Parameter estimation – Homework 2

Deadline: 26th of March, 23:59:59

Solutions should be submitted via Moodle

Task 1

Implement the Least Squares (LSQ) estimator function, which given the matrix of inputs $X \in \mathbf{R}^{n \times p}$ and the vector of outcomes $Y \in \mathbf{R}^n$ estimates the parameter vector $\theta \in \mathbf{R}^p$ (where n is the number of measurements and p is the number of parameters). Use the formula derived at the lecture:

$$\hat{\theta}_{LS} = (X^\top X)^{-1} X^\top Y$$

Task 2

Test the LSQ estimator function created in Task 1.

1. Generate n data points using the equation: $y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$. (Note: you can rewrite the equation as a vector product: $y = (1\ x_1\ x_2) \cdot (\theta_0\ \theta_1\ \theta_2)^\top = \mathbf{x} \cdot \theta$). You can choose arbitrary nonzero, distinct θ values.
2. Plot the generated data points (this should be a 3D plot).
3. Use least square estimation to estimate the parameters. You should get something similar to your values chosen in 1.
4. Calculate the loss function $L(\hat{\theta}_{LS}) = (Y - X\hat{\theta}_{LS})^\top (Y - X\hat{\theta}_{LS})$ for your estimation.
5. Show how the loss function depends on the amplitude of the noise. Run steps 1-4 for different σ_ϵ values, and plot the results.
6. Show how the loss function depends on the sample size. Run steps 1-4 for different n values and plot the results.
7. Create a 3D plot from the computations 5-6, showing the loss in function of the noise amplitude and sample size.