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CSE135A/EECS152A Exam #1 November 4, 2014

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This is an 80 minute, CLOSED BOOK exam. Calculators are not allowed. Show your work. GOOD LUCK!

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

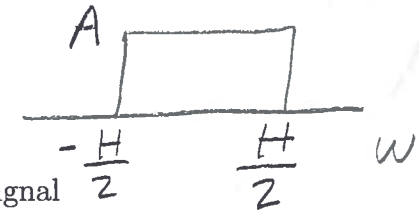
Question 6:

Question 7:

Question 8:

TOTAL:

(R3) $\frac{A(H/2)}{\pi(Ht/2)} \sin\left(\frac{Ht}{2}\right) \xleftrightarrow{\mathcal{F}}$



Question 1 (20 points) Let $x(t)$ be the continuous-time signal

$$x(t) = \frac{\sin(\pi t)}{\pi t}$$

$H \to 2\pi$, $A \to 1$

Suppose that $x(t)$ is sampled every two seconds to generate the sampled signal $x(n)$ where the $n = 0$ sample corresponds to $t = 0$.

a) Find the Fourier transform $X(w)$ of $x(t)$.

4

$X(w)$

2 b) Find $x(n)$.

$T = 2$

$$x(n) = \frac{\sin(2\pi n)}{2\pi n} = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

c) Let $x'(t)$ be the recovered continuous-time signal when we apply sinc interpolation to $x(n)$. Find $x'(t)$.

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$$x'(t) = \sum_{n=-\infty}^{\infty} x(n) \frac{\sin(\pi(t-nT)/T)}{\pi(t-nT)/T}$$

$$= \frac{\sin(\pi t/2)}{\pi t/2}$$

2

d) Do we have $x'(t) = x(t)$? (Answer Yes or No)

No.

e) Explain why your answer to part d is consistent with the sampling theorem.

ω_{max} for $x(t)$ is $\pi + \epsilon$ with ϵ small and positive.

4

The sampling theorem requires the condition $\omega_s \geq 2\omega_{max}$

We have $\omega_s = \frac{2\pi}{T} = \pi$

Therefore, the condition is not satisfied and $x'(t) \neq x(t)$.

f) Find the Fourier transform $X'(\omega)$ of $x'(t)$.

This is (R3) above with $H = \pi$ and $A = 2$.

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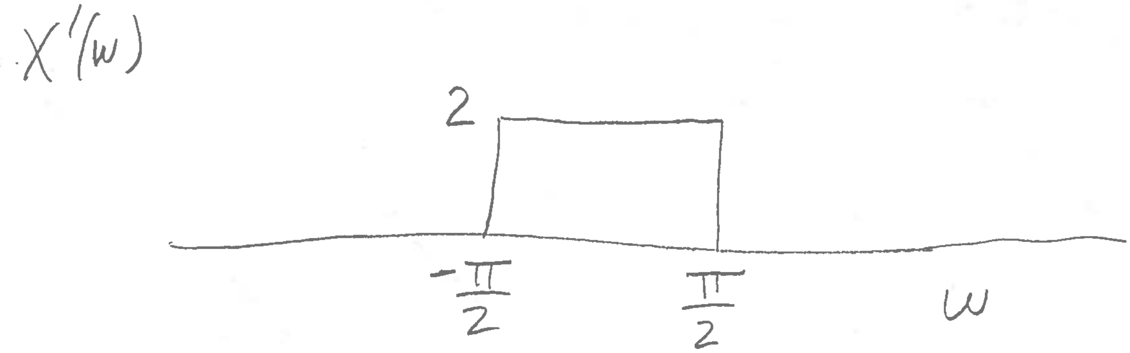

Question 2 (12 points) Consider the discrete-time system

$$y(n) = 0.5\,[x(n+3) - x(n+9)] = \sum_{K=-\infty}^{\infty} x(n-K)h(K)$$

where $x(n)$ is the input signal and $y(n)$ is the output signal.

2 a) Find the impulse response $h(n)$ of the system.

$h(-3) = 0.5 \quad h(-9) = -0.5$

2 b) Find the frequency response $H(\Omega)$ of the system.

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\Omega n} = \frac{1}{2} e^{3j\Omega} - \frac{1}{2} e^{9j\Omega}$$

c) Find the wrapped phase response (ARG$[H(\Omega)]$) of the system over $0 \leq \Omega \leq \pi$.

$$H(\Omega) = -j e^{6j\Omega} \sin(3\Omega) = e^{j(6\Omega - \frac{\pi}{2})} \sin(3\Omega)$$

$$\theta(\Omega) = \begin{cases} 6\Omega - \frac{\pi}{2} & 0 \leq \Omega \leq \frac{\pi}{3} \\ 6\Omega + \frac{\pi}{2} & \frac{\pi}{3} \leq \Omega \leq \frac{2\pi}{3} \\ 6\Omega - \frac{\pi}{2} & \frac{2\pi}{3} \leq \Omega \leq \pi \end{cases}$$

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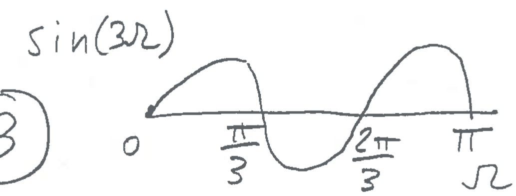

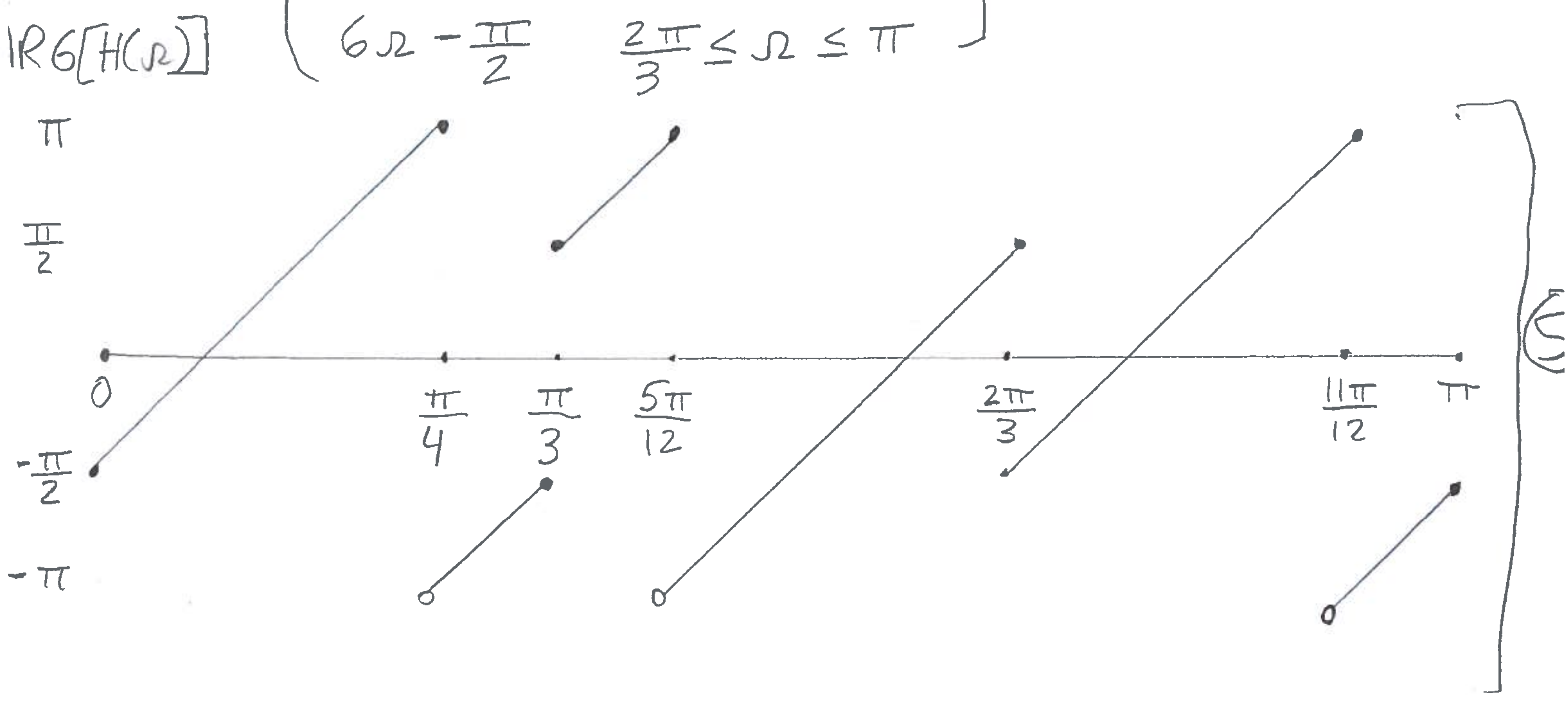

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Question 3 (12 points) Let T be the discrete-time system defined by

$$y(n) = x(n) - x(n-4) \;=\; \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

where $x(n)$ is the input signal and $y(n)$ is the output signal. Find the output $y(n)$ of the system if the input is

$$x(n) = \cos\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{4}\right)$$

Simplify your answer.

$h(0)=1 \quad h(4)=-1$

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\Omega n} = 1 - e^{-j\Omega 4}$$ 2

$$x(n) = \frac{e^{jn\pi/2} + e^{-jn\pi/2}}{2} + \frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}$$ 2

$$y(n) = \frac{H\left(\frac{\pi}{2}\right)e^{jn\pi/2} + H\left(\frac{-\pi}{2}\right)e^{-jn\pi/2}}{2} + \frac{H\left(\frac{\pi}{4}\right)e^{jn\pi/4} + H\left(\frac{-\pi}{4}\right)e^{-jn\pi/4}}{2}$$ 2

$H\left(\frac{\pi}{2}\right) = 0 \quad H\left(-\frac{\pi}{2}\right) = 0 \quad H\left(\frac{\pi}{4}\right) = 2 \quad H\left(-\frac{\pi}{4}\right) = 2$

$$y(n) = e^{jn\pi/4} + e^{-jn\pi/4} = 2\cos\left(\frac{n\pi}{4}\right)$$ 6

Question 4 (12 points) Consider a discrete-time LTI system that generates an output $y(n)$ from an input $x(n)$ according to

$$y(n) = 3x(n) - 6x(n-1) + 3x(n-2)$$

a) Find the magnitude response $|H(\Omega)|$ of the system.

6

$$Y(z) = 3X(z) - 6X(z)z^{-1} + 3X(z)z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = 3 - 6z^{-1} + 3z^{-2}$$

$$H(\Omega) = 3 - 6e^{-j\Omega} + 3e^{-2j\Omega} = e^{-j\Omega}(3e^{j\Omega} - 6 + 3e^{-j\Omega})$$

$$= e^{-j\Omega}(6\cos\Omega - 6)$$

$$|H(\Omega)| = 6|\cos\Omega - 1| = 6(1 - \cos\Omega)$$

b) Find the unwrapped phase response ($\arg[H(\Omega)]$) of the system.

6

$$H(\Omega) = |H(\Omega)| e^{j\theta(\Omega)}$$

$$e^{-j\Omega}6(\cos\Omega - 1) = 6(1 - \cos\Omega)e^{j\theta(\Omega)}$$

$$e^{j\theta(\Omega)} = e^{j\pi}e^{-j\Omega} = e^{j(\pi - \Omega)}$$

$$\theta(\Omega) = \pi - \Omega$$

Question 5 (10 points) We want to approximate the function

$$x_1(t) = \sin(t) + \sin(2t)$$

with a function of the form

$$x_2(t) = \sum_{k=-1}^{1} C_k e^{jkt}$$

Find constants $C_{-1},\ C_0,\ C_1$ so that

$$\frac{1}{2\pi}\int_0^{2\pi} |x_1(t) - x_2(t)|^2 dt$$

is as small as possible. Note that C_{-1}, C_0, C_1 may be complex.

$x_1(t)$ has period $T_0 = 2\pi$, fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = 1$

$x_2(t)$ is the truncated Fourier series for $x_1(t)$.

$$x_1(t) = \frac{e^{jt} - e^{-jt}}{2j} + \frac{e^{2jt} - e^{-2jt}}{2j}$$

$$x_2(t) = C_{-1} e^{-jt} + C_0 + C_1 e^{jt}$$

$$C_{-1} = \frac{-1}{2j} = \frac{j}{2} \qquad C_0 = 0 \qquad C_1 = \frac{1}{2j} = \frac{-j}{2}$$

Same result can be obtained using

$$C_K = \frac{1}{2\pi}\int_0^{2\pi} x_1(t) e^{-jKt} dt$$

3 points for writing C_K integral

Question 6 (12 points) Consider a discrete-time system defined by

$$y(n) - ay(n-1) = bx(n) + x(n-1)$$

where $x(n)$ is the input signal, $y(n)$ is the output signal, and a is a known real constant with $|a| < 1$. Let $H(\Omega)$ be the DTFT of the impulse response of the system and suppose that

$$|H(\Omega)| = 1 \qquad 0 \leq \Omega \leq 2\pi$$

a) Find b.

6

$$Y(z) - aY(z)z^{-1} = bX(z) + X(z)z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b + z^{-1}}{1 - az^{-1}}$$

We showed $\dfrac{z^{-1} - a}{1 - az^{-1}}$ is allpass.

Therefore, $b = -a$

b) Find the phase response of the system.

6

For $H(z) = \dfrac{-a + z^{-1}}{1 - az^{-1}}$

We showed $\theta(\Omega) = -\Omega - 2\arctan\left[\dfrac{a\sin\Omega}{1 - a\cos\Omega}\right]$

Question 7 (12 points) Suppose that a continuous-time signal $x(t)$ has the Fourier transform $X(\omega)$ given by

$$X(\omega) = \begin{cases} (\omega + 12\pi)/8\pi & -12\pi \le \omega \le -4\pi \\ 1 & -4\pi \le \omega \le 8\pi \\ 0 & \text{otherwise} \end{cases}$$

a) Plot $X(\omega)$.

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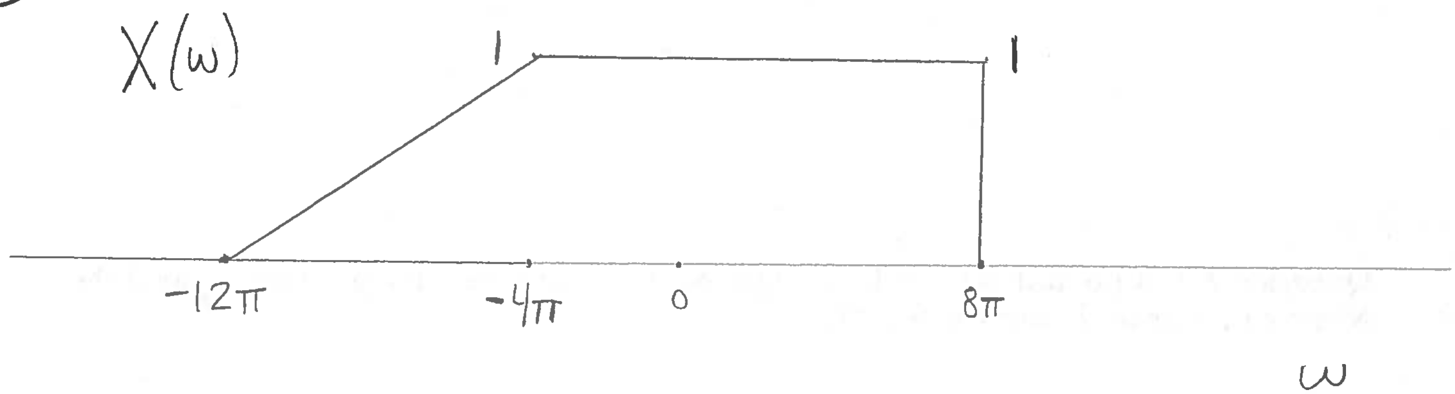

b) Define

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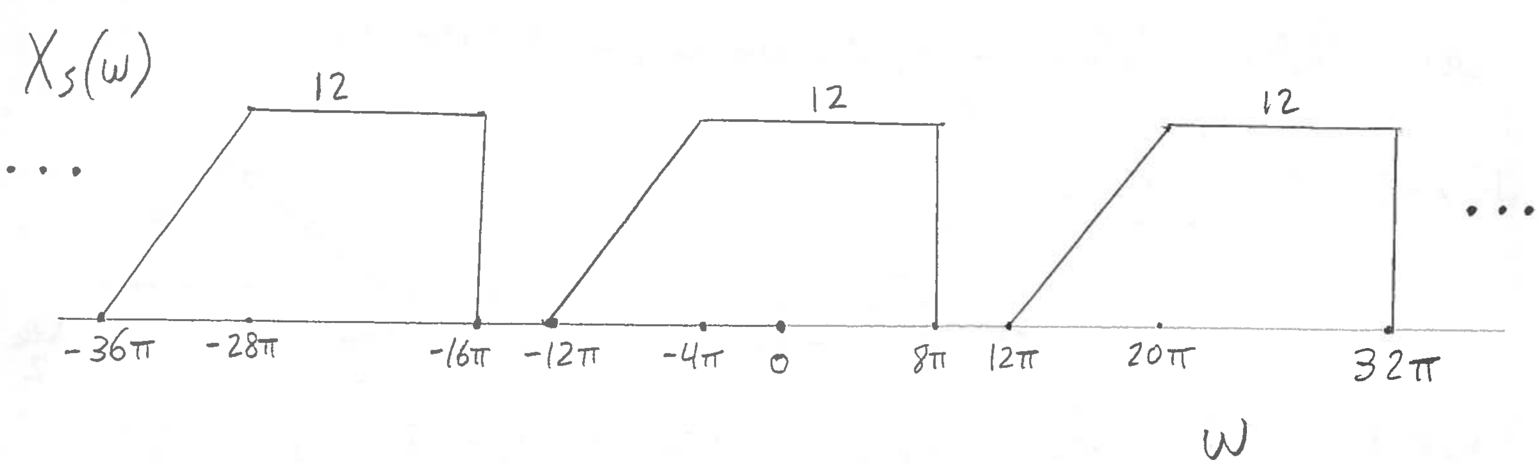
$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{n}{12}\right)$$

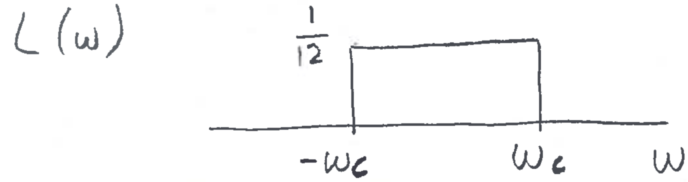
$T = \frac{1}{12}$

$\omega_s = \frac{2\pi}{T} = 24\pi$

Plot the Fourier transform $X_s(\omega)$ of $x_s(t)$.

$$X_s(\omega) = \frac{1}{T} \sum_{K=-\infty}^{\infty} X(\omega - K\omega_s) = 12 \sum_{K=-\infty}^{\infty} X(\omega - 24\pi K)$$





c) Consider an ideal low-pass filter that is defined in the frequency domain by

$$L(\omega) = \begin{cases} (1/12) & |\omega| \le \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

Find the range of values of ω_c for which this filter will recover $x(t)$ from $x_s(t)$ or state that no such range exists.

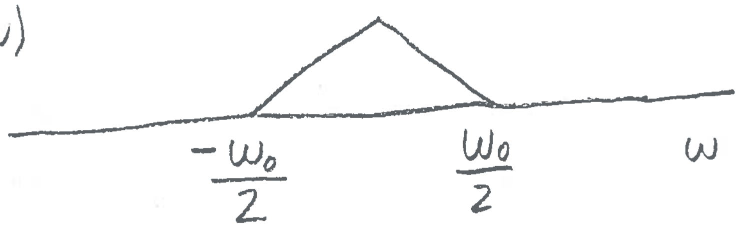
4 Can only use the single value $W_c = 12\pi$

Question 8 (10 points) Let $x(t)$ be a continuous-time signal with Nyquist rate ω_0. Find the Nyquist rate for the continuous-time signal

$$y(t) = x(t)\cos(\omega_0 t)$$

$W_0 = 2\omega_{max}$ where $X(\omega) = 0$ for $|\omega| \ge \omega_{max} = \frac{\omega_0}{2}$

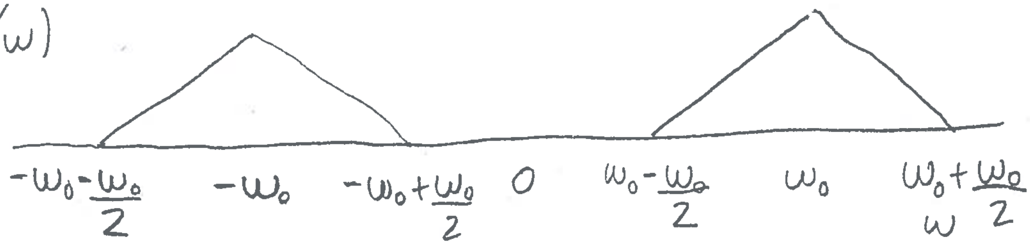
For example, $X(\omega)$



$$Y(\omega) = \mathcal{F}\left[x(t)\cos(\omega_0 t)\right] = \frac{1}{2\pi}\left[X(\omega) * \left[\pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)\right]\right]$$

Using (R4) $\quad Y(\omega) = \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$ 5

For the triangle, $Y(\omega)$



W_{max} for $y(t)$ is $\frac{3\omega_0}{2}$. Nyquist rate for $Y(\omega)$ is $3\omega_0$ 5