Parameter estimation – Homework 4

Deadline: 16th of April, 11:59:59 pm

Solutions should be submitted via Moodle

Task

Let us consider the following simple ARX system:

$$y(k) = \theta_1 y(k-1) + \theta_2 u(k-1) + e(k);$$

1. Let the simulation time horizon contain $N = 1000$ samples. Let $e(k)$ be a Gaussian white noise process with zero mean such that its variance changes at $k = 500$, i.e. $\sigma_e^2(k) = \sigma_1^2$ for $k = 1, \ldots, 499$, and $\sigma_e^2(k) = \sigma_2^2$ for $k = 500, \ldots, 1000$ such that $\sigma_1^2 \neq \sigma_2^2$. Choose arbitrary "true" parameter values θ_1 and θ_2. Also choose σ_1 and σ_2. Generate and fix (store) an input signal sequence $u(k)$ which is a stationary process (e.g., white noise, too). Simulate the system for $N = 1000$ samples with the input signal.

2. Implement a function called $log\_likelihood(u, y, \theta_1, \theta_2)$ for computing the log-likelihood of the joint probability density of the samples. Don't forget that the variance of $e(k)$ (and therefore the likelihood function) changes at $k = 500$.

3. Plot the log-likelihood surface as a function of θ_1 and θ_2 on an appropriate domain.

4. Estimate the parameters θ_1 and θ_2 using the maximum likelihood estimation.

5. Run 100 simulations of the model using the same $\theta_1, \theta_2, \sigma_1^2, \sigma_2^2$ and $u(k)$, but re-generate $e(k)$ (as written in point 1 above) for each simulation run. Perform a maximum likelihood estimation for each run and store the estimated parameter values. Also perform a simple least squares estimation for each run and store the estimated parameters. Compute and compare the covariance matrices of the estimates obtained by the two methods.