Perturbation Methods (MAT 1572)

Winter Semester 2017/2018

List 1

1. Write down an order relation that expresses the fact that $\varepsilon^2(1+\ln\varepsilon)$ decays to zero faster than ε as $\varepsilon \to 0$ and prove your assertion.

2. Verify the following order relations:

   (a) $e^{\varepsilon} - 1 = O(\varepsilon)$ as $\varepsilon \to 0$.

   (b) $x = O(x^2)$ as $x \to \infty$.

   (c) $\int_0^{\varepsilon} e^{-x^2}\,dx = O(\varepsilon)$ as $\varepsilon \to 0$.

   (d) $\sin\varepsilon = O(\varepsilon)$ as $\varepsilon \to 0$.

   (e) $\ln\varepsilon = o(\varepsilon^{-p})$ as $\varepsilon \to 0$ for all $p > 0$.

   (f) $e^{-\varepsilon} = o(\varepsilon^{-p})$ as $\varepsilon \to 0$ for all $p > 0$.

   (g) $e^{-1/\varepsilon} = o(\varepsilon^{p})$ as $\varepsilon \to 0$ for all $p \in \mathbb{R}$.

3. Consider the function $f(x) = (1+\varepsilon x)^{-3/2}$, $x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \ldots$. Expand f in powers of ε up to $O(\varepsilon^2)$.

4. Find an asymptotic expansion of $f(x) = e^{1/x^2}\cos(e^{1/x^2})$ for $x \to 0$. Consider the derivative of this expansion and compare it with $f'(x)$.

5. Consider the approximation of $f(x) = x + \varepsilon^{-x/\varepsilon}$ by $g(x) = x$ for $x \in [0,1]$. Is this approximation uniformly valid?