Laboratory 1

Convergence and asymptotic theory.

Exercise 1

- Generates a matrix Y (1000×1) of random normal variables with mean 2 and a variance 4. Suppose $Y_1 = 3 * (Y - 1)$. What is the distribution of Y_1? Plot its histogram.
- Create a variable $Y_2 = ((Y - 2)/2)^2$. What is the distribution of Y_2? Plot its histogram.
- (Convergence of a mean and a variance) Compute a sequence of means m_n and a sequence of variances σ_n^2 for the variable Y, where

$$m_n = \frac{1}{n}\sum_{i=1}^{n} Y_i,$$

$$\sigma_n^2 = \frac{1}{n}\sum_{i=1}^{n}(Y_i - m_n)^2.$$

- Plot the sequences. What do the sequences of means and variances converge to? What can you say about the variability of the sequences?

Exercise 2

- Simulate (10000 times) a random variable $X \sim N(0, I_2)$.
- Transform the variable X into the variable $Y \sim N(\mu, \Sigma)$: $\mu = [\begin{array}{cc} 0 & 1 \end{array}]'$ and

$$\Sigma = \left[\begin{array}{cc} 2 & 0.5 \\ 0.5 & 2 \end{array}\right].$$

  Hint: find a vector a and a matrix A, such that $Y = AX + a$; use the property $Var(AX) = AVar(X)A'$.
- Plot the 3-D histogram with bars colored according to height and 30 bins.
- Transform the variable Y into the variable $Z = (Y - \mu)'\Sigma^{-1}(Y - \mu)$. What is the distribution of the new variable? Plot its histogram.

Exercise 3

Let $\hat{\beta}$ be an estimator (a sequence of estimators) of a $(K \times 1)$ vector β, which is asymptotically normal with

$$\sqrt{N}(\hat{\beta} - \beta) \to_d N(0, \Sigma)$$

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- If $R \neq 0$ is an $(M \times K)$ matrix, what is the asymptotic distribution of $\sqrt{N}(R\hat{\beta} - R\beta)$?
- If $p\lim \hat{A} = A$, what is the asymptotic distribution of $\sqrt{N}\hat{A}(\hat{\beta} - \beta)$?
- If Σ is nonsingular and $p\lim \hat{\Sigma} = \Sigma$, prove that $N(\hat{\beta} - \beta)'\hat{\Sigma}^{-1}(\hat{\beta} - \beta) \to_d \chi^2(K)$.