Perturbation Methods (MAT 1572)

Winter Semester 2017/2018

List 2

1. Consider the algebraic equation $x^2 + \varepsilon x - 1 = 0$ with $0 < \varepsilon \ll 1$. Find the exact solution and compare the approximation. Is it uniform? Determine the first three terms in a perturbation series solution $x = x_0 + \varepsilon^{\alpha_1} x_1 + \varepsilon^{\alpha_2} x_2 + ...$ for each root. Compare with the exact roots.

2. To find approximations to the roots of the cubic equation $x^3 - 4.001x + 0.002 = 0$ why is it easier to examine the equation $x^3 - (4 + \varepsilon)x + 2\varepsilon = 0$?

3. Find a three-term perturbation expansion for the root of $x = 1 + \varepsilon x^2$, $0 < \varepsilon \ll 1$ near $x = 1$. Compare it to the exact solution for $\varepsilon = 0.1$ and $\varepsilon = 0.001$.

4. Consider the algebraic equation $g(x; \varepsilon) = 0$, $0 < \varepsilon \ll 1$, $x \in \mathbb{R}$, where $g \in C^\infty(\mathbb{R})$. Assuming $g(x; 0) = 0$ is solvable to obtain x_0, show how to find a three-term approximation of the form $x = x_0 + x_1\varepsilon + x_2\varepsilon^2$. What condition on g is required to determine x_1 and x_2? Find a three-term approximation to the roots of $e^{\varepsilon x} = x^2 - 1$.