Parameter estimation – Homework 3

Deadline: 2nd of April, 11:59:59 pm

Solutions should be submitted via Moodle

Task 1

Consider the ARMAX model from Homework 1:

$$y(k) = 1.5y(k-1) - 0.7y(k-2) - 0.25u(k-1) + 1.64u(k-2) - 1.6\epsilon(k-1) + 0.75\epsilon(k-2),$$

where u is the input, and ϵ is normally distributed noise with expected value 0 and $\sigma = 1.2$.

You should already have the following from Homework 1 (if not, do it now):

1. Load the input into the workspace from file:
   https://users.itk.ppke.hu/ csuba/parameterestimation/paramest_hw1_task1.mat
2. Calculate the system output y for N=100 timesteps (you can increase this number if you feel it necessary).

Using these results:

1. Estimate the parameters of the system using LSQ, using the know u and the calculated y values. Are your results similar to the real world parameters? (ie. 1.5, -0.7, -0.25, 1.64)
2. Simulate the system using the estimated parameters (without noise). Plot the output alongside the results from Homework 1.

Task 2

One of the simplest epidemiological model is the SIR (Susceptible-Infected-Recovered) model with three state variables: $S(t)$ represents the number of susceptibles who can get ill if they meet an infected individual, $I(t)$ denotes the number of infected and $R(t)$ is the number of recovered. (Note that in a continuous model the state variables can be interpreted as population densities rather than integer numbers.) The dynamical behavior of the system is quantified by the following system of ordinary differential equations:

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$
$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

Where β is the transmission rate from susceptible state to infected and γ is the recovery rate.

1. Discretize the model using the Euler method, ie.

$$\frac{dx(t)}{dt} \approx \frac{x(k+1) - x(k)}{t_s} \quad \Rightarrow \quad x_{k+1} = x_k + t_s \cdot \frac{dx(t)}{dt}$$

   Use sampling time $t_s = 0.01$.

2. To get an idea of how an epidemic curve should look, simulate the system using parameters $\beta = 1.1$ and $\gamma = 0.4$. You can use initial values $S(0) = 0.9999$, $I(0) = 0.0001$, $R(0) = 0$. (Here the value 1 means the total population).

3. Using the file *sir.csv* containing time series data of equidistant samples ($t_s = 0.01$) of S, I and R, try to estimate the parameters β and γ. (Note: they are **not** the same as in the previous excersise).