Advanced Micro II

Homework 1

Due: 1/12/2018

1. Prove the following: If a preference relation $\succeq$ over ΔC satisfies the independence axiom, then for all $\alpha \in (0,1)$ and $F, G, H \in \Delta C$ we have
   (a) $F \succ G \iff \alpha F + (1-\alpha)H \succ \alpha G + (1-\alpha)H$
   (b) $F \sim G \iff \alpha F + (1-\alpha)H \sim \alpha G + (1-\alpha)H$

2. Assume utility depends only on consumption c and there are two possible states of the world 1 and 2. π_1 is the probability of state 1. Which of the following utility functions have the expected utility form:
   (a) $u(c_1, c_2, \pi_1, \pi_2) = a(\pi_1 c_1 + \pi_2 c_2)$
   (b) $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1 + \pi_2 c_2^2$
   (c) $u(c_1, c_2, \pi_1, \pi_2) = \pi_1 \ln(c_1) + \pi_2 \ln(c_2) + 17$

3. An asset is a divisible claim to a financial return in the future. Suppose that there are two assets, a safe asset with a return of 1 dollar per dollar invested and a risky asset with a random return of z dollars per dollar invested. The random return z has a distribution function $F(z)$.
   (a) Assume that F satisfies $\int z dF(z) > 1$. Interpret this condition.
   (b) An individual has initial wealth w to invest, which can be divided in any way between the two assets. Let α and β denote the amounts of wealth invested in the risky and the safe asset, respectively. The individual's portfolio (α, β) pays $\alpha z + \beta$. Write the utility maximization problem of the individual who must choose α^* and β^* given wealth w and the distribution $F(z)$ of the asset's returns.
   (c) Show that regardless of $F(z)$, $\alpha^* = 0$ cannot be a solution. Interpret this result.

4. Consider the following table which summarizes 4 lotteries over prizes of \$0, \$1 million, and \$5 million. The entries in the table give the probability of each outcome under the lottery indicated. For example, lottery A gives \$0 with probability 0, \$1 million with probability 1, and \$5 million with probability 0. So it is a lottery which pays 1 million dollars for sure.

Lottery	\$0	\$1*million*	\$5*million*
A	0	1	0
B	.01	.89	.1
C	.89	.11	0
D	.9	0	.1

   Faced with a choice between lotteries A and B, Rick would choose A; the 1% risk of getting nothing and thereby "losing" the \$1 million makes it seem like the obvious choice for him. However, faced with the choice between lotteries C and D, Rick would choose D. He figures he probably will get nothing anyway and five million is so much more than one million, the extra risk is worth it.

   (a) Assuming Rick's preferences are expected utility preferences, use the fact that $A \succ B$ to derive an expression relating $u(1million)$, $u(0)$ and $u(5million)$ where u is the Bernoulli utility function underlying Rick's expected utility function.

(b) Assuming Rick's preferences are expected utility preferences, use the fact that $D \succ C$ to derive an expression relating $u(1million)$, $u(0)$ and $u(5million)$ where u is the Bernoulli utility function underlying Rick's expected utility function.

(c) Show that (a) and (a) cannot both be true.

(d) Given what you showed in (c), do Rick's preferences satisfy completeness, transitivity, continuity, and independence? Why or why not?