Perturbation Methods (MAT 1572)

Winter Semester 2017/2018

List 4

1. Show that regular perturbation fails on the boundary value problem

$$\varepsilon y'' + 2y' + y = 0, \quad 0 < t < 1, \quad 0 < \varepsilon \ll 1$$

with $y(0) = 0$, $y(1) = 1$. Find the exact solution and sketch it for $\varepsilon = 0.05$ and $\varepsilon = 0.005$. If $t = O(\varepsilon)$, show that $\varepsilon y''$ is large. If $t = O(1)$, show that $\varepsilon y'' = O(1)$. Find an inner and an outer approximation of the exact solution. Find a uniform approximation of the exact solution.

2. Use singular perturbation methods to obtain a uniform approximate solutions to the problems

   (a) $\varepsilon y'' + t^{\frac{1}{3}} y' + y = 0$, $y(0) = 0$, $y(1) = e^{-\frac{3}{2}}$

   (b) $\varepsilon y'' - (2t + 1)y' + 2y = 0$, $y(0) = 1$, $y(1) = 0$

   In each case consider $0 < t < 1$ and $0 < \varepsilon \ll 1$.

3. Use the singular perturbation method to obtain a uniform approximate solution to the following problems

   (a) $\varepsilon y'' + (t - \frac{1}{2})y = 0$, $y(0) = 1$, $y(1) = 2$,

   (b) $\varepsilon y'' - (2 - t^2)y = -1$, $y(-1) = 1$, $y(1) = 1$.

4. Find a uniformly valid approximation to

$$\varepsilon y''(t) - a(t)y(t) = f(t) \qquad 0 < t < 1$$

$$y(0) = 0, \; y(b) = -f(1)/a(1)\,,$$

where $0 < \varepsilon \ll 1$ and $a > 0$, and a and f have infinitely many derivatives on $\mathbb{R}$.