Life insurance models

List 1.

1. The probability of a fire in a certain structure in a given time period is 0.02. If a fire occurs, the damage to the structure is uniformly distributed over the interval form 0 to its total value, a. Calculate the mean and variance of fire damage to the structure within the time period.
2. Let us consider 2-years term life insurance in which the insurer agrees to pay 100, if the insured dies within two years and pays nothing if the insured survives. The probability of death during the two years is equal 0.005. Find the probability and distribution functions of the claim random variable. Calculate the expected value and variance of the claim random variable.
3. Let us consider a 2-years term life insurance paying an extra benefit in case the insurer dies within the first year of policy issue. To be specific, if death occurs within the first year of policy issue, the benefit amount is 4500. If the insured dies within the second year of the policy issue, the benefit amount is 5000. Let us assume that for the age, health and occupation of a specific individual, the probability of death within the first year is 0.004 while the probability of death during the second year is 0.005. Find the distribution of the indicator I and conditional distribution of B, given $I = 1$.
4. Let us consider an automobile insurance. Assume, that for a particular individual the probability of 1 claim in a period is 0.2 and the chance of more than 1 claim is 0. Let us assume that the claim amount B is a random variable, which conditional distribution (when the claim is occured) is given by:

$$P(B = 20|I = 1) = 0.2,\ f_{B|I=1}(x) = b(x-2)\mathbb{I}_{\{2<x<20\}}.$$

   - Find the parameter b.
   - Find the conditional distribution function $P(B \leqslant x|I = 1)$.
   - Find the distribution function of the claim random variable X
   - Find probability function of X.
   - Calculate $\mathbb{E}X$ and $\mathrm{Var}X$.
5. Consider a portfolio of 32 policies. For each policy, the probability q of a claim is $\frac{1}{6}$ and B, the benefit amount given that there is a claim, has p.d.f

$$f(y) = \begin{cases} 2(1-y) & \text{for } 0 < y < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

   Let S be the total claims for the portfolio. Using a normal approximation, find $P(S > 4)$.
6. A life insurance company issues 1-year term life contracts for benefit amounts of 1 and 3 units to individuals with probabilities of death of 0.1 and 0.03. The following table gives the

number of individuals n_k in each of the four classes created by a benefit amount b_k and a probability of claim q_k.

k	q_k	b_k	n_k
1	0.1	1	200
2	0.03	3	300
3	0.1	1	150
4	0.03	3	200

Find the relative security loading θ such that $P(S \leqslant (1+\theta)\mathbb{E}S) = 0.99$, where S is a total claims.

7. The policyholders of an automobile insurance company fall into two classes

Class k	**Number in Class** n_k	**Claim Probability** q_k
1	50	0.2
2	20	0.09

In each class the claim amount B_k has a d.f. of the following form

$$F(x) = \begin{cases} 0 & \text{for } x < 0, \\ \dfrac{x}{a} & \text{for } 0 \leqslant x < \frac{a}{2} \\ 1 & \text{for } x \geqslant \frac{a}{2} \end{cases}$$

For the two classes we have the following parameters a

Class k	a
1	2
2	1

Calculate θ-the relative security loading such that $P(S \leqslant (1+\theta)\mathbb{E}S) = 0.95$, where S is a total claims.

8. A fire insurance company covers 160 structures agains the fire damage up to an amount stated in the contract. The numbers of contracts at the different contract amounts are given in the table

Contract Amount	**Number of Contracts**
10	80
20	35
30	25
50	15
100	5

Assume that for each of the structures, the probability of 1 claim within a year is 0.04 and the probability of more than 1 claim is 0. Assume that fires in the structures are mutually independent events. Furthemore, assume that the conditional distribution of the claim size, given that a claim is occured, is uniformly distributed over the interval form 0 to the contract amount. Let S be the amount of claims in a $1-$year period. Calculate the mean and variance of S.