

Laboratory 5

Exercise 1

Consider the data from file data-Lab5, which consists of observations of two variables $[Y, X]$. Suppose, Y is generated by the process

$$y_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 + e_i,$$

where e_i is a white noise and is assumed to follow a normal distribution $e_i \sim N(0, \sigma^2)$.

- Verify a hypothesis $H_0 : \sigma^2 = 1$ versus the alternative $H_1 : \sigma^2 > 1$ with the three tests: LM, Wald and LR. Present formulas for the appropriate test statistics.
- Suppose, you want to test, if the polynomial $\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2$ has exactly one root. Derive and present the three tests statistics: LM, Wald and LR. What are their asymptotic distributions?
- Choose one of these tests to verify the null hypothesis and present the result.

Exercise 2 Suppose, you want to verify if in the model

$$y_t = \alpha x_t + e_t$$

the parameter α is different from zero. The hypothesis is typically tested with a t-Student test. The aim of this exercise is to check, how important is the stationarity assumption of residuals. In order to verify the test performance, propose and conduct a small Monte-Carlo experiment.

- Define the number of MC iterations (N) and the sample size (choose one sample size, for example $T = 200$).
- Choose values of the parameter α , for which the frequency of rejections will be computed (both under the null and the alternative).
- Generate N different realizations of the stationary exogenous variable x_t (it may come from a normal distribution $x_t \sim N(\mu_x, \sigma_x^2)$).
- Describe the test properties (size and power), for stationary residuals. In order to do so, for each MC iteration, generate a white noise $e_t \sim N(0, \sigma^2)$. Then, conditional on the parameter α , generate y_t and compute the t statistic. Compare it with the critical value from the t-Student distribution and check if the null is rejected or not.
- Now, repeat the procedure from pervious point for residuals, which come from the random walk.
- Compute the frequency of null rejections for different scenarios and parameter values.
- What are the empirical sizes and powers of the test, when the residuals are white noise/random walk. Comment on the results.