Laboratory 4

Exercise 1 Generate a time series of 20 observations according to

$$y_t = \alpha + \varepsilon_t,$$

where $\varepsilon_t \sim N(0, \sigma^2)$. Choose values of α and σ^2.

1. Write down the density function of a single observation, the likelihood and the log-likelihood for the whole sample, as the function of a parameter vector θ.
2. Plot the log-likelihood function for a range of parameter values.
3. Derive the FOC and the ML estimator of model parameters.
4. What is the variance-covariance matrix of model parameters? What is the sample and asymptotic covariance of $\hat{\alpha}$ and $\hat{\sigma^2}$?
5. What is the ML estimator and its variance of $1 + \alpha + \alpha^2$?

Exercise 2

Download a data set datalab4-1. Suppose, y, has a mix normal distribution, which depends on $\theta = [\mu, \sigma^2, p]$

$$y_n \sim \begin{cases} N(0,1) & p \\ N(\mu, \sigma^2) & 1-p \end{cases}$$

1. Write down the likelihood and the log-likelihood function.
2. Suppose, $p = 0.8$. Let's set $\mu = y_1$. Plot the log-likelihood function for different values of σ^2, particularly for σ^2 close to 0 (Hint: make sure that σ^2 approaches 0 fast enough, for example $\sigma = 1/\exp(N)$).
3. What can you say about the existence of the maximum of the log-likelihood function?

Exercise 3

Download a data set datalab4-2. The first column is y, the second column describes x. Suppose, y, has a Bernoulli distribution

$$y_n = \begin{cases} 1 & F(x_n;\theta) \\ 0 & 1 - F(x_n;\theta) \end{cases}$$

The probability of success is given by a logit function

$$F(x_n;\theta) = \frac{exp(\theta_1 + x_n\theta_2)}{1 + exp(\theta_1 + x_n\theta_2)}$$

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- What is the marginal effect of the variable x on the probability of success? (*Hint*: marginal effect is the first derivative of $prob(y = 1)$ with respect to x)
- Write down the log-likelihood function.
- Derive FOC. Do they have a closed form solution?
- Find a maximum of the log-likelihood function using a numerical optimization method (try various starting point).
- Estimate the variance-covariance matrix using the second derivative obtained during the numerical optimization.
- What is the estimated marginal effect of x on y. What is its variance?
- Which estimator: value of θ_1 or the marginal effect of x is more useful for empirical analysis?