**Perturbation Methods (MAT 1572)**

Winter Semester 2017/2018

**List 3**

1. Find the first three terms of the perturbation series solution to the initial value problem
$$y' = 1 + (1+\varepsilon)y^2, \quad y(0) = 1, \quad t > 0$$
where $0 < \varepsilon \ll 1$. Find the exact solution and compare the approximation. Is it uniform? Is it enough to consider only the first term of the asymptotic expansion to approximate the exact solution? Are the first two terms enough?

2. Consider the initial value problem
$$y'' + (1+\varepsilon)y = 0, \quad t > 0, \quad 0 < \varepsilon \ll 1$$
with $y(0) = 1$ and $y'(0) = 0$. Find the exact solution. Find a two-term perturbation approximation. Compare the exact solution to the perturbation approximation for large $t$.

3. The pendulum problem can be scaled to obtain the initial value problem
$$\begin{cases} y'' + \dfrac{\sin(\varepsilon y)}{\varepsilon} = 0 \quad t > 0,\, 0 < \varepsilon \ll 1 \\ y(0) = 1,\, y'(0) = 0\,. \end{cases}$$
Apply the regular perturbation method to find a two term expansion. Show that the correction term is secular term and comment on the validity of the approximation.

4. Apply the Poincaré-Lindstedt method to the scaled pendulum problem to obtain a two-term perturbation solution.