Life insurance models

List 2.

1. Show that

   a) ${}_{t|u}q_x = {}_tp_x\ {}_uq_{x+t}$

   b) ${}_{u+t}p_x = {}_up_x\ {}_tp_{x+u}$

   c)

$$\sum_{h=0}^{k} {}_{h|}q_x = {}_{k+1}q_x.$$

2. If $\mu_x = 0.001$ for $20 \leqslant x \leqslant 25$, evaluate ${}_{2|2}q_{20}$.
3. Let

$$\mu_{x+t} = \frac{1}{85-t} + \frac{3}{105-t} \quad 0 \leqslant t < 85.$$

   Calculate ${}_{20}p_x$.

4. If $s(x) = 1 - \frac{x}{100}$, $0 \leqslant x \leqslant 100$, calculate

   a) μ_x

   b) $F(x)$

   c) $f(x)$

   d) $P(10 < X \leqslant 40)$

5. Complete the entries below

$s(x)$	$F(x)$	$f(x)$	μ_x
			$\operatorname{tg} x,\ 0 \leqslant x < \frac{\pi}{2}$
$e^{-x},\ x \geqslant 0$			
	$1 - \frac{1}{1+x},\ x \geqslant 0$		

6. Calculate probability that (55) survives at least 10 years if analogous probability for (25) is equal to 0.8. We assume the force of mortality is given by

$$\mu_x = kx,\ x > 0.$$

7. The force of mortality is given by the following function

$$\mu_{x+t} = be^{x+t},\ b > 0.$$

   Calculate the b parameter for which the probability that (30) survives next 10 years and then dies withing next 5 years is equal to r. We know that ${}_{10}p_{30} = 5r$.

8. In the population the force of mortality is given by

$$\mu_{x+t} = k(x+t),\ \ k > 0.$$

Calculate $\frac{SD(T(0))}{E(T(0))}$, where $SD(X)$ is a standard deviation of X.

9. Confirm that the following can serve as a survival function. Exhibit the corresponding μ_x, $f(x)$ and $F(x)$,

$$s(x) = e^{-x^3/12},\quad x \geqslant 0.$$

10. Each of the following functions can serve as a force of mortality. Exhibit the corresponding survival functions. In each case $x \geqslant 0$.

a) $Bc^x,\quad B > 0,\ c > 1$

b) $kx^n,\quad n > 0,\ k > 0$

c) $a(b+x)^{-1},\quad a > 0,\ b > 0.$