definitionhidealllines=true,leftline=true,linewidth=3pt,linecolor=primaryColor,frametitlerule=true,frametitlebackgroundcolor=primaryColor,backgroundcolor=gray!10, frametitleaboveskip=2mm, frametitlebelowskip=2mm, innertopmargin=3mm,

definitionsection

theoremhidealllines=true,leftline=true,linewidth=3pt,linecolor=secondaryColor,frametitlerule=true,frametitlebackgroundcolor=secondaryColor,backgroundcolor=gray!10, frametitleaboveskip=2mm, frametitlebelowskip=2mm, innertopmargin=3mm,

theoremsection

blueBoxhidealllines=true,leftline=true,backgroundcolor=cyan!10,linecolor=secondaryColor,linewidth= nertopmargin=.66em,innerbottommargin=.66em,

notehidealllines=true,leftline=true,backgroundcolor=yellow!10,linecolor=ternaryColor,linewidth=3pt, nertopmargin=.66em,innerbottommargin=.66em,

statementhidealllines=true,leftline=true,backgroundcolor=primaryColor!10,linecolor=primaryColor,linewidth=3pt,innertopmargin=.66em,innerbottommargin=.66em,singleextra= let 1=(P), 2=(O) in $((2,0)+0.5*(0,1))$ node[rectangle, fill=primaryColor!10, draw=primaryColor, line width=2pt, overlay,] primaryColor**!**; ,

learnMoreTitle==Kitekintő calc,arrows,backgrounds excursus arrow/.style=line width=2pt, draw=secondaryColor, rounded corners=1ex, , excursus head/.style= font=**, anchor=base west, text=secondaryColor, inner sep=1.5ex, inner ysep=1ex,** ,

learnMoresingleextra=let 1=(P), 2=(O) in (2,1) coordinate (Q); let 1=(Q), 2=(O) in (1,2) coordinate (BL); let 1=(Q), 2=(P) in (2,1) coordinate (TR); [excursus head] (A) at $((Q) + (2.5em, 0))$; [excursus arrow, line width=2pt] $((BL) + (1pt, 0))$ |- $((Q) + (2em, 0))$; [excursus arrow, line width=2pt, fill=gray!10, -to] $((Q)+(1em, 0))$ -| (A.north west) -| (A.base east) – (TR) ; [excursus head] (A) at $((Q) + (2.5em, 0))$; , backgroundcolor=gray!10, middlelinewidth=0, hidealllines=true,topline=true, innertopmargin=2.5ex, innerbottommargin=1.5ex, innerrightmargin=2ex, innerleftmargin=2ex, skipabove=0.5nobreak=true,

examplehidealllines=true, leftline=true, backgroundcolor=magenta!10, linecolor=magenta!60!black, linewidth=3pt, innertopmargin=.66em, innerbottommargin=.66em,

Többváltozós analízis BMETE94BG02 13

ft bg bg,main,ft

Matematika G2

Integrálás I

Utoljára frissítve: 2025. április 22.

0.1 Elméleti Áttekintő

[style=blueBox, nobreak=true,] **Integrálás téglatartományon:**

Téglatartomány esetén az integrálás sorrendje tetszőleges.

[thick, scale=5/4] [draw = primaryColor, -to] (-0.35,0) – (3.35,0) node[below] x; [draw = primaryColor, -to] (0,-0.35) – (0,2.35) node[left] y;
[draw=secondaryColor, pattern = north east lines, pattern color = secondaryColor,] (.75,.5) rectangle (2.75,1.75);
[draw=ternaryColor, dashed] (.75,.5) – (0,.5) node[left] y_1 (.75,1.75) – (0,1.75) node[left] y_2 (.75,.5) – (.75,0) node[below] x_1 (2.75,.5) – (2.75,0) node[below] x_2 ;

$$\mathrm{I} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x;y) yx = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f(x;y) xy$$

[style=blueBox, nobreak=true,] **Integrálás normáltartományon:**

[thick, scale=5/4] [draw = primaryColor, -to] (-0.35,0) – (3.35,0) node[below] x; [draw = primaryColor, -to] (0,-0.35) – (0,2.35) node[left] y; [draw=ternaryColor, dashed] (.75,2) – (.75,0) node[below] x_1 (2.75,2) – (2.75,0) node[below] x_2 ; [draw = secondaryColor, name path = f1] plot[domain=0.75:2.75, smooth, samples=150] (, 0.25+0.125*sin(540*)+*0.125) node[right] $g_1(x)$; [draw = secondaryColor,name path = f2] plot[domain=0.75:2.75, smooth, samples=150] (, 1.75+0.125*sin(540*)-*0.125) node[right] $g_2(x)$; [of=f1 and f2, on layer = bg,] pattern = north east lines, pattern color = secondaryColor,

$$\mathrm{I} = \int^{x_2}_{x_1} \int\limits_{g_1(x)}^{g_2(x)} f(x;y)yx = \int\limits_{y_1}^{y_2} \int\limits_{g_1^{-1}(y)}^{g_2^{-1}(y)} f(x;y)xy$$

[thick, scale=5/4] [draw = primaryColor, -to] (-0.35,0) – (3.35,0) node[below] x; [draw = primaryColor, -to] (0,-0.35) – (0,2.35) node[left] y; [draw=ternaryColor, dashed] (3,.5) – (0,.5) node[left] y_1 (3,1.75) – (0,1.75) node[left] y_2 ; [draw = secondaryColor, name path = f1] plot[domain=1.75:0.5, smooth, samples=150] (0.75+0.125*sin(540*)+*0.125,) ; [above] at (1,1.75) $g_1(y)$; [draw = secondaryColor,name path = f2] plot[domain=0.5:1.75, smooth, samples=150] (2.75+0.125*sin(540*)-*0.125,) node[above] $g_2(y)$; [of=f1 and f2, on layer = bg,] pattern = north east lines, pattern color = secondaryColor,

$$\mathrm{I} = \int^{y_2}_{y_1} \int\limits_{g_1(y)}^{g_2(y)} f(x;y)xy = \int\limits_{x_1}^{x_2} \int\limits_{g_1^{-1}(x)}^{g_2^{-1}(x)} f(x;y)yx$$

0.2 Feladatok

1. Számolja ki az alábbi függvények integrálját a megadott tégla tartományokon!

   a) $f(x;y) = 2x^2 + 3xy + 4y^2$ $\quad 1 \leq x \leq 2 \quad 0 \leq y \leq 3$

   b) $g(x;y) = xy\sin(x^2 + y^2)$ $\quad 0 \leq x \leq \pi 2 \quad 0 \leq y \leq \pi 2$

   c) $h(x;y) = y\cos(2xy)$ $\quad 1 \leq x \leq 2 \quad 1 \leq y \leq 3$

2. Határozza meg az alábbi hármasintergrált! $\int_1^2 \int_0^3 \int_0^1 z\, x\, \sqrt{x^2+y} xyz$

3. Határozza meg az alábbi integrálokat, majd írja fel az integrációs határokat, ha először az x, majd a y változó szerint integrálnánk! 2

   a) $\displaystyle\int_0^1 \int_{x^2}^{\sqrt{x}} \left(x^2 + y^2\right) yx$

   b) $\displaystyle\int_1^3 \int_0^{1/x} (2y + x + 2)\, yx$

4. Határozza meg az alábbi felületi integrálok értékét!

a) ${}_T x^2 + y^2 T$

$T = \{(x;y) \mid 0 \leq y \leq 1 \ \wedge \ y \leq x \leq 3 - y\}$

[thick] [draw = primaryColor, -to] (-0.35,0) – (3.35,0) node[below] x; [draw = primaryColor, -to] (0,-0.85) – (0,1.85) node[left] y; [draw=ternaryColor, dashed] (1,1) – (0,1) node[left] 1 (1,1) – (1,0) node[below] 1 (2,1) – (2,0) node[below] 2 ; [below] at (3,0) 3; [below right] at (0,0) 0;
[draw = secondaryColor, pattern = north east lines, pattern color = secondaryColor,] (0,0) – (1,1) – (2,1) – (3,0) – cycle;
[fill = white, fill opacity=.5, text opacity=1] at (1.5,0.5) T;

b) $_{T}xyT$

$$T = \{(x;y) \mid x^2 + y^2 \leq R^2 \ \wedge \ x;y \geq 0\}$$

[thick] [draw = primaryColor, -to] (-1.85,0) – (1.85,0) node[below] x; [draw = primaryColor, -to] (0,-1.35) – (0,1.35) node[left] y; [ternaryColor] (0,0) circle (1); [draw = secondaryColor, pattern = north east lines, pattern color = secondaryColor,] (0,0) – (1,0) arc (0:90:1) – cycle; [fill = white, fill opacity=.5, text opacity=1] at (0.4,0.4) T; [above right=-.5mm] at (1,0) R; [above right=-.5mm] at (0,1) R;

c) $_{T}y\, e^{(x-1)^2}T$

$$T = \{(x;y) \mid x \leq 1 + y^2 \ \wedge \ y \geq 0 \ \wedge \ x \leq 5\}$$

[thick] [draw = primaryColor, -to] (-0.85,0) – (2.85,0) node[below] x; [draw = primaryColor, -to] (0,-0.85) – (0,1.85) node[left] y; [draw = secondaryColor, pattern = north east lines, pattern color = secondaryColor, scale=1/2,] plot[domain=0:2, smooth, samples=150] (1+*,) coordinate(T) |- (0,0); [fill = white, fill opacity=.5, text opacity=1] at (1.7,0.35) T; [below] at (2.5,0) 5; [below] at (0.5,0) 1; [draw=ternaryColor, dashed] (T) – ++(-2.5,0) node[left] 2;

5. Adja meg az integrációs intervallumokat, ha az alábbi felületeken kell integrálni:

   a) R sugarú körfelület,

   b) ha az $x = 0$, $y = x^2$ és $y = 2 - x$ görbék által határolt felület!

6. Adja meg a $f(x; y) = xy$ függvény a $P_1(1; 1)$, $P_2(4; 5)$ és $P_3(4; 2)$ pontok által meghatározott háromszög terület fölötti integrálját!