Natematilia G2 - GY10

1. feladat
$$f(x_iy) = \begin{cases} \frac{2xy}{x^2+y^2} & x^2+y^2 > 0 \\ x^2+y^2 & x^2+y^2 = 0 \end{cases}$$

[Natematilia G2 - GY10]

FAtviteli elv.: 
$$5: \mathbb{R}^n \rightarrow \mathbb{R}^k$$
,  $\lim_{x \to a} f(x) = A$ , ha  $\forall x \rightarrow a:$   $f(x \rightarrow a) \rightarrow A$ 

$$\sum_{n} = (1/n; 1/n) = 1$$

•! 
$$x_n = (1/n; 1/n) =$$
  $\lim_{n \to \infty} \frac{2 \cdot \frac{1}{n} \cdot \frac{1}{n}}{\frac{1}{n^2} + \frac{1}{n^2}} = \frac{2}{2} = 1$   
•!  $x_n = (1/n; 1/n) =$   $\lim_{n \to \infty} \frac{2 \cdot \frac{1}{n} \cdot \frac{1}{n}}{\frac{1}{n^2} + \frac{1}{n^2}} = \frac{2}{20}$ 

$$| \times_{\overline{n}}^{(2/n)} | = 0$$

$$0(x_{i}y) = \frac{x-y}{x-y}$$

$$g(x,y) = \frac{x-y}{x+y}$$

$$g(x_i y) = \frac{\lambda}{x + y}$$
ilonboző görbe m

TKülönböző görbe mentén odatartás 
$$J$$
 $x=y$  egyenes:  $\lim_{x\to x} \frac{x-x}{x+x} = \lim_{x\to 0} \frac{0}{2x} = \lim_{x\to 0} \frac{0}{2x}$ 

2. Seradat

 $\lim_{x\to 3^+} \frac{x^{\frac{1}{x-3}-1}}{\frac{1}{x-3}+1}$ 

$$x=y$$
 egyenes:  $\lim_{x\to 0} x+x - \lim_{x\to 0} 2x = 2$   
 $y=0$  egyenes:  $\lim_{x\to 0} x = 1$ 

e mentén odatartás J
$$\lim_{x\to 0} \frac{x-x}{x+x} = \lim_{x\to 0} \frac{0}{2x} = \frac{0}{2} = 0$$

 $\lim_{x\to 3} \frac{xy-1}{y+1} = 3$   $y = \frac{1}{x-3}$  , ha  $x\to 3^+$ :  $y\to \infty$   $y\to \infty$ 

 $- = \lim_{x \to 3^+} \frac{x - x + 3}{1 + x - 3} = \frac{3}{1} = 3$ 

Ell:  $\left| \frac{xy-1}{y+1} - 3 \right| = \left| \frac{xy-1-3y-3}{y+1} \right| = \left| \frac{(x-3)y-4}{y+1} \right| < \left| \frac{(x-3)y}{y} \right| = \left| \frac{x-3}{y} < \epsilon \right|$ 

$$\lim_{X\to 0} \frac{x+y+2z}{x-z+xy}$$

$$\lim_{Z\to 0} \lim_{X\to 0} \lim_{Y\to 0} \lim_{X\to 0}$$

e lim lim  $\frac{x+y+2z}{x-z+xy} = \lim_{z\to 0} \frac{2z}{-z} = -2$ 

•  $\lim_{x \to 0} \lim_{y \to 0} \lim_{z \to 0} \frac{x + y + 2z}{x - z + xy} = \lim_{x \to 0} \frac{x}{x} = 1$ 

The konvergens, az origóba tartas sorrendje mindenu lenne. mindegy lenne.

3. feladat  $f(x_iy) = \frac{x^2y^2}{x^2+y^2} \quad x_0 = 0$ 

b,  $\lim_{x\to 0} x \cos^2 y \sim \text{rend\'or elv} = > 0$   $\lim_{y\to \infty} x \cos^2 y \sim \text{rend\'or elv} = > 0$ 

 $\lim_{r \to 0} \frac{r_n^4 \cos^2 f_n \sin^2 f_n}{r_n^2} = 0$ 

 $g(x,y) = \frac{x^3y}{x^6 + y^2} \quad x_0 = 0 \quad y = mx^k$ 

 $\lim_{x\to 0} \frac{mx^6}{x^6 + m^2x^6} = \lim_{x\to 0} \frac{m}{1 + m^2}$ 

4. feradat

a,  $\lim_{\substack{x \to \infty \\ y \to \infty}} \frac{x+y}{x^2-xy+y^2} = \lim_{\substack{x \to \infty \\ xy \to \infty}} \frac{\frac{x}{xy} + \frac{y}{xy}}{\frac{x^2}{xy} - 1 + \frac{y}{xy}} = \lim_{\substack{x \to \infty \\ xy \to \infty}} \frac{\frac{x}{xy} + \frac{y}{xy}}{\frac{x}{xy} - 1 + \frac{y}{xy}} = 0$ 

x = rn cos Pn

y=rn sin In

 $y = mx^3$ 

C, 
$$\lim_{x \to 0} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 4y^2}} = \lim_{x \to 0} \sqrt{x^2 + y^2 + 4y^2 + 4y^2} = \lim_{x^2 + y^2 + 4y^2 + 4y^2 + 4y^2 + 4y^2} = \lim_{x \to \infty} (1 + 1/x)^{\frac{x^2}{x^2 + y^2}} = \lim_{x \to \infty} (1 + 1/x)^{\frac{x^2}{x^2 + y^2}} = e$$

J. Schalace
$$f(x;y) = x^{2} - 2xy - 4y^{2} \quad P(1;-1) \quad \nu(1;-1)$$

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$$f(x;y) = x^{2} - 2xy - 4yy - 4y$$

5. feladat

$$\lim_{x \to 0} \frac{(x+\lambda)^2 - 2(x+\lambda)(y-\lambda) - 4(y-\lambda)^2 - x^2 + 2xy + 4y}{2}$$

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$$\frac{1}{\lambda \rightarrow 0} \frac{1}{\lambda} \sim \text{iranyment: derivalt}$$

$$\frac{1}{\lambda \rightarrow 0} \sim \frac{1}{\lambda} \left( \frac{1}{\lambda} + \frac{1}{\lambda} \right)^2 - \frac{1}{\lambda^2} + \frac{1}{\lambda} + \frac{1}$$

6. feladat  

$$f(x;y) = x^3 - 5x^2y + 3xy^2 - 12y^3 + 5x - 6y + 7$$

$$\frac{0f}{0x} = 3x^2 - 10xy + 3y^2 + 5$$

$$\frac{0f}{0x} = -5x^2 + 6xy - 36y^2 - 6$$

$$\frac{\partial f}{\partial y} = -5x^2 + 6xy - 36y^2 - 6$$

$$\frac{\partial^2 f}{\partial x^2} = 6x - 10$$

$$\frac{\partial^2 f}{\partial x \partial y} = -10x + 6y$$

$$\frac{\partial^2 f}{\partial y^2} = 6x - 72y$$

$$\frac{\partial^2 f}{\partial y \partial x} = -10x + 6y$$

$$g(x_iy) = x^y$$

$$\frac{\partial f}{\partial x} = y^{y-1}$$

$$\frac{\partial f}{\partial y} = x^y \ln x$$

$$\frac{\partial^{2} f}{\partial x} = y^{x^{3}} \qquad \frac{\partial^{3} f}{\partial y} = x^{3} \ln x$$

$$\frac{\partial^{2} f}{\partial x^{2}} = y(y-\ell) \times y^{-2} \qquad \frac{\partial^{2} f}{\partial y^{2}} = x^{3} \ln^{2} x$$

$$\frac{\partial^{2} f}{\partial x^{2}} = y^{-1} (1+i\gamma \ln x) \qquad \frac{\partial^{2} f}{\partial x^{2}} = y^{-1} \ln x + x^{3}$$

$$\frac{\partial^{2}f}{\partial x^{2}} = y(y-1) \times y^{-2} \qquad \frac{\partial^{2}f}{\partial y^{2}} = x^{y} \ln^{2}x$$

$$\frac{\partial^{2}f}{\partial x^{2}} = x^{y-1} \left(1 + y \ln x\right) \qquad \frac{\partial^{2}f}{\partial y \partial x} = y x^{y-1} \ln x + x^{y} \frac{1}{x}$$

$$\frac{\partial^2 S}{\partial x^2} = y(y-1) \times 3$$

$$\frac{\partial^2 S}{\partial y^2} = x^{y-1}(1+y^2nx)$$

$$\frac{\partial^2 S}{\partial y \partial x} = y^{y-1}2nx + x^{y-1}x$$

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$$h(x) = e^{x^{2}y} - 2x^{2}y^{3} \sin(2nx+y)$$

$$\frac{\partial h}{\partial x} = 2xye^{x^{2}y} - 4xy^{3} \sin(...) - 2x^{2}y^{3} \cos(...) \frac{1}{x}$$

$$h(x) = e^{xy} - 2x^{2}y^{3} \sin(2nx+y)$$

$$\frac{\partial h}{\partial x} = 2xye^{x^{2}y} - 4xy^{3} \sin(...) - 2x^{2}y^{3} \cos(...) + \frac{\partial h}{\partial y} = x^{2}e^{x^{2}y} - 6x^{2}y^{3} \sin(...) - 2x^{2}y^{3} \cos(...)$$

7. Seladat
$$f(x;y) = x \ln(x+y)$$

$$P(-2;3) \quad \text{grad flp} = ?$$

$$P(-2;3)$$

$$\frac{\partial f}{\partial x} = 1$$

$$P(-2.3)$$
 grad  $f|_{p} = \frac{1}{2}$   
 $\frac{\partial f}{\partial x} = \ln(x+y) + \frac{x}{x+y} = \frac{p}{p} > \ln 1 + \frac{-2}{1} = 0 - 2 = -2$ 

$$\frac{\partial f}{\partial y} = \frac{x}{x+y}$$

$$\operatorname{grad} f | p = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2} (x^2)$$

05 = 1

$$\frac{\partial f}{\partial x} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \cdot 2x = 3 - \frac{1}{5} \cdot 3 = -\frac{3}{5}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \cdot 2y = 3 - \frac{1}{5} (-4) = 4|5$$

$$\frac{\partial f}{\partial x} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \cdot 2x = 3 - \frac{1}{5} \cdot 3 = \frac{-3}{5}$$

grad  $g|_p = \begin{bmatrix} -3|5\\ 4|5 \end{bmatrix}$ 

$$\rangle = Z - \sqrt{x^2 + y^2}$$

$$z = z - \sqrt{x^2 + y^2}$$

$$g(x_iy_iz) = z - (x^2+y^2)$$
  
 $P(3_i-4_i7)$  grad glp=?

8. feradat  

$$f(x_i y) = 3x^2 - 4xy + 2x + y^2 + 1$$
  
 $\frac{\partial f}{\partial x} = 6x - 4y + 2 = 0$   
 $\frac{\partial f}{\partial y} = -4x + 2y = 0$   
 $-2x + 2 = 0$   $x = +1$   
 $x = +2$  =>  $P = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$