefficiency using methods of moment (MOM) or quasi-likelihood estimation (QLE), We can use JLMS estimator proposed by Jondrow et al. (1982) to estimate the firm-specific inefficiencies (Johnson & Kuosmanen, 2015). Under the assumption of a normally distributed error term and a half-normally distributed inefficiency term, they formulate the conditional distribution of inefficiency  $u_i$ , given  $\varepsilon_i$ , and propose the inefficiency estimator as the conditional mean  $E(u_i|\varepsilon_i)$ .

1.  $E(u_i | \varepsilon_i)$ : Following Kumbhakar & Lovell (2000), the conditional expected value of inefficiency are

- Production function

$$\begin{aligned} E(u_i | \varepsilon_i) &= \mu_{*i} + \sigma_* \left[ \frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[ \frac{\phi(\varepsilon_i \lambda / \sigma)}{1 - \Phi(\varepsilon_i \lambda / \sigma)} - \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

where  $\mu_* = -\varepsilon \sigma_u^2 / \sigma^2$  and  $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$ .

- Cost function

$$\begin{aligned} E(u_i | \varepsilon_i) &= \mu_{*i} + \sigma_* \left[ \frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right] \\ &= \sigma_* \left[ \frac{\phi(\varepsilon_i \lambda / \sigma)}{1 - \Phi(-\varepsilon_i \lambda / \sigma)} + \frac{\varepsilon_i \lambda}{\sigma} \right] \end{aligned}$$

where  $\mu_* = \varepsilon \sigma_u^2 / \sigma^2$  and  $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$ .

2. Technical inefficiency (TE):

- Production function
  - Logged Dependent Variable:  $TE = \exp(-E(u_i | \varepsilon_i))$
  - Otherwise,  $TE = \frac{f_{tt} - E(u_i | \varepsilon_i)}{f_{tt}}$
- Cost function
  - Logged Dependent Variable:  $TE = \exp(E(u_i | \varepsilon_i))$
  - Otherwise,  $TE = \frac{f_{tt} + E(u_i | \varepsilon_i)}{f_{tt}}$

## References

- Johnson, A. L. & Kuosmanen, T. (2015), An Introduction to CNLS and StoNED Methods for Efficiency Analysis: Economic Insights and Computational Aspects, *in* S. C. Ray, S. C. Kumbhakar & P. Dua (eds), *Benchmarking for Performance Evaluation: A Production Frontier Approach*, Springer, chapter 3, pp. 117–186.
- Jondrow, J., Lovell, C. A. K., Materov, I. S. & Schmidt, P. (1982), On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics* 19, 233–238.
- Kumbhakar, S. C. & Lovell, C. A. K. (2000), *Stochastic Frontier Analysis*, Cambridge University Press.