Kinematics

Sarah Ziselman

Convert a Desired Twist to Wheel Velocities

Given $\dot{q}$, which represents a twist corresponding to a frame located at the body frame {b} but oriented at the world frame {w}, we want to find the wheel velocities u_L and u_R required to achieve that desired twist. First, we must use the following relationship between $\dot{q}$ and $\mathcal{V}_b$,

$$\begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

Multiplying the matrices above yields the following equations,

$$\omega_{bz} = \dot{\theta} \tag{1}$$

$$v_{bx} = \dot{x}\cos\theta + \dot{y}\sin\theta \tag{2}$$

$$v_{by} = \dot{y}\cos\theta - \dot{x}\sin\theta \tag{3}$$

Next, we use the following relation to get wheel speeds from a given twist.

$$u = H(0)\mathcal{V}_b$$

We need to solve for the H matrix. Let r represent the radius of the wheels and $2d$ represent the distance between wheels. We solve for $H(\theta)$ by using the following equation provided by the Modern Robotics textbook.

$$h_i(\theta) = \frac{1}{r\cos\gamma_i} \begin{bmatrix} x_i \sin(\beta_i + \gamma_i) - y_i \cos(\beta_i + \gamma_i) \\ \cos(\beta_i + \gamma_i + \theta) \\ \sin(\beta_i + \gamma_i + \theta) \end{bmatrix}^T$$

Since the differential drive robot uses conventional wheels, we know for each wheel that $\beta_i = 0$ and $\gamma_i = 0$. Therefore, the H matrix can be represented as follows,

$$H(\theta) = \begin{bmatrix} -\frac{d}{r} & \frac{\cos\theta}{r} & \frac{\sin\theta}{r} \\ \frac{d}{r} & \frac{\cos\theta}{r} & \frac{\sin\theta}{r} \end{bmatrix}$$

When we solve for $H(0)$, we get the following,

$$H(0) = \begin{bmatrix} -\frac{d}{r} & \frac{1}{r} & 0 \\ \frac{d}{r} & \frac{1}{r} & 0 \end{bmatrix}$$

We can now use the following relation to solve for wheel velocities,

$$\begin{bmatrix} u_L \\ u_R \end{bmatrix} = \begin{bmatrix} -\frac{d}{r} & \frac{1}{r} & 0 \\ \frac{d}{r} & \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} \tag{4}$$

Multiplying the matrices above and plugging in Equations 1, 2 and 3, we get the following for u_L and u_R

$$u_L = -\frac{d}{r}(\dot{\theta}) + \frac{1}{r}(\dot{x}\cos\theta + \dot{y}\sin\theta) \tag{5}$$

$$u_R = \frac{d}{r}(\dot{\theta}) + \frac{1}{r}(\dot{x}\cos\theta + \dot{y}\sin\theta) \tag{6}$$

Update Configuration Given Updated Wheel Angles

Let θ_L represent the angle of the left wheel and θ_R represent the angle of the right wheel. We want to be able to update the configuration of the robot given updated wheel angles assuming constant velocity between updates. We know that,

$$\Delta\theta_L = \theta_{L,new} - \theta_{L,old}$$
$$\Delta\theta_R = \theta_{R,new} - \theta_{R,old}$$

We can use the following equation provided from the Modern Robotics textbook to solve for $\mathcal{V}_b$.

$$\mathcal{V}_b = r \begin{bmatrix} -\frac{1}{2d} & \frac{1}{2d} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix}$$

Now that we have solved for the twist that results from the updated wheel angles, we can now integrate it to find $T_{bb'}$ where $\{b\}$ is the original body frame and $\{b'\}$ is the body frame after the latest movement. We note the following,

$$T_{bb'} = T(\Delta\theta_b, \Delta x_b, \Delta y_b)$$

From $T_{bb'}$, we can get the displacement in the body frame Δq_b. Lastly, we convert the twist to the desired placement in the body frame using the following,

$$\Delta q = A(\theta, 0, 0)\Delta q_b$$