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# Assignment 6

## 6.1

For simplicity, we denote $u^\theta(w,h) = u$, and $q(w), q(\bar{w}) = q$

Then,

$$\begin{aligned}
\nabla L_{NCE} &= \sum_w \nabla(\tilde{p}(w|h)\log\frac{u}{u+kq} + kq\log\frac{kq}{u+kq}) \\
&= \sum_w [\tilde{p}(w|h)\nabla\log\frac{u}{u+kq} + kq\nabla\log\frac{kq}{u+kq}] \\
&= \sum_w [\tilde{p}(w|h)\cdot\frac{u+kq}{u}\cdot\frac{kq\nabla u}{(u+kq)^2} - kq\frac{u+kq}{kq}\frac{-kq\nabla u}{(u+kq)^2}] \\
&= \sum_w [\tilde{p}(w|h)\cdot\frac{kq\nabla u}{u(u+kq)} + \frac{kq\nabla u}{u+kq}] \\
&= \frac{kq}{u+kq}\sum_w ((\tilde{p}(w|h) - u)\frac{\nabla u}{u}) \\
&= \frac{kq}{u+kq}\sum_w ((\tilde{p}(w|h) - u)\nabla(\log u)) \\
&\approx \sum_w ((\tilde{p}(w|h) - p^\theta(w|h))\nabla(\log u)) \\
&= \nabla L_{MLE}
\end{aligned}$$

The "≈" sign achieves when$k \to \infty$, and $p^\theta(w|h) = u$