Diffusion processes in complex networks

Digression - parallel computing in Python

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Outlook:

References

- J.R. Johansson's lectures (http://github.com/jrjohansson/scientific-python-lectures (http://github.com/jrjohansson/scientific-python-lectures))
- Multiprocessing: https://docs.python.org/2/library/multiprocessing.html (https://docs.python.org/2/library/multiprocessing.html)
- "Using IPython for parallel computing": http://ipython.org/ipython-doc/dev/parallel/ (http://ipython.org/ipython-doc/dev/parallel/)
- MPI for Python: http://mpi4py.scipy.org/ (http://mpi4py.scipy.org/)
- PyOpenCL: http://documen.tician.de/pyopencl/ (http://documen.tician.de/pyopencl/)

In [1]:

```
%matplotlib inline
import matplotlib.pyplot as plt
```

Multiprocessing

`Multiprocessing` module is a part of Python's standard library. It may be used for simple parallel tasks.

In [2]:

```
import multiprocessing
import os
import time
import numpy
```

In [3]:

```
#simple function that gets the process id
def task(args):
    print("PID =", os.getpid(), ", args =", args)
    return os.getpid(), args
```

In [4]:

```
#ordinary call of the function
task("test")
```

```
PID = 11812 , args = test
```

Out[4]:

```
(11812, 'test')
```

In [5]:

```
#now the same in multiprocessing approach

#create the pool of processes first
pool = multiprocessing.Pool(processes=4)

#then map the function to arguments within the pool
result = pool.map(task, [1,2,3,4,5,6,7,8])
```

```
PID = 11835 , args = 2
PID = 11836 , args = 3
PID = 11837 , args = 4
PID = 11837 , args = 6
PID = 11836 , args = 5
PID = 11837 , args = 7
PID = 11835 , args = 8
PID = 11834 , args = 1
```

In [6]:

```
#wresults in a compact form
print(result)
```

```
[(11834, 1), (11835, 2), (11836, 3), (11837, 4), (11836, 5), (11837,
6), (11837, 7), (11835, 8)]
```

The module is useful for problems with little or no need for communication between the tasks. Such a type of problems is called **embarrassingly parallel problems** in parallel computing.

Examples:

- computer simulations (e.g. independent runs in Monte Carlo simulations, parameter sweeping)
- numerical integration
- genetic algorithms
- distributed database queries
- rendering of computer graphics
- brute-force attacks in cryptography
- serving static files on a webserver to multiple users at once
- large scale facial recognition systems

Let us consider now a more useful example than simply getting the process id. We want to calculate a function at many points:

In [7]:

```
def f(x):
    time.sleep(1) #sleep instead of complicated calculations
    return x*x

p = multiprocessing.Pool()
p.map(f,[1,2,3,4])
```

Out[7]:

```
[1, 4, 9, 16]
```

In [8]:

```
%timeit p.map(f,range(10))
```

```
1 loop, best of 3: 2 s per loop
```

In [9]:

```
%timeit [f(x) for x in range(10)]
```

```
1 loop, best of 3: 10 s per loop
```

We see that the parallel approach is faster that the traditional one. In general, parallel computing is worth considering if the time needed to proceed a single task is much larger as the overhead related to the multiple processes handling. Otherwise one should rather go for the traditional version:

In [10]:

```
def g(x):
    return x*x #similar to f, but no delay

p = multiprocessing.Pool()
```

In [11]:

```
%timeit p.map(g,range(100))
```

```
1000 loops, best of 3: 478 µs per loop
```

In [12]:

```
%timeit [g(x) for x in range(10)]
```

```
1000000 loops, best of 3: 1.17 µs per loop
```

The serial execution of the tasks is indeed much faster in this case.

Parallel computing in IPython

IPython environment offers an easy-to-use mechanism for parallel computing based on the concept of engines and controllers, which may proceed some tasks.

First, we have to install the `ipyparallel` extension,

```
pip3 install ipyparallel
```

In order to start parallel computing in IPython, we have to create a claster of engines. We can do that in the command line:

```
$ ipcluster start -n 4
```

Other possibility is the tab 'Clusters' in a jupyter notebook, provided the notebook extension was enabled with the command

```
ipcluster nbextension enable
```

The above `ipcluster` command will start 4 engines. If we work on a single machine, such a cluster of engines is useful only if it is a multicore machine. Otherwise it would rather slow down the computations. It should be noted however that the engines may be distributed over several machines (see https://ipyparallel.readthedocs.io/en/latest/ (https://ipyparallel.readthedocs.io/en/latest/) for more details).

After starting the engine cluster we import a client from the `ipyparallel` module:

In [13]:

```
from ipyparallel import Client
```

In [14]:

```
cli = Client()
```

With `ids` attribute we can check the ids of the engines:

In [15]:

```
cli.ids
```

Out[15]:

```
[0, 1, 2, 3, 4, 5, 6, 7]
```

Each of these engines may process different tasks. The tasks may be assigned to some of the engines or all of them:

In [16]:

```
def getpid():
    """ Return ID of current process """
    import os
    return os.getpid()
```

In [17]:

```
# we test the function locally
getpid()
```

Out[17]:

```
11812
```

In [18]:

```
# now we run it on a particular engine
cli[0].apply_sync(getpid)
```

Out[18]:

```
11732
```

In [19]:

```
# and on all engines in parallel
cli[:].apply_sync(getpid)
```

Out[19]:

```
[11732, 11734, 11736, 11738, 11740, 11749, 11765, 11774]
```

We can use this cluster of IPython engines to execute tasks in parallel. The easiest way to dispatch a function to different engines is to define the function with the decorator

```
@view.parallel(block=True)
```

(look at http://pythonconquerstheuniverse.wordpress.com/2012/04/29/python-decorators/ (http://pythonconquerstheuniverse.wordpress.com/2012/04/29/python-decorators/) for a nice explanation of decorators).

Here, `view` is the pool of engines we want to dispatch the function.

Let us check how it works:

In [20]:

```
dview = cli[:]
```

In [21]:

```
@dview.parallel(block=True)
def dummy_task(delay):
    """ do nothing for 'delay' seconds and finish """
    import os, time

    t0 = time.time()
    pid = os.getpid()
    time.sleep(delay)
    t1 = time.time()

    return [pid, t0, t1]
```

Once our function is defined this way we can dispatch it to the engine using the `map` method in the resulting class:

In [22]:

```
# random delay times
delay_times = numpy.random.rand(8)
```

In [23]:

```
#now we simply map the times to the function
dummy_task.map(delay_times)
```

Out[23]:

```
[[11732, 1495541901.9115462, 1495541902.095794],
 [11734, 1495541901.9117837, 1495541902.6235466],
 [11736, 1495541901.911783, 1495541902.079649],
 [11738, 1495541901.9143817, 1495541902.135188],
 [11740, 1495541901.9117632, 1495541902.0465705],
 [11749, 1495541901.9135165, 1495541902.2227879],
 [11765, 1495541901.915408, 1495541902.0197375],
 [11774, 1495541901.9134102, 1495541902.8327844]]
```

Let us repeat that with more tasks. We will use `matplotlib` to visualize the results:

In [24]:

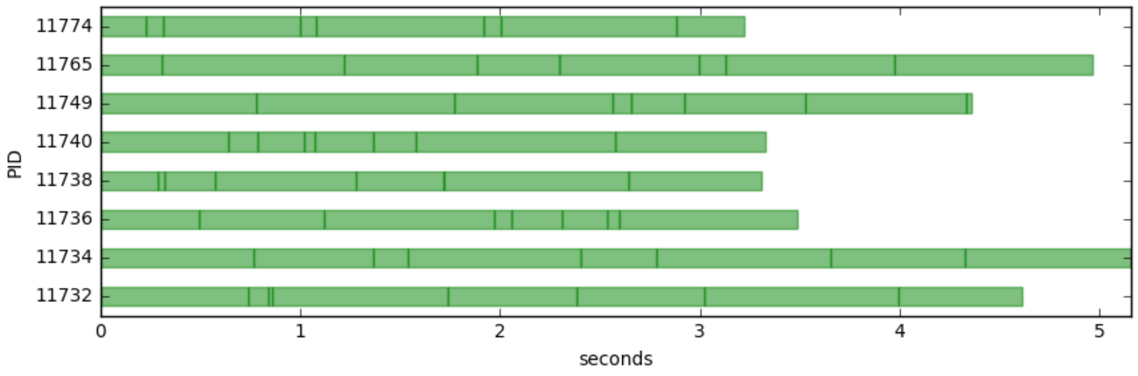
```
def visualize_tasks(results):
    res = numpy.array(results)
    fig, ax = plt.subplots(figsize=(10, res.shape[1]))

    yticks = []
    yticklabels = []
    tmin = min(res[:,1])
    for n, pid in enumerate(numpy.unique(res[:,0])):
        yticks.append(n)
        yticklabels.append("%d" % pid)
        for m in numpy.where(res[:,0] == pid)[0]:
            ax.add_patch(plt.Rectangle((res[m,1] - tmin, n-0.25),
                         res[m,2] - res[m,1], 0.5, color="green", alpha=0.5))

    ax.set_ylim(-.5, n+.5)
    ax.set_xlim(0, max(res[:,2]) - tmin + 0.)
    ax.set_yticks(yticks)
    ax.set_yticklabels(yticklabels)
    ax.set_ylabel("PID")
    ax.set_xlabel("seconds")
```

In [25]:

```
delay_times = numpy.random.rand(64)
```

In [26]:

```
result = dummy_task.map(delay_times)
visualize_tasks(result)
```



We utilized all engines quite well. However, the tasks seem to be not balanced, i.e. one engine might be idle while others still have tasks to work on.

To obtain a load balanced view of the engines we simply use the `load_balanced_view` decorator:

In [27]:

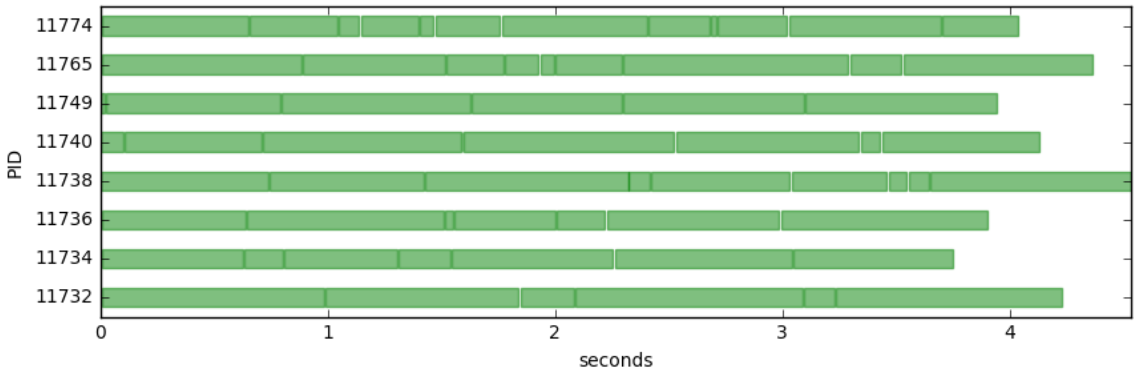
```
lbview = cli.load_balanced_view()
```

In [28]:

```
@lbview.parallel(block=True)
def dummy_task_load_balanced(delay):
    """ do nothing for 'delay' seconds and finish """

    import os, time

    t0 = time.time()
    pid = os.getpid()
    time.sleep(delay)
    t1 = time.time()

    return [pid, t0, t1]
```

In [29]:

```
result = dummy_task_load_balanced.map(delay_times)
visualize_tasks(result)
```



This time the loads of the engines are similar. Moreover, the time to complation is now shorter than in the previous case.

And now a more useful example:

In [30]:

```
import sympy

def factorit(n):
    x = sympy.var('x')
    return sympy.factor(x**n - 1,x)
```

In [31]:

```
factorit(2)
```

Out[31]:

```
(x - 1)*(x + 1)
```

In [32]:

```
factorit(5)
```

Out[32]:

```
(x - 1)*(x**4 + x**3 + x**2 + x + 1)
```

In [33]:

```
#let us make it harder
%timeit f = [factorit(i) for i in range(100,110)]
```

```
10 loops, best of 3: 47.8 ms per loop
```

In [34]:

```
cli = Client()
dview = cli[:]
#we have to import sympy on every engine
dview.execute('import sympy')
```

Out[34]:

```
<AsyncResult: execute>
```

In [35]:

```
%timeit f = dview.map(factorit,range(100,110))
```

```
The slowest run took 4.85 times longer than the fastest. This could
 mean that an intermediate result is being cached.
100 loops, best of 3: 4.46 ms per loop
```

In [36]:

```
f = dview.map(factorit,range(100,110))
```

In [38]:

```
f[-1]
```

Out[38]:

```
(x - 1)*(x**108 + x**107 + x**106 + x**105 + x**104 + x**103 + x**10
2 + x**101 + x**100 + x**99 + x**98 + x**97 + x**96 + x**95 + x**94
 + x**93 + x**92 + x**91 + x**90 + x**89 + x**88 + x**87 + x**86 + x
**85 + x**84 + x**83 + x**82 + x**81 + x**80 + x**79 + x**78 + x**77
+ x**76 + x**75 + x**74 + x**73 + x**72 + x**71 + x**70 + x**69 + x*
*68 + x**67 + x**66 + x**65 + x**64 + x**63 + x**62 + x**61 + x**60
 + x**59 + x**58 + x**57 + x**56 + x**55 + x**54 + x**53 + x**52 + x
**51 + x**50 + x**49 + x**48 + x**47 + x**46 + x**45 + x**44 + x**43
+ x**42 + x**41 + x**40 + x**39 + x**38 + x**37 + x**36 + x**35 + x*
*34 + x**33 + x**32 + x**31 + x**30 + x**29 + x**28 + x**27 + x**26
 + x**25 + x**24 + x**23 + x**22 + x**21 + x**20 + x**19 + x**18 + x
**17 + x**16 + x**15 + x**14 + x**13 + x**12 + x**11 + x**10 + x**9
 + x**8 + x**7 + x**6 + x**5 + x**4 + x**3 + x**2 + x + 1)
```

MPI for Python

When more communication between processes is required, sophisticated solutions as MPI or OpenMP are often needed.

MPI is a standardized and portable message-passing system designed by a group of researchers from academia and industry to function on a wide variety of parallel computing architectures. It defines the syntax and semantics of a core of library routines useful to a wide range of users writing portable message-passing programs in C, C++, and Fortran. There are several well-tested and efficient implementations of MPI, many of which are open-source or in the public domain.

MPI can be used in Python programs through the `mpi4py` package (http://mpi4py.scipy.org/ (http://mpi4py.scipy.org/)). To use it we usually include `MPI` from the corresponding module,

```
from mpi4py import MPI
```

An MPI Python program must be started using

```
$mpirun -n N
```

where `N` is the number of processes that should be included in the process pool (can be larger as the number of available cores/processors).

Example 1 - sending data from one process to the other

In [39]:

```
%%file mpitest.py

# -*- coding: utf-8 -*-

from mpi4py import MPI

#initialize process pool
comm = MPI.COMM_WORLD
#get process id
rank = comm.Get_rank()



if rank == 0:                            #master
   data = [1.0, 2.0, 3.0, 4.0]           #create some data
   comm.send(data, dest=1, tag=11)       #send data to node 1
elif rank == 1:                          #slave
   data = comm.recv(source=0, tag=11)    #receive data from 0

print("rank =", rank, ", data =", data) #print to screen (all processes)
```

```
Writing mpitest.py
```

In [40]:

```
!mpirun -n 2 python3 mpitest.py
```

```
rank = 0 , data = [1.0, 2.0, 3.0, 4.0]
rank = 1 , data = [1.0, 2.0, 3.0, 4.0]
```

Przykład 2 - two processes, sending numpy array from one to other

In [41]:

```
%%file mpi-numpy-array.py

# -*- coding: utf-8 -*-

from mpi4py import MPI
import numpy

comm = MPI.COMM_WORLD
rank = comm.Get_rank()

if rank == 0:                        #master
   data = numpy.random.rand(10)      #data to send
   comm.Send(data, dest=1, tag=13)   #(dest - destination id, tag - data label)
elif rank == 1:
   data = numpy.empty(10, dtype=numpy.float64) #container for data
   comm.Recv(data, source=0, tag=13)           #(source - sender id, tag - data
 label)

print("rank =", rank, ", data =", data)
```

```
Writing mpi-numpy-array.py
```

Note, that in this case we used `Send` and `Recv` instead of `send` and `recv`. The latter ones are for built-in Python types only.

In [42]:

```
!mpirun -n 2 python3 mpi-numpy-array.py
```

```
rank = 0 , data = [ 0.01115279  0.95229013  0.85597378  0.06067628
 0.57811044  0.4065776
  0.0230081   0.38120197  0.44154346  0.85905686]
rank = 1 , data = [ 0.01115279  0.95229013  0.85597378  0.06067628
 0.57811044  0.4065776
  0.0230081   0.38120197  0.44154346  0.85905686]
```

Example 3 - matrix-vector multiplication

In [43]:

```
import numpy
# prepare data and save it to files
N = 16
A = numpy.random.rand(N, N)
numpy.save("random-matrix.npy", A)
x = numpy.random.rand(N)
numpy.save("random-vector.npy", x)
```

In [44]:

```
%%file mpi-matrix-vector.py

# -*- coding: utf-8 -*-

from mpi4py import MPI
import numpy

#initialize MPI cluster
comm = MPI.COMM_WORLD
#get process id
rank = comm.Get_rank()
#get cluster size
p = comm.Get_size()

def matvec(comm, A, x):
    m = A.shape[0] // p
    #every process gets a part of the data
    y_part = numpy.dot(A[rank * m:(rank+1)*m], x)
    #container for the result
    y = numpy.zeros_like(x)
    #collect results from the pool, write them to container y
    comm.Allgather([y_part,  MPI.DOUBLE], [y, MPI.DOUBLE])
    return y

A = numpy.load("random-matrix.npy")
x = numpy.load("random-vector.npy")
y_mpi = matvec(comm, A, x)

if rank == 0:
    #test
    y = numpy.dot(A, x)
    print(y_mpi)
    #compare the local and MPI results
    print("sum(y - y_mpi) =", (y - y_mpi).sum())
```

```
Writing mpi-matrix-vector.py
```

In [45]:

```
!mpirun -n 4 python3 mpi-matrix-vector.py
```

```
[ 5.69327703  4.85358127  4.46203991  5.67797315  6.05630264  4.6817
4254
  3.89666749  4.81103283  6.64800639  4.6662954   5.54068971  4.7007
1352
  4.14724973  4.32205529  4.89661218  4.13669822]
sum(y - y_mpi) = 0.0
```

Example 4 - sum of the elements in a vector

In [46]:

```
# create data and save to file
N = 128
a = numpy.random.rand(N)
numpy.save("random-vector.npy", a)
```

In [47]:

```
%%file mpi-psum.py

# -*- coding: utf-8 -*-

from mpi4py import MPI
import numpy as np

def psum(a):
    #process id
    r = MPI.COMM_WORLD.Get_rank()
    #cluster size
    size = MPI.COMM_WORLD.Get_size()
    #calculate data size for each process
    m = len(a) // size
    #local sum (different on each host)
    locsum = np.sum(a[r*m:(r+1)*m])
    #container for results
    rcvBuf = np.array(0.0, 'd')
    #collect results and sum them
    MPI.COMM_WORLD.Allreduce([locsum, MPI.DOUBLE], [rcvBuf, MPI.DOUBLE], op=MPI.
SUM)
    return rcvBuf

a = np.load("random-vector.npy")
s = psum(a)

if MPI.COMM_WORLD.Get_rank() == 0:
    #mpi vs local
    print("sum =", s, ", numpy sum =", a.sum())
```

```
Writing mpi-psum.py
```

In [48]:

```
!mpirun -n 4 python3 mpi-psum.py
```

```
sum = 59.954215672639656 , numpy sum = 59.9542156726
```

Cython i OpenMP

OpenMP (*Open Multi-Processing*) is a standard and widely used **thread**-based parallel API. It cannot be use directly in Python, because the CPython implementation uses a global interpreter lock, making it impossible to simultaneously run several Python threads. This is clearly a limitation in the Python interpreter, and as a consequence all parallelization in Python must use processes (not threads).

However, when calling out to compiled code the GIL is released, and it is possible to write Python-like code in Cython where we can selectively release the GIL and do OpenMP computations.

In [49]:

```
import multiprocessing
N_core = multiprocessing.cpu_count()

print("Number of cores: %d" % N_core)
```

```
Number of cores: 8
```

Here is a simple example that shows the usage of OpenMP via Cython:

In [50]:

```
%load_ext Cython
```

In [51]:

```
%%cython -f -c-fopenmp --link-args=-fopenmp -c-g

# -*- coding: utf-8 -*-

cimport cython
cimport numpy
from cython.parallel import prange, parallel
cimport openmp

def cy_openmp_test():

    cdef int n, N

    # release GIL so that we can use OpenMP
    with nogil, parallel():
        N = openmp.omp_get_num_threads()
        n = openmp.omp_get_thread_num()
        with gil:
            print("Number of threads: %d, thread number: %d\n" % (N, n))
```

In [52]:

```
cy_openmp_test()
```

```
Number of threads: 8, thread number: 0

Number of threads: 8, thread number: 2
Number of threads: 8, thread number: 5
Number of threads: 8, thread number: 4
Number of threads: 8, thread number: 3
Number of threads: 8, thread number: 7
Number of threads: 8, thread number: 6







Number of threads: 8, thread number: 1
```

Example - matrix-vector multiplication

In [53]:

```
# prepare some random data
N = 4 * N_core

M = numpy.random.rand(N, N)
x = numpy.random.rand(N)
y = numpy.zeros_like(x)
```

We start with a simple implementation in Cython (without OpenMP):

In [54]:

```
%%cython

cimport cython
cimport numpy
import numpy

@cython.boundscheck(False) #
@cython.wraparound(False)  #

def cy_matvec(numpy.ndarray[numpy.float64_t, ndim=2] M,
              numpy.ndarray[numpy.float64_t, ndim=1] x,
              numpy.ndarray[numpy.float64_t, ndim=1] y):

    cdef int i, j, n = len(x)

    for i from 0 <= i < n:
        for j from 0 <= j < n:
            y[i] += M[i, j] * x[j]

    return y
```

In [55]:

```
# check that we get the same results
y = numpy.zeros_like(x)
cy_matvec(M, x, y)
numpy.dot(M, x) - y
```

Out[55]:

```
array([ -1.77635684e-15,  -1.77635684e-15,   0.00000000e+00,
         0.00000000e+00,  -1.77635684e-15,   0.00000000e+00,
        -8.88178420e-16,   8.88178420e-16,  -8.88178420e-16,
        -8.88178420e-16,   0.00000000e+00,  -8.88178420e-16,
         3.55271368e-15,  -8.88178420e-16,   1.77635684e-15,
        -8.88178420e-16,   0.00000000e+00,  -3.55271368e-15,
         0.00000000e+00,  -8.88178420e-16,   1.77635684e-15,
         8.88178420e-16,  -1.77635684e-15,  -8.88178420e-16,
         0.00000000e+00,   3.55271368e-15,   1.77635684e-15,
         8.88178420e-16,   8.88178420e-16,  -1.77635684e-15,
        -8.88178420e-16,   8.88178420e-16])
```

In [56]:

```
#and now some performance tests
%timeit numpy.dot(M, x)
```

```
The slowest run took 26.68 times longer than the fastest. This could
mean that an intermediate result is being cached.
1000000 loops, best of 3: 558 ns per loop
```

In [57]:

```
%timeit cy_matvec(M, x, y)
```

```
The slowest run took 7.56 times longer than the fastest. This could
 mean that an intermediate result is being cached.
1000000 loops, best of 3: 1.55 µs per loop
```

The Cython implementation is a bit slower that `numpy.dot`. We expect to improve the performance by making use of multiple cores:

In [58]:

```
%%cython -f -c-fopenmp --link-args=-fopenmp -c-g

cimport cython
cimport numpy
from cython.parallel import parallel
cimport openmp

@cython.boundscheck(False)
@cython.wraparound(False)
def cy_matvec_omp(numpy.ndarray[numpy.float64_t, ndim=2] M,
                  numpy.ndarray[numpy.float64_t, ndim=1] x,
                  numpy.ndarray[numpy.float64_t, ndim=1] y):

    cdef int i, j, n = len(x), N, r, m

    # release GIL, so that we can use OpenMP
    with nogil, parallel():
        N = openmp.omp_get_num_threads()
        r = openmp.omp_get_thread_num()
        m = n // N
        #every thread gets its own data chunk
        for i from 0 <= i < m:
            for j from 0 <= j < n:
                y[r * m + i] += M[r * m + i, j] * x[j]

    return y
```

In [59]:

```
# check the results
y = numpy.zeros_like(x)
cy_matvec_omp(M, x, y)
print(numpy.dot(M, x) - y)
```

```
[ -1.77635684e-15  -1.77635684e-15   0.00000000e+00   0.00000000e+00
  -1.77635684e-15   0.00000000e+00  -8.88178420e-16   8.88178420e-16
  -8.88178420e-16  -8.88178420e-16   0.00000000e+00  -8.88178420e-16
   3.55271368e-15  -8.88178420e-16   1.77635684e-15  -8.88178420e-16
   0.00000000e+00  -3.55271368e-15   0.00000000e+00  -8.88178420e-16
   1.77635684e-15   8.88178420e-16  -1.77635684e-15  -8.88178420e-16
   0.00000000e+00   3.55271368e-15   1.77635684e-15   8.88178420e-16
   8.88178420e-16  -1.77635684e-15  -8.88178420e-16   8.88178420e-1
6]
```

In [60]:

```
#performance tests once again
%timeit numpy.dot(M, x)
```

```
The slowest run took 31.11 times longer than the fastest. This could
mean that an intermediate result is being cached.
1000000 loops, best of 3: 558 ns per loop
```

In [61]:

```
%timeit cy_matvec_omp(M, x, y)
```

```
The slowest run took 113.12 times longer than the fastest. This coul
d mean that an intermediate result is being cached.
10000 loops, best of 3: 56 µs per loop
```

The OpenMP implementation is even slower for the given problem size. It is due to the overhead associated with OpenMP and threading. But let us look at larger matrix sizes:

In [62]:

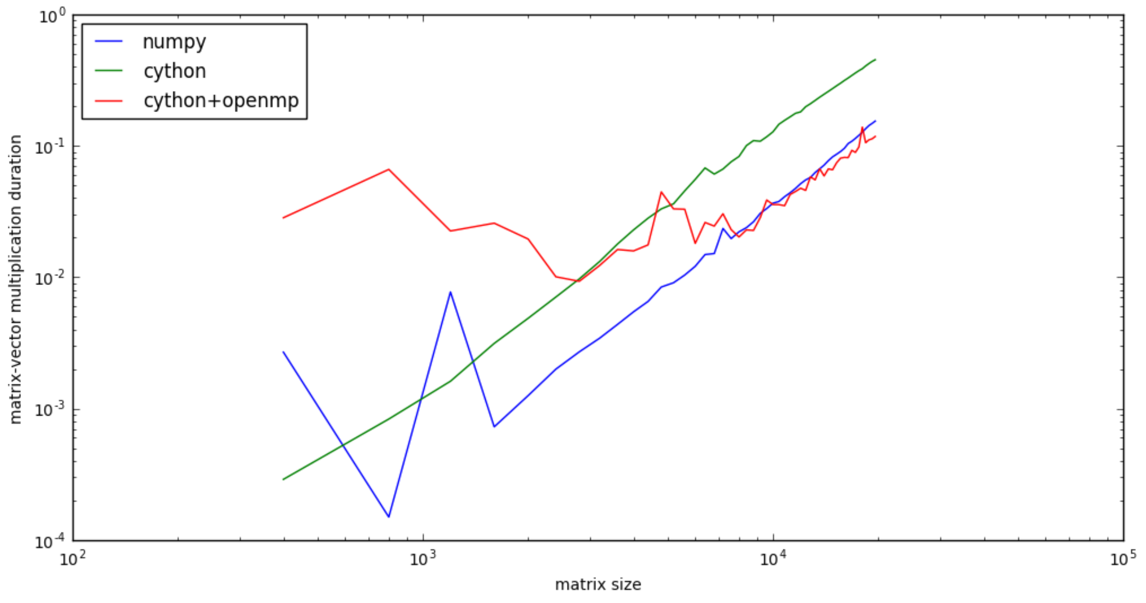
```
N_vec  = numpy.arange(50, 2500, 50) * N_core
```

In [63]:

```
import time
duration_ref = numpy.zeros(len(N_vec))
duration_cy = numpy.zeros(len(N_vec))
duration_cy_omp = numpy.zeros(len(N_vec))

for idx, N in enumerate(N_vec):

    M = numpy.random.rand(N, N)
    x = numpy.random.rand(N)
    y = numpy.zeros_like(x)

    t0 = time.time()
    numpy.dot(M, x)
    duration_ref[idx] = time.time() - t0

    t0 = time.time()
    cy_matvec(M, x, y)
    duration_cy[idx] = time.time() - t0

    t0 = time.time()
    cy_matvec_omp(M, x, y)
    duration_cy_omp[idx] = time.time() - t0
```

In [64]:

```
fig, ax = plt.subplots(figsize=(12, 6))

ax.loglog(N_vec, duration_ref, label='numpy')
ax.loglog(N_vec, duration_cy, label='cython')
ax.loglog(N_vec, duration_cy_omp, label='cython+openmp')

ax.legend(loc=2)
ax.set_yscale("log")
ax.set_ylabel("matrix-vector multiplication duration")
ax.set_xlabel("matrix size");
```



For large matrix sizes the OpenMP implementation is faster than `numpy.dot`. The speed-up is about:

In [65]:

```
((duration_ref / duration_cy_omp)[-10:]).mean()
```

Out[65]:

```
1.2226171108325758
```

However, we a still far away from the theoretical limit of the speed-up:

In [66]:

```
N_core
```

Out[66]:

```
8
```

Python and OpenCL

OpenCL (*Open Computing Language*) is an API for heterogenous computing, for example using both CPUs and GPUs for numerical computations. The python package `pyopencl` allows OpenCL code to be compiled, loaded and executed on the compute units completely from within Python: