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definitionsection

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theoremsection

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statementhidealllines=true,leftline=true,backgroundcolor=primaryColor!10,linecolor=primaryColor,linewidth=3pt,innertopmargin=.66em,innerbottommargin=.66em,singleextra= let 1=(P), 2=(O) in $((2,0)+0.5*(0,1))$ node[rectangle, fill=primaryColor!10, draw=primaryColor, line width=2pt, overlay,] primaryColor**!**; ,

learnMoreTitle==Kitekintő calc,arrows,backgrounds excursus arrow/.style=line width=2pt, draw=secondaryColor, rounded corners=1ex, , excursus head/.style= font=**, anchor=base west, text=secondaryColor, inner sep=1.5ex, inner ysep=1ex,** ,

learnMoresingleextra=let 1=(P), 2=(O) in (2,1) coordinate (Q); let 1=(Q), 2=(O) in (1,2) coordinate (BL); let 1=(Q), 2=(P) in (2,1) coordinate (TR); [excursus head] (A) at $((Q) + (2.5em, 0))$; [excursus arrow, line width=2pt] $((BL) + (1pt, 0))$ |- $((Q) + (2em, 0))$; [excursus arrow, line width=2pt, fill=gray!10, -to] $((Q)+(1em, 0))$ -| (A.north west) -| (A.base east) – (TR) ; [excursus head] (A) at $((Q) + (2.5em, 0))$; , backgroundcolor=gray!10, middlelinewidth=0, hidealllines=true,topline=true, innertopmargin=2.5ex, innerbottommargin=1.5ex, innerrightmargin=2ex, innerleftmargin=2ex, skipabove=0.5nobreak=true,

examplehidealllines=true, leftline=true, backgroundcolor=magenta!10, linecolor=magenta!60!black, linewidth=3pt, innertopmargin=.66em, innerbottommargin=.66em,

Sorok BMETE94BG01 14

Matematika G1

Numerikus sorok

Utoljára frissítve: 2024. november 11.

1. Bizonyítsa be a konvergencia definíciója alapján, hogy az alábbi sorok konvergensek vagy divergensek! 2

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

   c) $\displaystyle\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

   d) $\displaystyle\sum_{n=1}^{\infty} \frac{a^n}{n^k}$

2. A Cauchy-féle konvergenciakritérium alapján bizonyítsa be, hogy az alábbi sor konvergens!

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + n^2 + 1}$$

3. Vizsgálja meg az alábbi sorok konvergenciáját! 3

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3 - 16}{n^5 + n}$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{(\cos \pi 2)^n}{n^n + 1}$

   c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

d) $\displaystyle\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n}$

e) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n(n+1)}}$

f) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2}{(2+1n)^n}$

g) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$

h) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{e^n}$

i) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

j) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{\sqrt[3]{n^4}}$

k) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{n-1}{n(n+1)}$

l) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{n-1}{n(n+1)}$

4. Konvergens-e a $\sum a_n$ és $\sum b_n$ összegsora, ha

$$\sum a_n = \sum_{n=1}^{\infty}\frac{1+n}{3^n} \qquad \sum b_n = \sum_{n=1}^{\infty}\frac{(-1)^n-n}{3^n}.$$

5. Konvergens-e a $\sum a_n$ és $\sum b_n$ különbségsora, ha

$$\sum a_n = \sum_{n=1}^{\infty}\frac{1}{2n-1} \qquad \sum b_n = \sum_{n=1}^{\infty}\frac{1}{2n}.$$

6. Konvergens-e a $\sum a_n$ és $\sum b_n$ Cauchy-szorzata, ha

$$\sum a_n = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}} \qquad \sum b_n = \sum_{n=1}^{\infty}\frac{1}{2^{n-1}}.$$