Life insurance models

List 3.

1. Let for each x the random variable $T(x)$ has p.d.f. given by $g(t) = ce^{-ct}$ for $t \geqslant 0$ and $c > 0$. Calculate $\overset{\circ}{e}_x$.

2. If for the age x the force of mortality $\mu_{x+t} = \frac{t}{3600}$, $t \geqslant 0$, calculate density of $T(x)$ and $\overset{\circ}{e}_x$.

3. If for each x the future life-time $T(x)$ is a random variable with d.f given by

$$G(t) = \begin{cases} \frac{t}{100-x} & \text{for } 0 \leqslant t < 100 - x \\ 1 & \text{if } t \geqslant 100 - x, \end{cases}$$

   calculate $\overset{\circ}{e}_x$ and $\mathrm{Var}(T(x))$.
4. Consider a modification of de Moivre's law given by

$$s(x) = \left(1 - \frac{x}{\omega}\right)^{\alpha}, \; 0 \leqslant x < \omega, \; \alpha > 0.$$

   Calculate μ_x and $\overset{\circ}{e}_x$.
5. If the force of mortality μ_{x+t}, will be changed to $\mu_{x+t} + c$, where c is constant, find the value of c for which the probability that (x) will survive one year will be halved?
6. For the Moivre's law with parameter ω calculate the survival function.
7. Let us consider the population with Weibull's law

$$\mu_x = kx, \; k > 0.$$

   Find x_{max} for which the p.d.f. of the random variable X takes the maximum value.
8. Let us consider two independent populations with the following mortality functions (the Gompertz's law): $\mu_x^{(1)} = B2^x, \mu_x^{(2)} = 2B8^x$. If we know that $P(X^{(2)} \leqslant 50) = 1/3$, calculate ${}_{150}p_1^{(1)}$.
9. Let us consider the population with $\mu_x = 0.0001 \times 1,1^x$. Calculate the probability that the couple (40) and (50) survives 10 years.
10. In A population the force of mortality is given by

$$\mu^A = \frac{1}{100 - x}, \quad x < 100.$$

   In B population we have

$$\mu^B = \frac{n}{100 - x}, \quad x < 100,$$

   where n is a parameter. It is known that individuals in the population A have an average of 10% more life than individuals from populations B in the same age. Find n.

11. The force of mortality is given by the following formula

$$\mu_x = \frac{0.6}{100 - x}, \ x < 100.$$

Find probability that newborn will survive to age $x = \overset{\circ}{e}_0$.