Life insurance models

List 5.

1. Show that $A_x = vq_x + vp_x A_{x+1}$.

2. If $l_x = 100 - x$ for $0 \leqslant x \leqslant 100$ and $i = 0.05$, evaluate $A_{25:\overline{40}|}$.

3. If $A_x = 0.25$, $A_{x+20} = 0.40$, and $A_{x:\overline{20}|} = 0.55$, calculate $A^1_{x:\overline{20}|}$ and ${A_{x:\overline{20}|}}^1$.

4. Express A_{x+t} ($x \in N$, $0 \leqslant t \leqslant 1$) as a function of A_x, v and life table parameters under the assumpion of a uniform distribution of deaths.

5. Calculate $A^1_{25:\overline{5}|}$ with benefit 100000 payable at the end of the year of death. We assume $i = 3\%$. Use the life tables. Calculate also $A^1_{60:\overline{5}|}$.
6. Let us consider two populations P and P^M with forces of mortality μ_{x+t} and μ^M_{x+t}, respectively. We know that $\mu^M_{x+t} = \mu_{x+t} + M$ and M is a random number from uniform distribution over interval $[0.01; 0.02]$. For population P we have
$$ {A_{x:\overline{35}|}}^1 = 0.195276. $$
Calculate ${A_{x:\overline{35}|}}^1$ for population P^M.
7. The p.d.f. of the future lifetime T for (x) is assumed to be
$$ g(t) = \begin{cases} \dfrac{1}{80} & \text{for } 0 < t < 80, \\ \\ 0 & \text{elsewhere.} \end{cases} $$
At a force of interest, δ, calculate for Z (the present value random variable for a whole life insurance of unit amount issued to (x)) the net single premium and the variance.
8. Calculate ${}_{3|}A_x$ if we know
$$ A_x = \alpha_0, \ (IA)^1_{x:\overline{k}|} = \alpha_k, \ k = 1, 2, 3. $$
Hint: ${}_{3|}A_x = \sum_{k=3}^{\infty} v^{k+1}{}_k p_x q_{x+k}$, $(IA)^1_{x:\overline{n}|} = \sum_{k=0}^{n-1}(k+1)v^{k+1}{}_k p_x q_{x+k}$.
9. Under the assumption $A^1_{x:\overline{20}|} = 0.2$ and $(IA)^1_{x:\overline{20}|} = 0.75(DA)^1_{x:\overline{20}|}$ calculate $(DA)^1_{x:\overline{20}|}$.
Hint: $(DA)^1_{x:\overline{n}|} = \sum_{k=0}^{n-1}(n-k)v^{k+1}{}_k p_x q_{x+k}$.
10. We consider two 10−years policy issues for (40) payable at the moment of death. The policy issue $ROSS$ pays $10000t$ if the insured dies at the age of $40 + t$, $t < 10$. The policy issue MAL pays $10000(10 - t)$ if the insured dies at the age of $40 + t$, $t < 10$. Calculate the net single premium for policy $ROSS$ if it is known that it is 11 times higher than the net single premium for policy MAL. Moreover we know $\overline{A}^1_{40:\overline{10}|} = 0.05$.