Lectures and exercises

Estimation theory

- A contact:
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  - all the additional information on the web site www.ioz.pwr.wroc.pl/Pracownicy/Maciejowska
- Grading (exercise classes):
  - Maximum 100 points: Laboratories (50), two tests (50).

Lectures and exercises

- Textbook: "*Econometric Analysis*", William H. Greene
- Course content:
  - Matrix algebra and distribution theory.
  - Properties of estimators and large-sample distribution theory.
  - Estimation methods: Least Squares(LS), Maximum Likelihood (ML),Quintile regression (QR).
  - Model verification: tests based on ML approach.
  - Simulations and test properties.
  - Forecasting and prediction evaluation.

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Matrix algebra - revision

Notation

Matrix

$$A = [a_{ik}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1K} \\ a_{21} & a_{22} & \cdots & a_{2K} \\ & & \cdots & \\ a_{N1} & a_{N2} & \cdots & a_{NK} \end{bmatrix}$$

What is the matrix dimension ($N \times K$) for

- row vector
- column vector
- square matrix

Notation

Some types of square matrices

- a symmetric matrix: $a_{ik} = a_{ki}$
- a diagonal matrix
- a scalar matrix: diagonal matrix with $a = a_{ii} = a_{kk}$
- an identity matrix(denoted by I_K):a scalar matrix with ones on the diagonal.

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Transposition

The transpose of matrix A

$$B = A' \Leftrightarrow b_{ik} = a_{ki}$$

Some laws

- A is symmetric $\Leftrightarrow A' = A$
- $(A')' = A$

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Multiplication

The inner product of two vectors: a and b

$$a'b = a_1b_1 + a_2b_2 + ... + a_Nb_N$$

The matrix multiplication of A: $(N \times K)$ and B: $(K \times M)$

$$C = AB \Rightarrow c_ik = a_i'b_k$$

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Multiplication

Some generale rules

- Associative law: $(AB)C = A(BC)$
- Distributive law: $A(B + C) = AB + AC$
- Transpose of a product: $(AB)' = B'A'$
- Transpose of an extended product: $(ABC)' = C'B'A'$

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Sums of values

Denote by i a column vector of ones. Write in the form of inner product

- $\sum_{i=1}^{N} x_i$
- $\sum_{i=1}^{N} x_i^2$
- $\sum_{i=1}^{N} x_i y_i$
- $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$

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Idempotent matrix

The matrix M is idempotent if

$$M^2 = MM = M$$

If M is symmetric then

$$M'M = M$$

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Idempotent matrix

Write down $x = \bar{x}$ as the product of matrices: $x - \bar{x} = Mx$

- What can you say about the matrix M?
- Compute $\sum_{i=1}^{N}(x_i - \bar{x})$.
- Compute $\sum_{i=1}^{N}(x_i - \bar{x})^2$.
- Compute $\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})$.

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Trace of a matrix

The trace of a square $K \times K$ matrix A

$$tr(A) = \sum_{i=1}^{K} a_{ii}$$

Some basic laws

- $tr(cA) = c(tr(A))$
- $tr(A') = tr(A)$
- $tr(A+B) = tr(A) + tr(B)$
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Trace of a matrix

Questions:

- How much is $tr(I_K)$?
- Express $\sum_{i=1}^{K} a_{ik}^2$ with the trace function.

Rank of a matrix

The rank of a matrix is the dimension of the vector space that is spanned by its column vectors (or by its row vectors).

Some general rules

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- $rank(AB) \leq min(rank(A), rank(B))$
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Determinant of a matrix

Lets denote by $|A|$ as the determinant of the square matrix A: $(K \times K)$.

Some rule

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- $|AB| = |A||B|$

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Inverse matrices

If the square matrix A is non-singular then there exists its inverse: A^{-1} such that

$$A^{-1}A = I.$$

Show that

- $AA^{-1} = I$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $|A^{-1}| = \frac{1}{|A|}$

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Characteristic roots and vectors

For a symmetric, square $(K \times K)$ matrix A lets denote

- $c_1, c_2, ..., c_K$ - characteristic, $(K \times 1)$ vectors
- $\lambda_1, \lambda_2, ..., \lambda_K$ - characteristic roots

Characteristic roots and vectors

Then

$$C = [c_1, c_2, ..., c_K]$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & \lambda_K \end{bmatrix}$$

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- $C' = C^{-1}$
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Characteristic roots and vectors

Some (more general)laws:

- The $rank(A)$ is the number of non-zero characteristic roots of A.
- The $rank(A)$ is the number of non-zero characteristic roots of $A'A$.
- The non-zero characteristic roots of $A'A$ and AA' are the same.

Show that

- $|A| = |\Lambda| = \prod_{i=1}^{N} \lambda_i$.
- $tr(A) = tr(\Lambda) = \sum_{i=1}^{N} \lambda_i$.

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Suppose A is symmetric

- What are the characteristic roots of A^2?
- What are the characteristic roots of A^K, $K > 0$ and $K = 1, 2, ...$?
- What is $A^{1/2}$ and what are its characteristic roots?
- If A is *positive definite*, what it is A^r?

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