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definitionsection

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theoremsection

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learnMoresingleextra=let 1=(P), 2=(O) in (2,1) coordinate (Q); let 1=(Q), 2=(O) in (1,2) coordinate (BL); let 1=(Q), 2=(P) in (2,1) coordinate (TR); [excursus head] (A) at ((Q) + (2.5em,0)) ; [excursus arrow, line width=2pt] ((BL) + (1pt,0)) |- ((Q) + (2em,0)); [excursus arrow, line width=2pt, fill=gray!10, -to] ((Q) + (1em,0)) -| (A.north west) -| (A.base east) - (TR) ; [excursus head] (A) at ((Q) + (2.5em,0)) ; , backgroundcolor=gray!10, middlelinewidth=0, hidealllines=true,topline=true, innertopmargin=2.5ex, innerbottommargin=1.5ex, innerrightmargin=2ex, innerleftmargin=2ex, skipabove=0.5nobreak=true,

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Sorok BMETE94BG01 14

Matematika G1

Numerikus sorok

Utoljára frissítve: 2024. november 11.

1. Bizonyítsa be a konvergencia definíciója alapján, hogy az alábbi sorok konvergenssek vagy divergenssek! 2

a) $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$

b) $\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

c) $\sum_{n=2}^{\infty} \ln \left(1 - \frac{1}{n^2} \right)$

d) $\sum_{n=1}^{\infty} \frac{a^n}{n^k}$

2. A Cauchy-féle konvergenciakritérium alapján bizonyítsa be, hogy az alábbi sor konvergens!

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + n^2 + 1}$$

3. Vizsgálja meg az alábbi sorok konvergenciáját! 3

a) $\sum_{n=1}^{\infty} \frac{2n^3 - 16}{n^5 + n}$

b) $\sum_{n=1}^{\infty} \frac{(\cos \pi 2)^n}{n^n + 1}$

c) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

$$\text{d)} \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$$

$$\text{e)} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$$

$$\text{f)} \sum_{n=1}^{\infty} \frac{2n^2}{(2+1n)^n}$$

$$\text{g)} \sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$$

$$\text{h)} \sum_{n=1}^{\infty} \frac{n}{e^n}$$

$$\text{i)} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$$

$$\text{j)} \sum_{n=1}^{\infty} \frac{\sin n}{\sqrt[3]{n^4}}$$

$$\text{k)} \sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n(n+1)}$$

$$\text{l)} \sum_{n=1}^{\infty} (-1)^n \frac{n-1}{n(n+1)}$$

4. Konvergens-e a $\sum a_n$ és $\sum b_n$ sorozata, ha

$$\sum a_n = \sum_{n=1}^{\infty} \frac{1+n}{3^n} \qquad \sum b_n = \sum_{n=1}^{\infty} \frac{(-1)^n - n}{3^n}.$$

5. Konvergens-e a $\sum a_n$ és $\sum b_n$ különbségsora, ha

$$\sum a_n = \sum_{n=1}^{\infty} \frac{1}{2n-1} \qquad \sum b_n = \sum_{n=1}^{\infty} \frac{1}{2n}.$$

6. Konvergens-e a $\sum a_n$ és $\sum b_n$ Cauchy-szorzata, ha

$$\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} \qquad \sum b_n = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}}.$$