Asymptotic theory

Stochastic convergence

Stochastic convergence

Let $x_1, x_2, \ldots$ be a sequence of a scalar random variable that converge to a random variable x. Both x_N and x are defined on probability space $(\Omega, \mathcal{F}, Pr)$. Let us denote the cumulative distribution functions of x_N and x by F_N and F respectively.

Questoin: What if x is a fixed, real number?

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Questoin: What if x is a fixed, real number?

Convergence in probability

The sequence x_n converges in probability to x if for every $\epsilon > 0$

$$\lim_{n \to \infty} Pr(|x_n - x| < \epsilon) = 1$$

It is abbreviated as

$$p\lim x_n = x$$

or

$$x_n \to^p x$$

Rules

If x_n and y_n are random variables with $p\lim x_n = c$ and $p\lim y_n = d$, then

- Sum rule: $p\lim(x_n + y_n) = c + d$
- Product rule: $p\lim x_n y_n = cd$
- Ratio rule: $p\lim x_n / y_n = c/d$, if $d \neq 0$

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If X_n and Y_n are random matrices with $p\lim X_n = A$ and $p\lim Y_n = B$, then

- Inverse rule: if A is square and non-degenerated then $p\lim X_n^{-1} = A^{-1}$
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Convergence almost surely

The sequence x_N converges almost surely or with a probability one to x if for every $\epsilon > 0$

$$Pr(\lim_{N \to \infty} |x_N - x| < \epsilon) = 1$$

It is abbreviated as

$$x_N \to^{a.s.} x$$

Sometimes it is called a strong convergence.

Convergence in distribution

The sequence x_N converges in distribution to x if for every real number c

$$\lim_{N \to \infty} F_N(c) = F(c)$$

It is abbreviated as

$$x_N \to^d x$$

Sometimes it is called a weak convergence.

Remark: It does not require convergence of p.d.f.s

Rules

If $x_n \to^d x$ and $p\lim y_n = c$, then

- $x_n + y_n \to^d x + c$
- $x_n y_n \to^d cx$
- $x_n / y_n \to^d x/c$, if $c \neq 0$

Rules

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Convergence properties

Convergence properties

1. $x_N \to^p x \Rightarrow x_N \to^d x$
2. Suppose x is a fixed, real number then

$$x_N \to^p x \Leftrightarrow x_N \to^d x$$

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Convergence properties

Slutsk'y theorem

If $g : R \to R$ is a continuous functions then

- $x_N \to^p x \Rightarrow g(x_N) \to^p g(x)$
- $x_N \to^d x \Rightarrow g(x_N) \to^d g(x)$
- $x_N \to^{a.s.} x \Rightarrow g(x_N) \to^{a.s.} g(x)$

where

$$\lim g(x_N) = g(\lim x_N)$$

Questions

Suppose $x_n \rightarrow^d N(\alpha, \sigma^2)$

- if $\hat{\alpha}_n \rightarrow^d \alpha \in R$, does $\hat{\alpha}_n$ converge in probability?
- $p\lim \hat{\alpha} = \alpha$, what is the asymptotic distribution of $x_n - \hat{\alpha}$?
- $p\lim \hat{\sigma}^2 = \sigma^2$, what is $p\lim 1/\sqrt{\hat{\sigma}^2}$ and the asymptotic distribution of $(x_n - \hat{\alpha})/\sqrt{\hat{\sigma}^2}$?

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Asymptotic theory

Laws of Large Number and Central Limit Theorem

Laws of large numbers

Khinchine's Theorem

Let x_n be a sequence of i.i.d. random variables with $E(x_n) = \mu < \infty$. Then

$$\bar{x_N} := \frac{1}{N} \sum_{n=1}^{N} x_n \to^p \mu$$

Laws of large numbers

Chebyshev's Theorem

Let x_n be a sequence of independent random variables with $E(x_i) = \mu_i < \infty$ and $Var(x_i) = \sigma_i^2 < \infty$, such that $\bar{\sigma}_n^2 = 1/n^2 \sum_{i=1}^n \sigma_i^2 \to 0$, then

$$\bar{x}_n \to^p \bar{\mu}_n$$

Question

Suppose, x_n is a sequence of independent random variables with $E(x_n) = \mu < \infty$ and $Var(x_n) \leq c < \infty$ for some finite constant c.

- Are the conditions of Chebyshev's Theorem satisfied?
- What is the $p\lim \bar{x_N}$?

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Central Limit Theorem

Lindberg-Levy CLT

Suppose x_n be a sequence of independent random variables with finite means μ_i and a finite variances σ_i^2. If no single element dominates

$$\lim_{n\to\infty} max(\sigma_i^2)/(n^2\bar{\sigma}_n^2) = 0$$

and there exists a finite constant $\bar{\sigma}^2$

$$\bar{\sigma}^2 = \lim_{n\to\infty} \bar{\sigma}_i^2 < \infty$$

then

$$\sqrt{n}(\bar{x}_n - \bar{\mu}_n) \to^d N(0, \bar{\sigma}^2))$$

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Question

Suppose x_n is a sequence of i.i.d. random variables with a mean μ and a finite variance σ^2.

- Are the conditions of Lindberg-Levy CLT satisfied?
- Formulate the CLT.

$$\sqrt{n}(\bar{x}_n - \mu) \rightarrow^d N(0, \sigma^2)$$

Question

Suppose x_n is a sequence of i.i.d. random variables with a mean μ and a finite variance σ^2.

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Question

Suppose $x_n \sim N(\mu, \sigma^2)$ is a sequence of i.i.d. random variables. Let $\hat{\mu}_n = 1/n \sum_{i=1}^{n} x_i$.

- What is the asymptotic distribution of $\hat{\mu}_n$ (use CLT)?
- Does it meter, if x_n does not have a normal distribution?

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Limiting distribution of a function

Suppose $\sqrt{n}(x_n - \mu) \rightarrow^d N(0, \sigma^2)$ and if $g(z)$ is a continuous and continuously differentiable function with $g'(z) \neq 0$, then

$$\sqrt{n}(g(x_n) - g(\mu)) \rightarrow^d N(0, g'(\mu)^2\sigma^2)$$

Question

Suppose $\sqrt{n}(x_n - 1) \rightarrow^d N(0, 5)$. What is the asymptotic distribution of $g(x_n)$, when

- $g(z) = z + 2$
- $g(z) = z^2$

Central Limit Theorem

Lindberg-Levy CLT

Let x_n be a sequence of K-dimensional i.i.d random variables with a mean μ and a covariance matrix Σ. Then

$$\sqrt{N}(\bar{x_N} - \mu) \rightarrow^{d} N(0, \Sigma)$$

Limiting distribution of a function

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$$\sqrt{n}(g(x_n) - g(\mu)) \rightarrow^d N(0, G(\mu)'\Sigma G(\mu))$$

Question

Suppose $\sqrt{n}x_n \to^d N(0, I_2)$. What is the asymptotic distribution of $g(x_n)$, when

- $g(z) = z_1 + z_2$
- $g(z) = [z_1, z_1 - z_2 + 1]'$

Asymptotic theory

Asymptotic properties of estimators

Properties of estimators

Properties of an estimator $\hat{\beta}$:

- $\hat{\beta}$ is unbias estimator of β if $E(\hat{\beta}) = \beta$
- $\hat{\beta}$ is (weakly) consistent if $\hat{\beta}_N \rightarrow^p \beta$
- $\hat{\beta}$ is strongly consistent if $\hat{\beta}_N \rightarrow^{a.s.} \beta$
- $\hat{\beta}$ is asymptotically normal if

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Linear combination of estimators

Suppose $\hat{\beta}$ is an estimator of a $(K \times 1)$ vector β with

$$\sqrt{N}(\hat{\beta} - \beta) \to^{d} N(0, \Sigma)$$

- Let $A \neq 0$. What is the asymptotic distribution of $\sqrt{N}A(\hat{\beta} - \beta)$?
- Suppose, $p\lim \hat{A} = A$. Does it change the previous result?

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Example 1

Suppose $K = 2$ and $\beta = (2, 2)'$ and

$$\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 3 \end{bmatrix}$$

What is the asymptotic distribution of $\hat{\beta}_1 - \hat{\beta}_2$?

$$\sqrt{N}(\hat{\beta}_1 - \hat{\beta}_2) \rightarrow^d N(0, 3)$$

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Delta method

Suppose $g(\beta) = (g_1(\beta), ..., g_m(\beta))'$ is a vector-valued continuously differentiable function with $\partial g(\beta)/\partial\beta' \neq 0$ at β, then

$$\sqrt{N}(g(\hat{\beta}) - g(\beta)) \to^d N(0, \frac{\partial g(\beta)}{\partial\beta'}\Sigma\frac{\partial g(\beta)'}{\partial\beta})$$

Remark: if $\partial g(\beta)/\partial\beta' = 0$ then

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Remark: if $\partial g(\beta)/\partial\beta' = 0$ then

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Example 2

Suppose the true parameter $\beta = 1$ is a single valued scalar and $\sqrt{N}(\hat{\beta} - \beta) \rightarrow^{d} N(0, 2)$

What is an asymptotic distribution of $g(\hat{\beta}) = \hat{\beta}^3 + \hat{\beta}^2 - 2$?

$$\sqrt{N}(g(\hat{\beta}) - g(\beta)) = \sqrt{N}g(\hat{\beta}) \rightarrow^{d} N(0, 50)$$

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$$\sqrt{N}(g(\hat{\beta}) - g(\beta)) = \sqrt{N}g(\hat{\beta}) \to^d N(0, 50)$$

Quadratic form

Suppose, Σ is nonsingular, $(K \times K)$ matrix and

$$\sqrt{N}(\hat{\beta} - \beta) \to^{d} N(0, \Sigma)$$

- What is the asymptotic distribution of $N(\hat{\beta} - \beta)'\Sigma^{-1}(\hat{\beta} - \beta)$?
- Suppose, $p\lim \hat{\Sigma} = \Sigma$. Does it change the previous result?

$$N(\hat{\beta} - \beta)'\Sigma^{-1}(\hat{\beta} - \beta) \to^{d} \chi^2(K)$$

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Example 3

Suppose, $\sqrt{N}(\hat{\beta} - \beta) \to^d N(0, \Sigma)$, with a nonsingular, $(K \times K)$ matrix Σ.

- Find a matrix M such that a quadratic form of $A(\hat{\beta} - \beta)$ will converge to $\chi^2(K)$

What is the asymptotic distribution of a quadratic form of $A(\hat{\beta} - \beta)$, where A is a quadratic,nonsingular matrix?

$$N(\hat{\beta} - \beta)' A' (A \Sigma A')^{-1} A(\hat{\beta} - \beta) \to^d \chi^2(K)$$

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Example 4

What happens if the matrix A is not quadratic, but has dimension $(M \times K)$, where $M < K$ and $rank(A) = M$?

Similarly to the previous example, the quadratic form will have a χ^2 distribution. It will have M degrees of freedom.

$$N(\hat{\beta} - \beta)' A' (A \Sigma A')^{-1} A (\hat{\beta} - \beta) \to^d \chi^2(M)$$

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