

1. feladat

$m \times n$ A	$\text{rg } \underline{\underline{A}}$	$\text{rg}(\underline{\underline{A}} b)$	
$M_{4 \times 3}$	3	4	$\sim$ nincs mo. ( $\text{rg } \underline{\underline{A}} + \text{rg}(\underline{\underline{A}} b) = 7$ )
$M_{3 \times 3}$	2	2	$\sim \infty$ mo $3-2=1$ paraméter
$M_{6 \times 8}$	6	6	$\sim \infty$ mo $8-6=2$ paraméter
$M_{8 \times 6}$	6	6	$\sim$ 1 mo
$M_{3 \times 2}$	0	0	$\sim \infty$ mo $2-0=2$ paraméter

 $\underline{\underline{A}}$  nullmátrix2. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\underline{\underline{b}} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 1 & 4 & 3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-S_1} \left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array} \right] \xrightarrow{-3S_2 - 3S_3} \left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array} \right]$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array} \right] \xleftarrow{\quad}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right]$$

$$\underline{x} = \underline{\underline{A}}^{-1} \underline{\underline{b}} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 7 \cdot 3 - 3 \cdot 6 - 3 \cdot 9 \\ -3 + 9 \\ -3 + 6 \end{bmatrix} = \begin{bmatrix} -24 \\ 6 \\ 3 \end{bmatrix}$$

### 3. se2adat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad \underline{\underline{Ax}} = \underline{\underline{B}} + 2\underline{x}$$

$$\underline{\underline{Ax}} - 2\underline{x} = \underline{\underline{B}}$$

$$(\underline{\underline{A}} - 2\underline{\underline{E}})\underline{x} = \underline{\underline{B}}$$

$$\underline{x} = (\underline{\underline{A}} - 2\underline{\underline{E}})^{-1} \underline{\underline{B}}$$

$$\underline{\underline{A}} - 2\underline{\underline{E}} = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix} = \underline{\underline{A}}^*$$

$$\left. \begin{array}{l} \det \underline{\underline{A}}^* = -1 \cdot 1 - (-2) \cdot 2 = 3 \\ \text{adj } \underline{\underline{A}}^* = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \end{array} \right\} \underline{\underline{A}}^{*-1} = \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$\underline{x} = \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5 & -2 \\ 10 & -1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

### 4. se2adat

$$\underline{\underline{A}} = \begin{bmatrix} 2 & -3 & 1 \\ -3 & 4 & -2 \\ 5 & 0 & 4 \end{bmatrix} \quad \det \underline{\underline{A}} = 5 \begin{vmatrix} -3 & 1 \\ 4 & -2 \end{vmatrix} + 4 \begin{vmatrix} 2 & -3 \\ -3 & 4 \end{vmatrix} = 5 \cdot 2 + 4 \cdot (-1) = 6$$

$$\underline{\underline{b}} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \quad \underline{\underline{A}}_1 = \begin{bmatrix} 0 & -3 & 1 \\ 1 & 4 & -2 \\ -3 & 0 & 4 \end{bmatrix} \quad \underline{\underline{A}}_2 = \begin{bmatrix} 2 & 0 & 1 \\ -3 & 1 & -2 \\ 5 & -3 & 4 \end{bmatrix} \quad \underline{\underline{A}}_3 = \begin{bmatrix} 2 & -3 & 0 \\ -3 & 4 & 1 \\ 5 & 0 & -3 \end{bmatrix}$$

$$\left. \begin{array}{l} \det \underline{\underline{A}}_1 = 6 \\ \det \underline{\underline{A}}_2 = 0 \\ \det \underline{\underline{A}}_3 = -12 \end{array} \right\} \underline{x} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$x_1 = 6/6 = 1$

$x_2 = 0$

$x_3 = -12/6 = -2$

## 5. Seladat

$$\underline{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 5 \\ 1 \\ 1 \\ -5 \end{bmatrix}$$

$$\underline{A} \underline{x} = \underline{b} \quad x_1 - x_2 = ?$$

matrixcalc :)

$$\left[ \begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 1 \\ 3 & 4 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 & -5 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 13 & 213 & -6 \\ 0 & 1 & -4/3 & 5/3 & 3 \\ 0 & 0 & 1 & 15/12 & 3/4 \\ 0 & 0 & 10/3 & 3/3 & -10 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 4 & 2 & 14 \\ 1 & 2 & 1 & 7 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 1 & 7 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

## 6. feladat

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & -1 & 2 & 7 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 1 & 7 \end{array} \right] \xrightarrow{-3S_1} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -7 & 5 & 1 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 3 & 3 \end{array} \right] \xrightarrow{-3S_2} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -5 & -13 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 3 & 3 \end{array} \right] \xrightarrow{-2S_2} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & 10 & -30 \\ 0 & 0 & -12 & -36 \end{array} \right] \xrightarrow{+2S_2} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & 10 & -30 \\ 0 & 0 & -12 & -36 \end{array} \right] \xrightarrow{+3S_2} \left[ \begin{array}{ccc|c} 1 & 2 & -1 & 28 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & 10 & -30 \\ 0 & 0 & -12 & -36 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{+9S_3} \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right] \quad \begin{aligned} x_1 &= 1 \\ x_2 &= 2 \\ x_3 &= 3 \end{aligned}$$

## 7. feladat

$$\left[ \begin{array}{cccc|c} 5 & -1 & 2 & 1 & 7 \\ 2 & 1 & 4 & -2 & 1 \\ 1 & -3 & -6 & 5 & 0 \end{array} \right] \xrightarrow{+2S_2 - S_1} \left[ \begin{array}{cccc|c} \cdot & \cdot & \cdot & \cdot & 7 \\ \cdot & \cdot & \cdot & \cdot & 1 \\ 0 & 0 & 0 & 0 & 9 \end{array} \right]$$

$\text{rg } \underline{\underline{A}} < \text{rg } (\underline{\underline{A}} | \underline{b}) \Rightarrow \text{nincs mo.}$

## 8. feladat

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ -2 & -2 & -6 & -8 & 0 & 10 \\ 2 & 2 & -3 & -1 & 4 & 17 \\ -1 & -1 & 1 & 0 & 4 & 9 \\ 3 & 3 & 6 & 9 & 8 & 15 \end{array} \right] \xrightarrow{+2S_1} \left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & -4 & -4 & 2 & 14 \\ 0 & 0 & -5 & -5 & 1 & 13 \\ 0 & 0 & 2 & 2 & 5 & 11 \\ 0 & 0 & 3 & 3 & 5 & 9 \end{array} \right] \xrightarrow{-S_3} \left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 6 & 18 \\ 0 & 0 & 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array} \right]$$

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -5 & -5 & 1 & 13 \\ 0 & 0 & 2 & 2 & 5 & 11 \\ 0 & 0 & 3 & 3 & 5 & 9 \end{array} \right] \xrightarrow{+5S_2} \left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 6 & 18 \\ 0 & 0 & 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array} \right] \xrightarrow{+6} \left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 6 & 18 \\ 0 & 0 & 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array} \right]$$

$$\left[ \begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

paraméterek:  $x_4$  és  $x_5$

$$(3) \rightarrow x_5 = 3$$

$$(2) \rightarrow x_3 + x_4 + x_5 = 1$$

$$x_3 = 1 - x_4 - x_5 = -2 - x_4$$

$$(1) \rightarrow x_1 + x_2 + \underline{x_3 + 2x_4 + x_5} = 2$$

$$x_1 + x_2 + \underline{1 + x_4} = 2$$

$$x_1 = 1 - x_2 - x_4$$

### 9. feladat

$$\underline{A} = \left[ \begin{array}{ccc} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 1 & 7 \end{array} \right] - 2S_1$$

$$\left[ \begin{array}{ccc} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{array} \right] + 2S_2$$

$$\left[ \begin{array}{ccc} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$\text{rg } \underline{A} = 2 \neq 3 \Rightarrow$  nem csak triviális mo!

1 paraméter:  $x_3$

$$-7x_2 - x_3 = 0 \quad x_2 = -x_3/7$$

$$x_1 + 3x_2 + 2x_3 = 0 \quad x_1 = \frac{+3x_3}{7} - 2x_3 = \frac{-11x_3}{7}$$

### 10. feladat

$$\underline{A} = \left[ \begin{array}{ccc} 1 & 1 & -4 \\ 4 & 7 & 5 \\ 3 & 5 & 2 \\ 2 & 9 & 6 \end{array} \right] - 4S_1$$

$$\left[ \begin{array}{ccc} 1 & 1 & -4 \\ 0 & 3 & 21 \\ 0 & 2 & 14 \\ 0 & 7 & 14 \end{array} \right] \xrightarrow{13} \downarrow$$

$$\left[ \begin{array}{ccc} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 7 & 14 \\ 0 & 2 & 14 \end{array} \right] \xrightarrow{17} \downarrow$$

$$\left[ \begin{array}{ccc} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 1 & 7 \\ 0 & 1 & 2 \end{array} \right]$$

$$\left[ \begin{array}{ccc} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{array} \right]$$

$$\text{rg } \underline{A} = 3$$

↓

csak triviális

## 11. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & 6 \end{bmatrix}$$

$$\det \underline{\underline{A}} = 2 \cdot 5 \cdot 6 + 1(-7)4 + (-4)3(-5) - 4 \cdot 5(-4) - (-5)(-7)2 - 6 \cdot 1$$

$$= 60 - 28 + 60 + 80 - 70 - 18 = 84$$

$\Rightarrow \text{rg } \underline{\underline{A}} = 3 \Rightarrow$  csak triviális mű.

## 12. feladat

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & -3 & -2 \\ -3 & -5 & 4 & 1 \end{array} \middle| \begin{array}{c} 2 \\ 4 \\ G \end{array} \right] - 2S_1$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & -4 \\ 0 & 1 & 1 & 4 \end{array} \middle| \begin{array}{c} 2 \\ 0 \\ 6+G \end{array} \right] + S_2$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{array} \middle| \begin{array}{c} 2 \\ 0 \\ 6+G \end{array} \right]$$

Megoldható, ha  $G = -6$  ( $\text{rg } \underline{\underline{A}} = \text{rg } (\underline{\underline{A}} | b)$ )

$n = 4$  és  $\text{rg } \underline{\underline{A}} = 2 \Rightarrow 2$  paraméter

### 13. feladat

$$\operatorname{rg} \underline{\underline{A}} \leq 4 \text{ és } n = 5$$



Létezik nem triviális mo.

### 14. feladat

$$\left[ \begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 1 & u & 2 & 2 \\ 1 & 9 & -5 & v \end{array} \right]$$

$$\det \underline{\underline{A}} = -15u + 10 - 9 + u - 54 + 25 = \\ = -14u - 28$$

$$\det \underline{\underline{A}} = 0 \Leftrightarrow u = -2 \Rightarrow \operatorname{rg} \underline{\underline{A}} = 2$$

$$\det \underline{\underline{A}} \neq 0 \Leftrightarrow u \neq -2 \Rightarrow 1 \text{ db mo} \quad (\operatorname{rg} \underline{\underline{A}} = \operatorname{rg} (\underline{\underline{A}} | \underline{b}))$$

$$\text{Ha } u = -2$$

$$\left[ \begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 1 & -2 & 2 & 2 \\ 1 & 9 & -5 & v \end{array} \right] \xrightarrow[-S_1|3]{-S_1|3}$$

$$\left[ \begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 0 & \frac{-11}{3} & \frac{7}{3} & 5|3 \\ 0 & \frac{22}{3} & -\frac{14}{3} & v - \frac{1}{3} \end{array} \right] \xrightarrow[-S_2|3]{-S_3|3}$$

$$(-2) \frac{5}{3} = v - \frac{1}{3}$$



$$v = -3$$



$$\operatorname{rg} (\underline{\underline{A}} | \underline{b}) = 2$$



$\infty$  mo.

$$\text{Ha } v \neq -2 \Rightarrow \text{nincs mo}$$