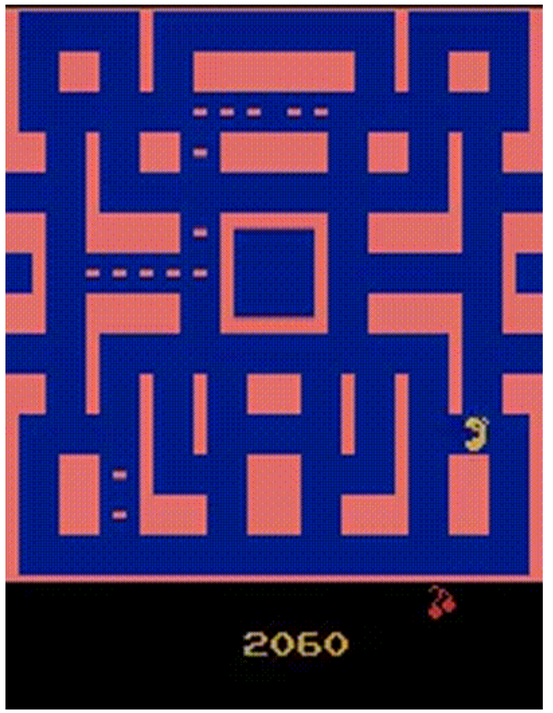
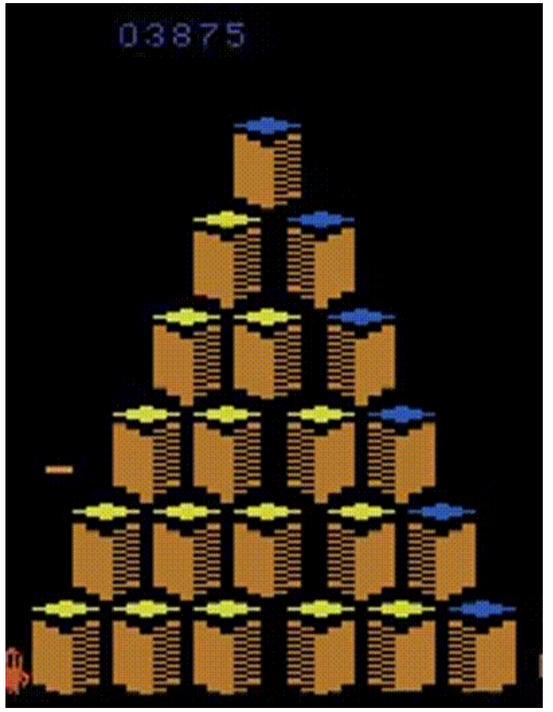
CS 224R Tutorial

Review of Q-Learning





Anikait Singh

Outline of Tutorial

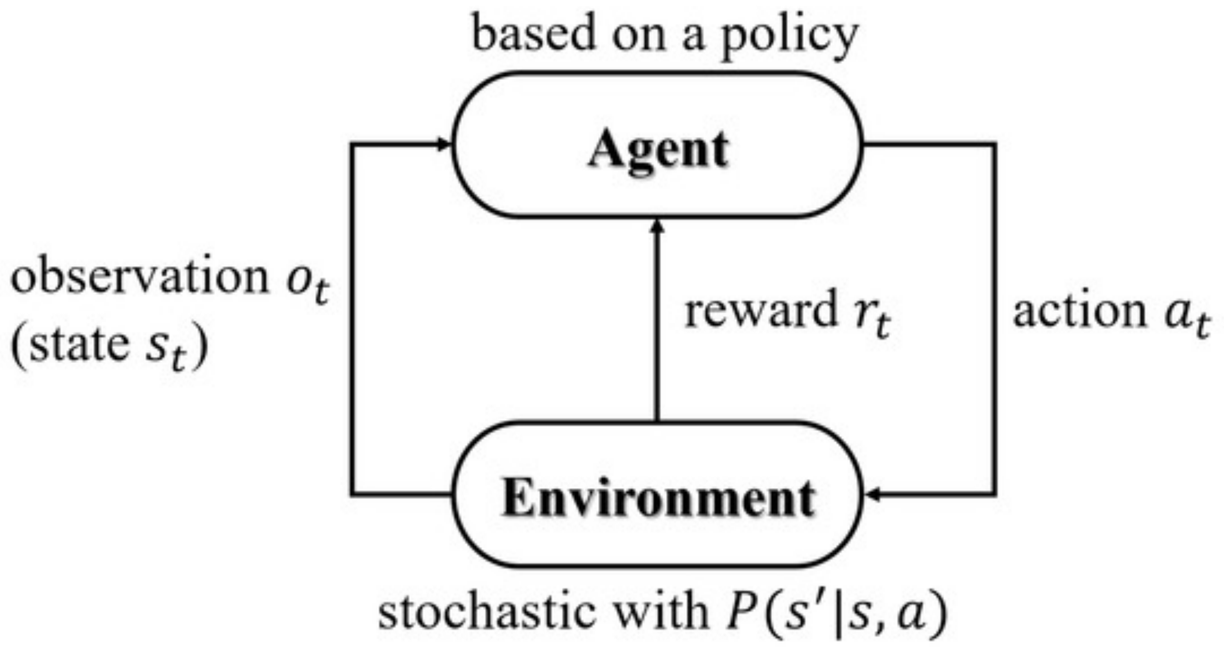
- Review of Markov Decision Processes (MDP)
- Exact MDPs
  - Fitted Q Iteration
- Parametric Q-Learning
  - Bias Variance Tradeoff (TD vs MC)
- Practical Details
  - Replay Buffer, Overestimation, TD Gradients
- Q-Learning/Actor Critic Algorithm Walkthrough (Preview of Homework 2)

Many thanks to Pieter Abbeel, David Silver, Aviral Kumar, and Justin Fu!

MDP Formulation

An MDP is defined by:

- Set of states s_t
- Set of actions a_t
- Transition function $p(s_{t+1} \mid s_t, a_t)$
- Reward Function $r(s_t, a_t)$
- Start state s_1
- Discount Factor γ
- Horizon T



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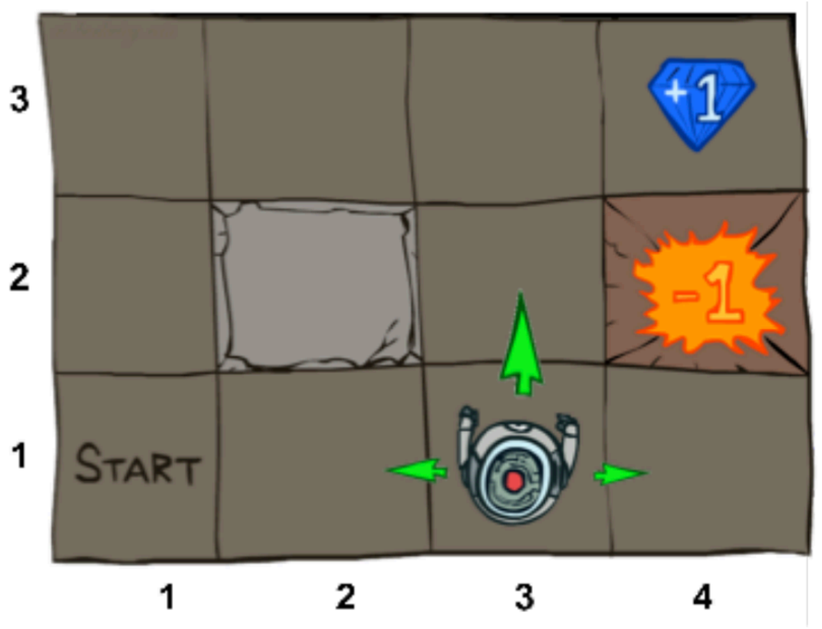
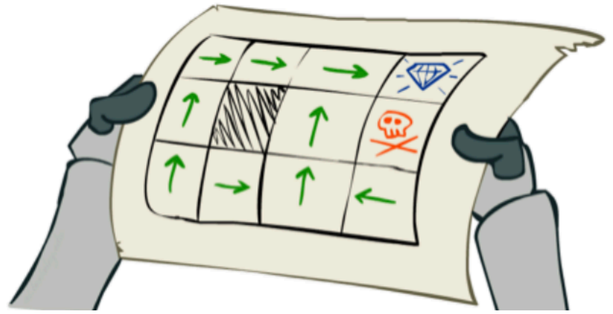


Goal: $\max_{\pi} \mathbb{E}\left[\sum_{t=1}^{T} \gamma^t r(s_t, a_t) \mid \pi\right]$

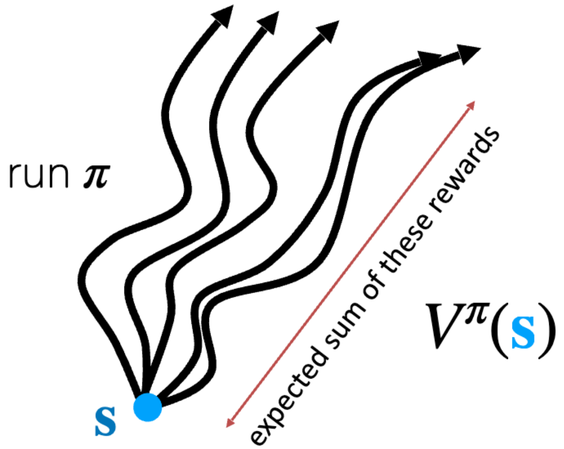
π:

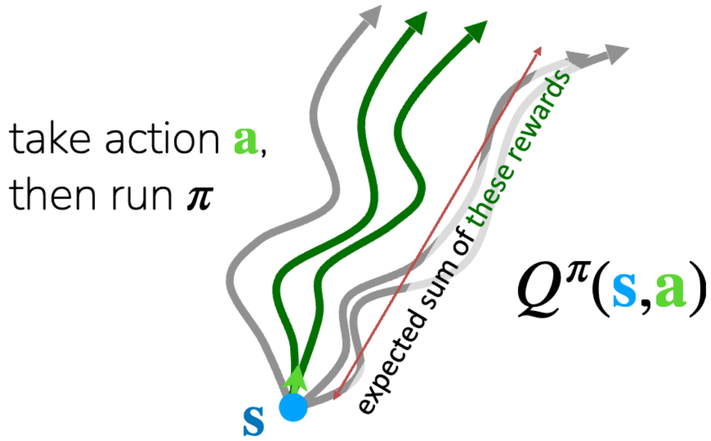


Revisiting Some Useful Objects

value function $V^{\pi}(\mathbf{s})$ - future expected rewards starting at $\mathbf{s}$ and following π

Q-function $Q^{\pi}(\mathbf{s}, \mathbf{a})$ - future expected rewards starting at $\mathbf{s}$, taking $\mathbf{a}$, then following π





Useful relation: $V^{\pi}(\mathbf{s}) = \mathbb{E}_{\mathbf{a}\sim\pi(\cdot|\mathbf{s})}\left[Q^{\pi}(\mathbf{s}, \mathbf{a})\right]$

advantage $A^{\pi}(\mathbf{s}, \mathbf{a})$ - how much better it is to take $\mathbf{a}$ than to follow policy π at state $\mathbf{s}$

$$A^{\pi}(\mathbf{s}, \mathbf{a}) = Q^{\pi}(\mathbf{s}, \mathbf{a}) - V^{\pi}(\mathbf{s})$$

Tabular Learning - Fitted Q Iteration

Q-Function $Q^{\pi}(\mathbf{s}, \mathbf{a})$ - future expected rewards starting at state $\mathbf{s}$, taking action $\mathbf{a}$, then following π

Bellman Equation

$$Q^*(s,a) = \sum_{s_{t+1}} p(s_{t+1} \mid s_t, a_t) \left(r(s_t, a_t) + \gamma \max_{a_t} Q^*(s_{t+1}, a_{t+1} \right)$$

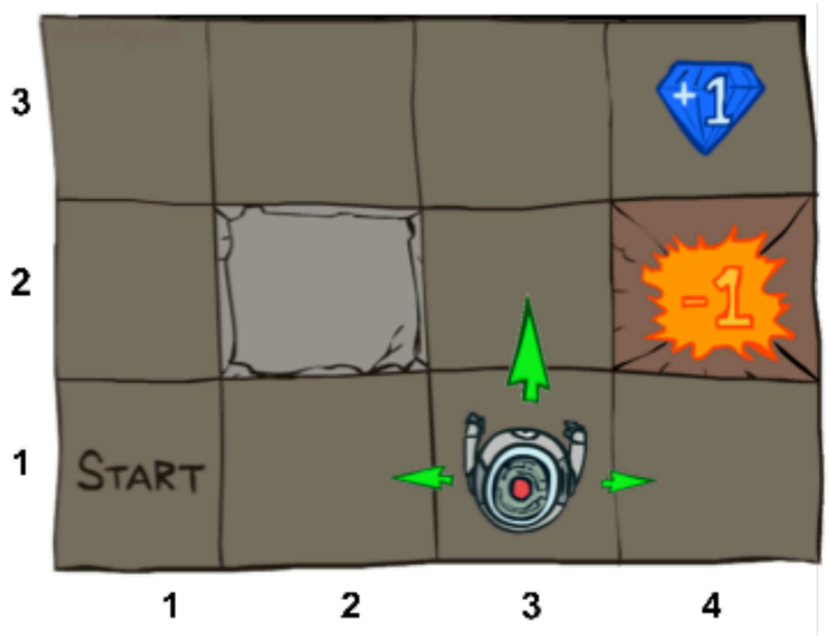
Q-Value Iteration

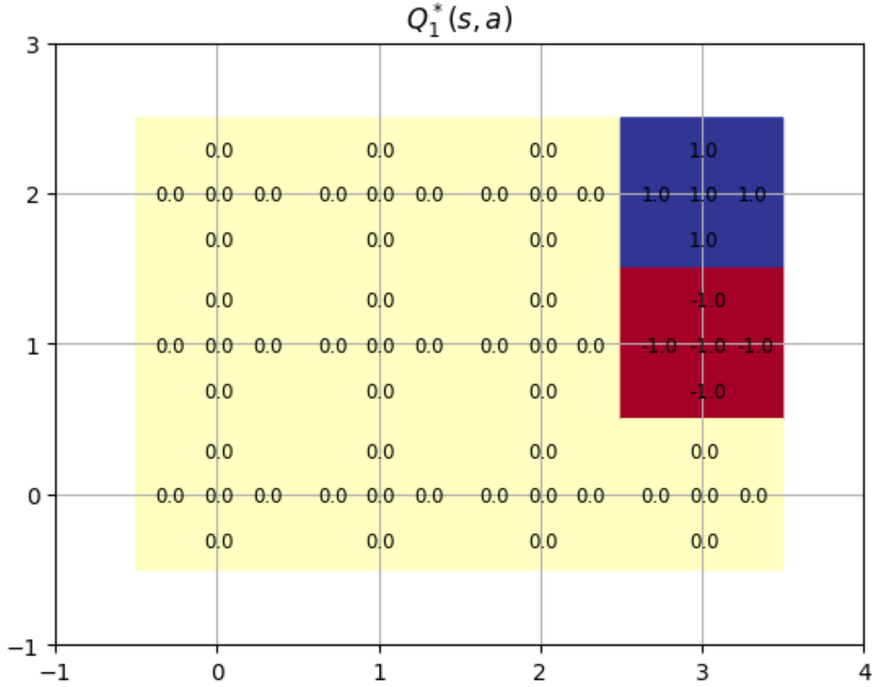
$$Q^*_{k+1}(s,a) = \sum_{s_{t+1}} p(s_{t+1} \mid s_t, a_t) \left(r(s_t, a_t) + \gamma \max_{a_t} Q^*_k(s_{t+1}, a_{t+1} \right)$$

Fitted Q Iteration with the MDP

Discount = 0.9

$$Q^*_{k+1}(s,a) = \sum_{s_{t+1}} p(s_{t+1} \mid s_t, a_t) \left(r(s_t, a_t) + \gamma \max_{a_t} Q^*_k(s_{t+1}, a_{t+1}) \right)$$





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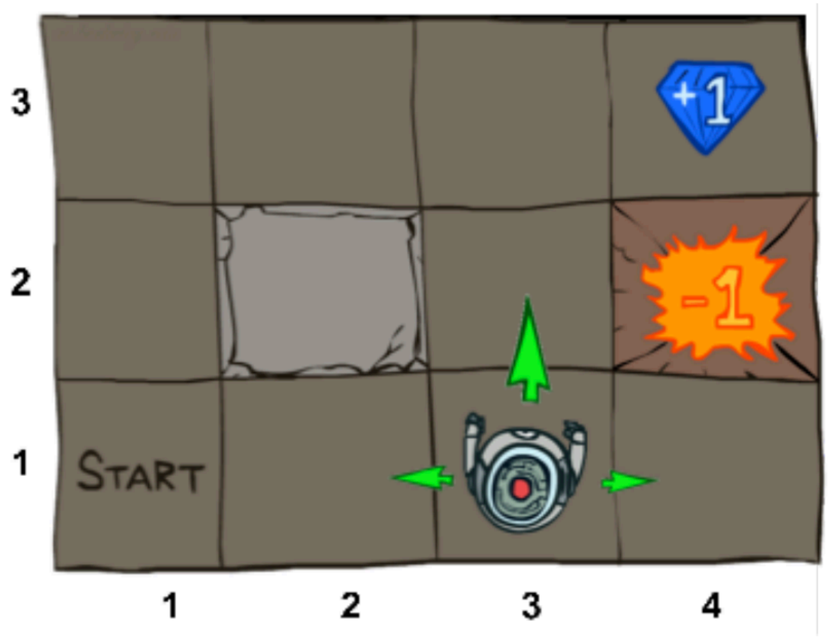

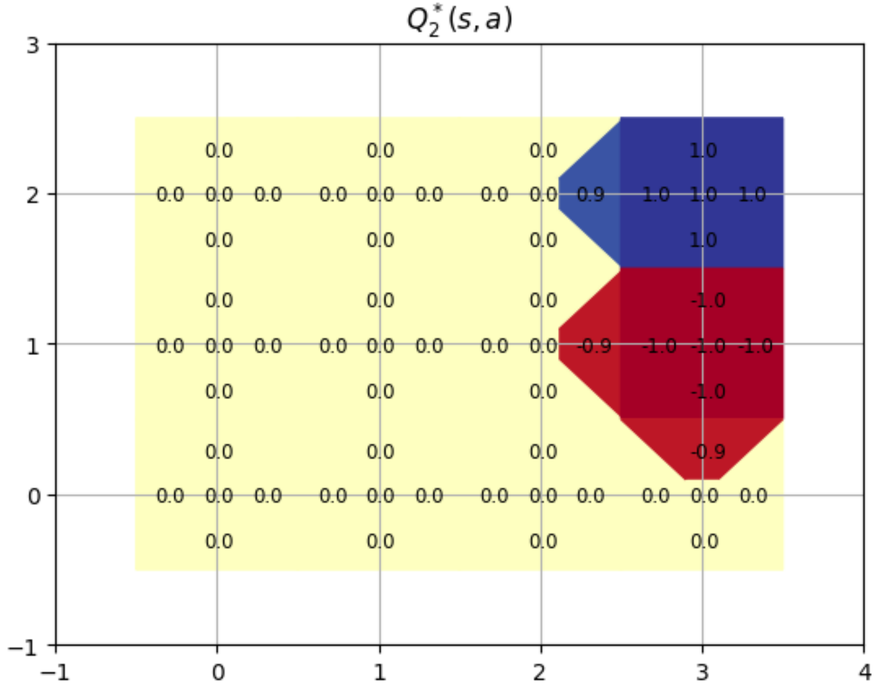

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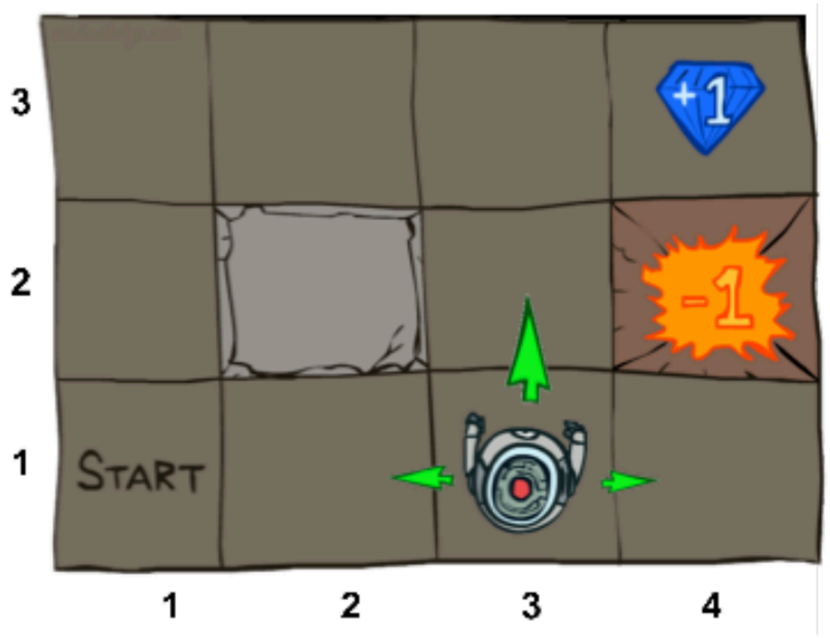

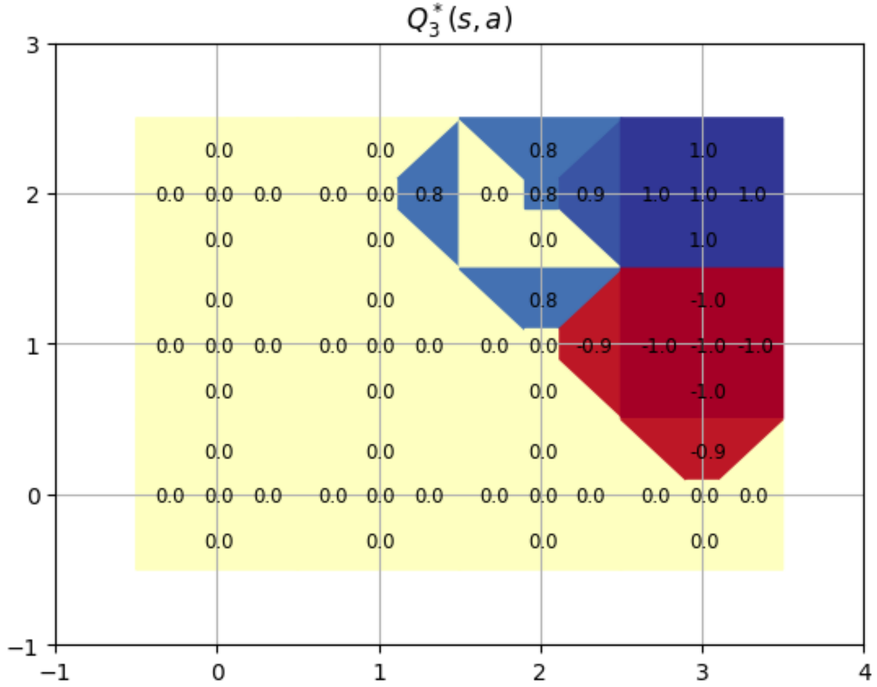

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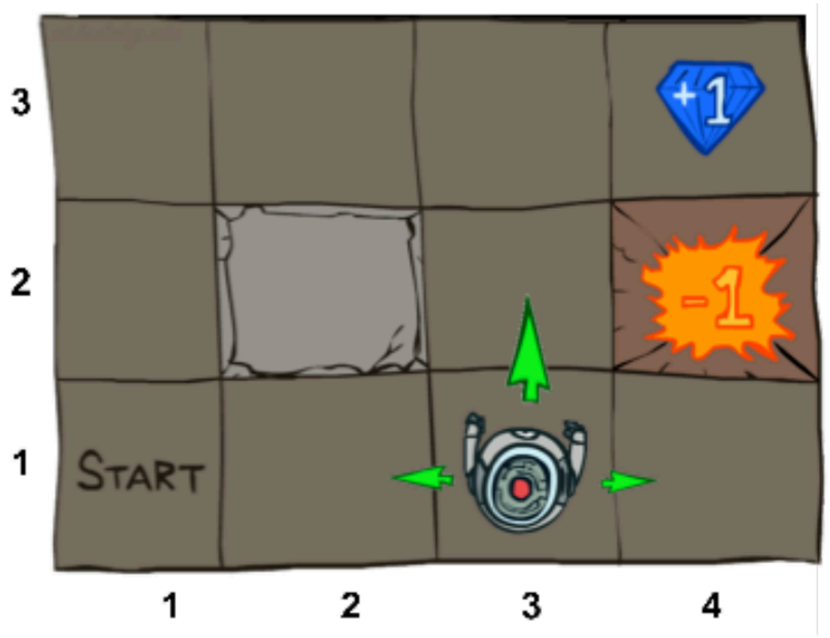

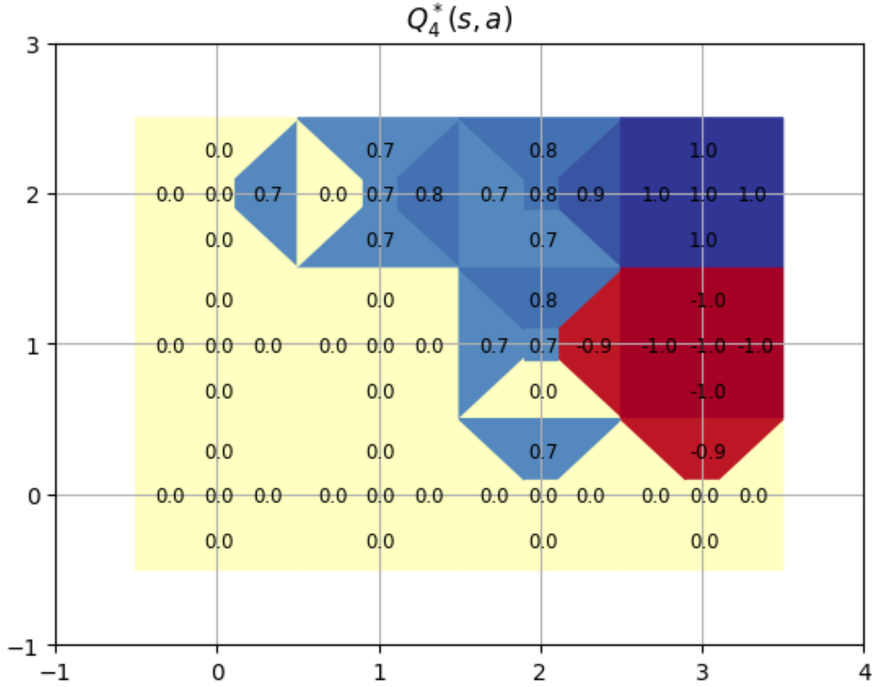

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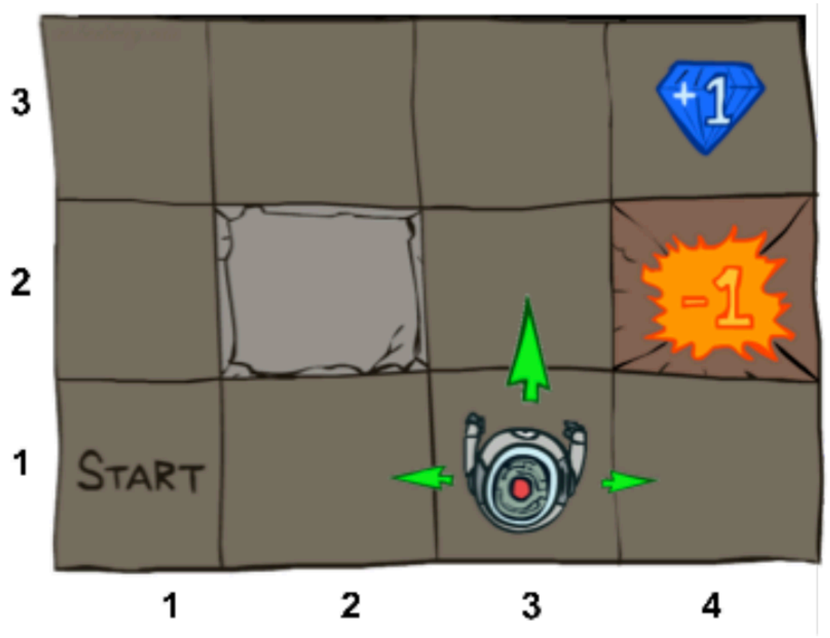

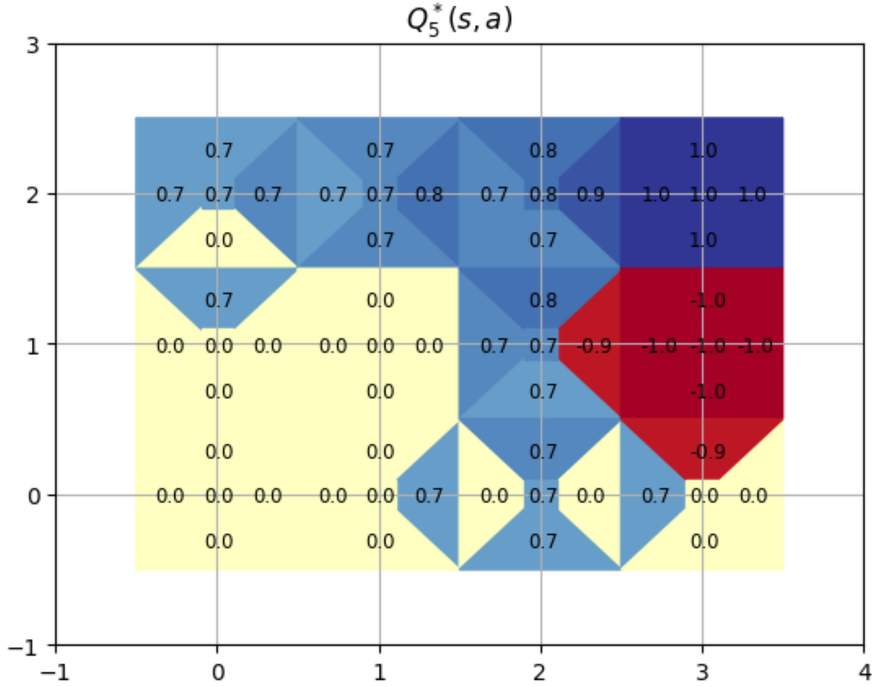

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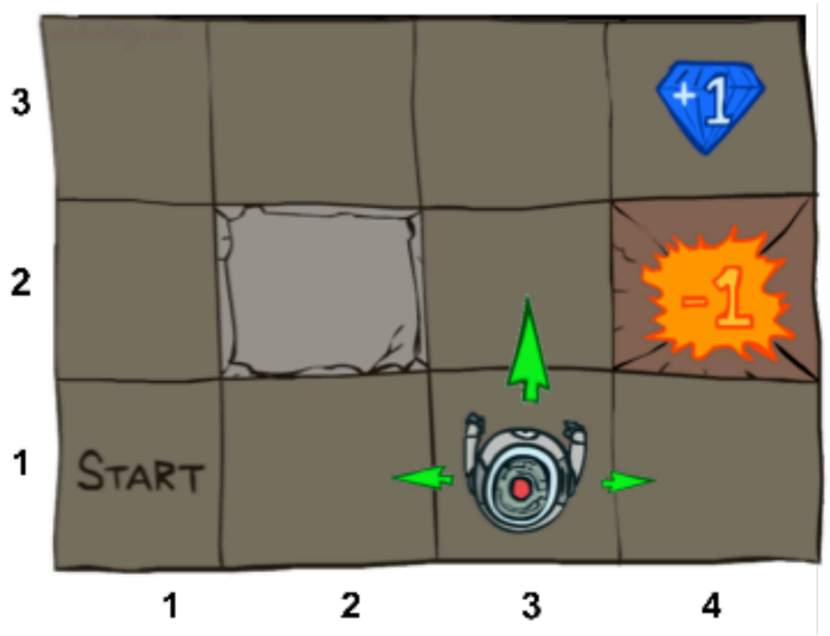
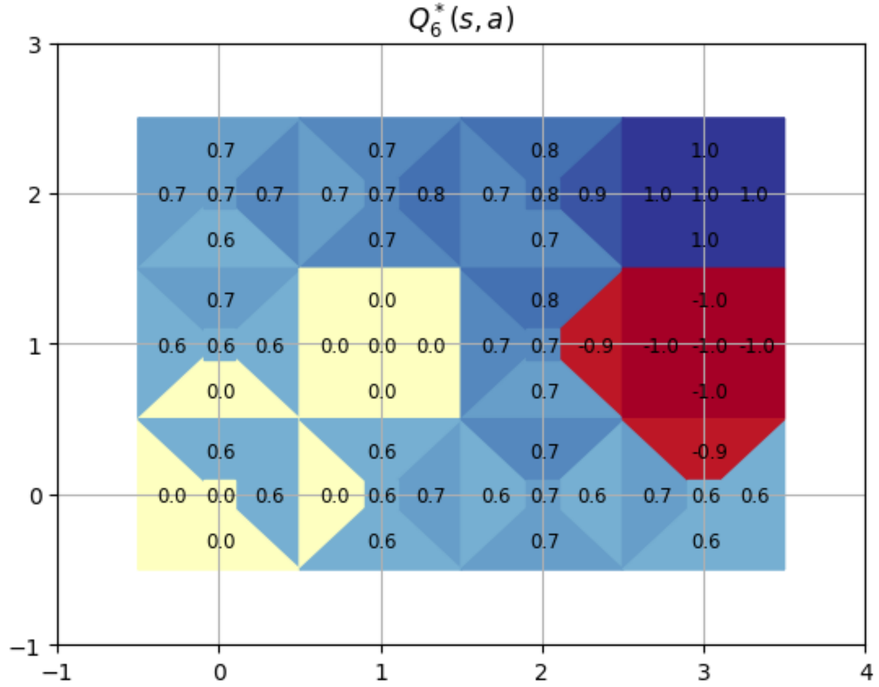
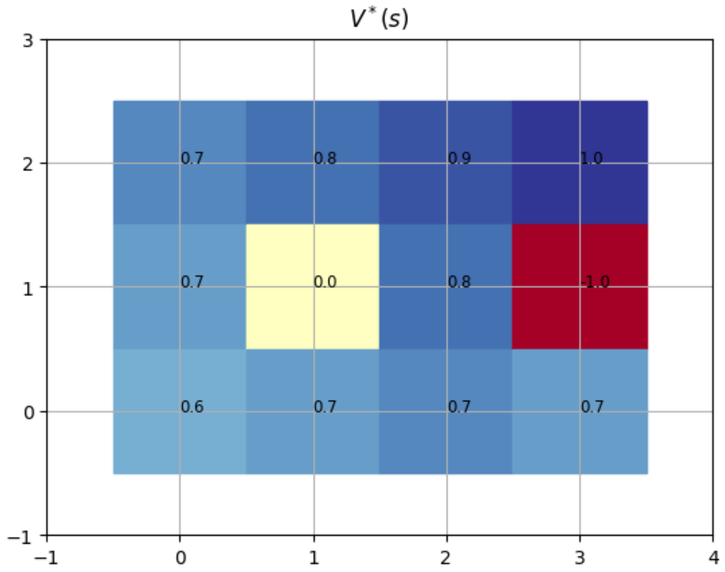
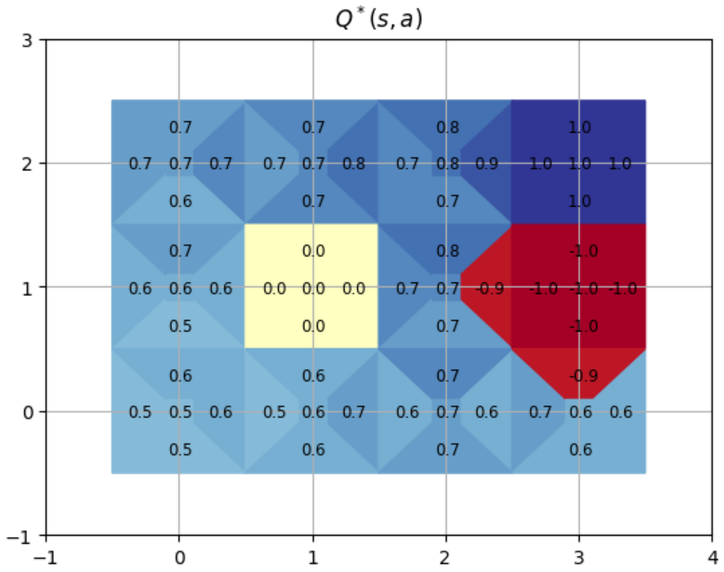
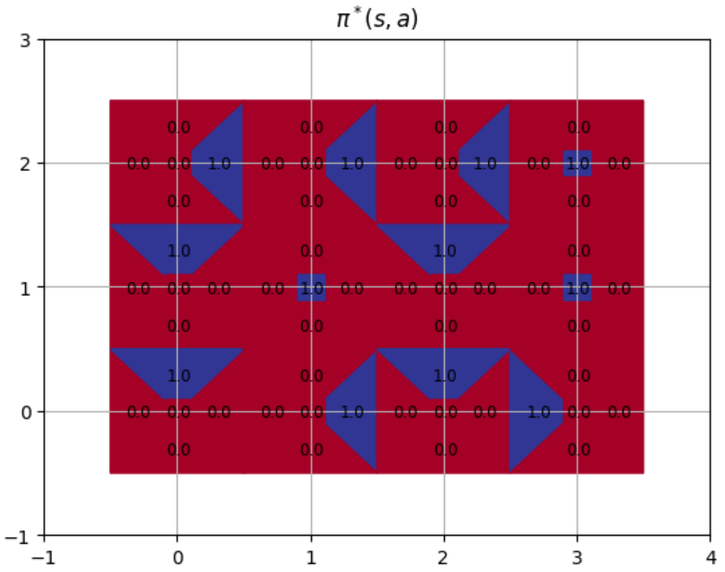




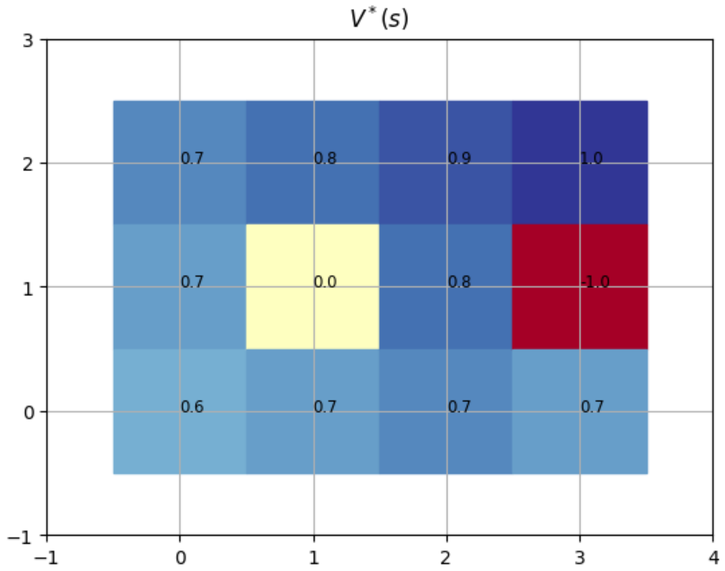
Optimal Value, Q-Value, Policy

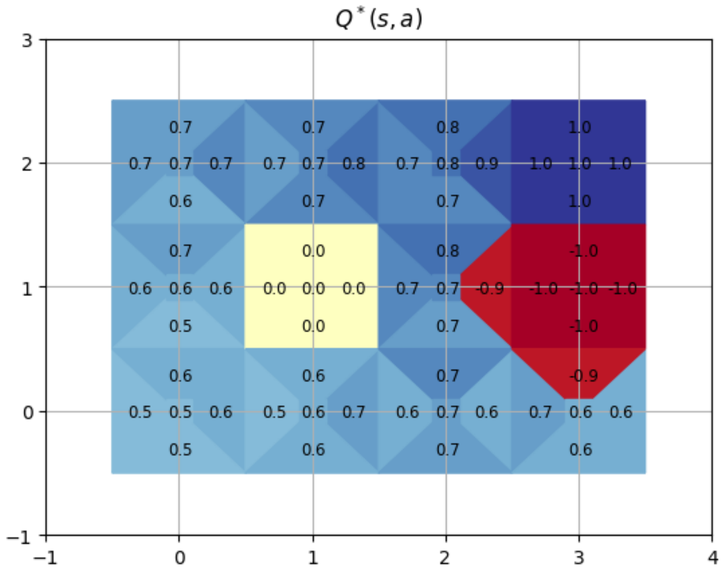


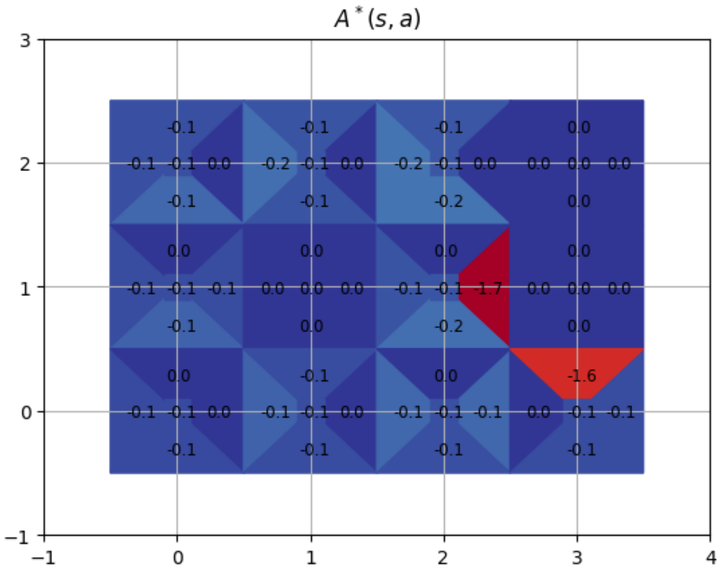




Advantage (Optimal Policy)



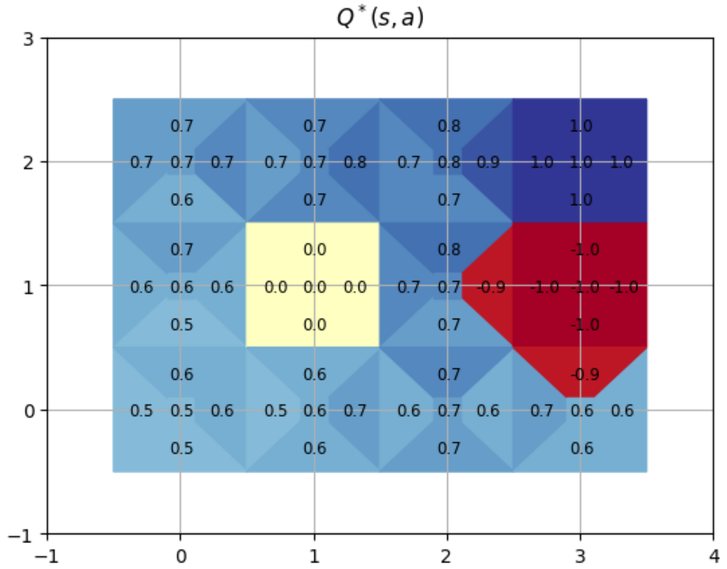


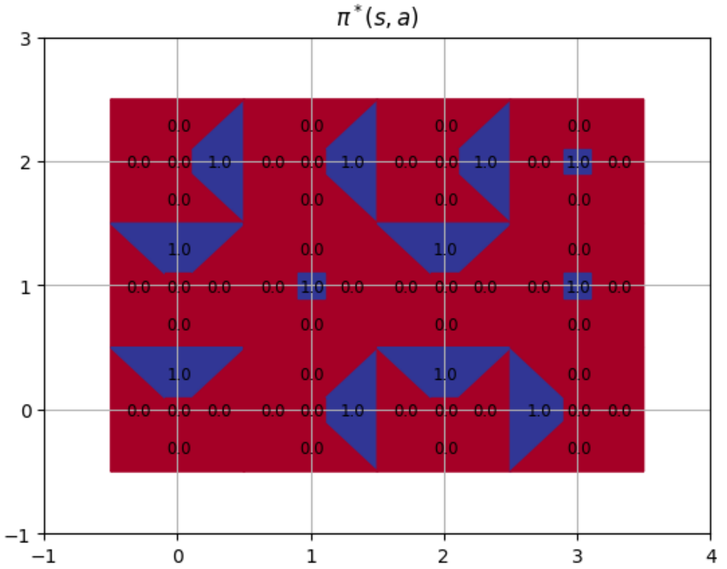


$$A^{\pi}(\mathbf{s}_t, \mathbf{a}_t) = Q^{\pi}(\mathbf{s}_t, \mathbf{a}_t) - V^{\pi}(\mathbf{s}_t)\text{: how much better } \mathbf{a}_t \text{ is}$$

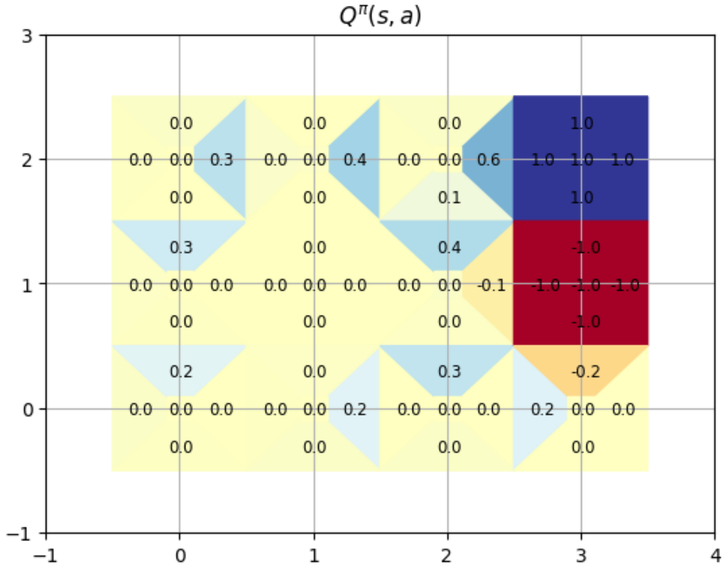
Q^* vs Q^π

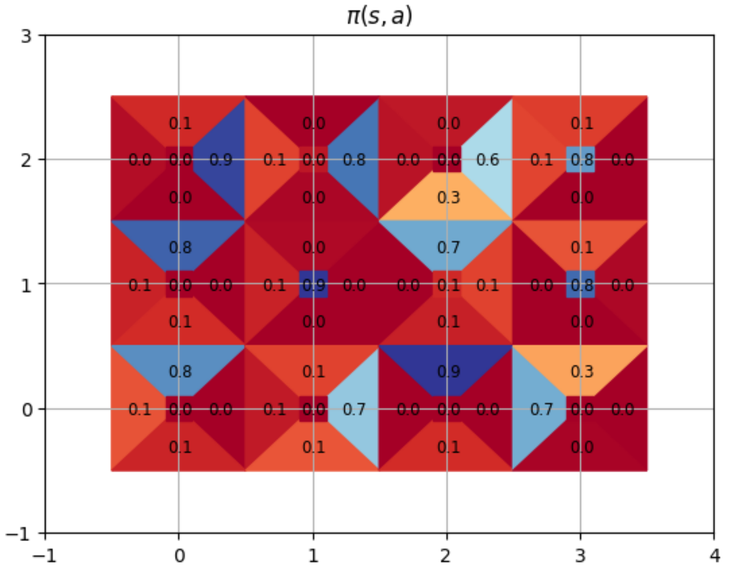
Optimal Policy
(Deterministic for MDP)





Suboptimal Policy





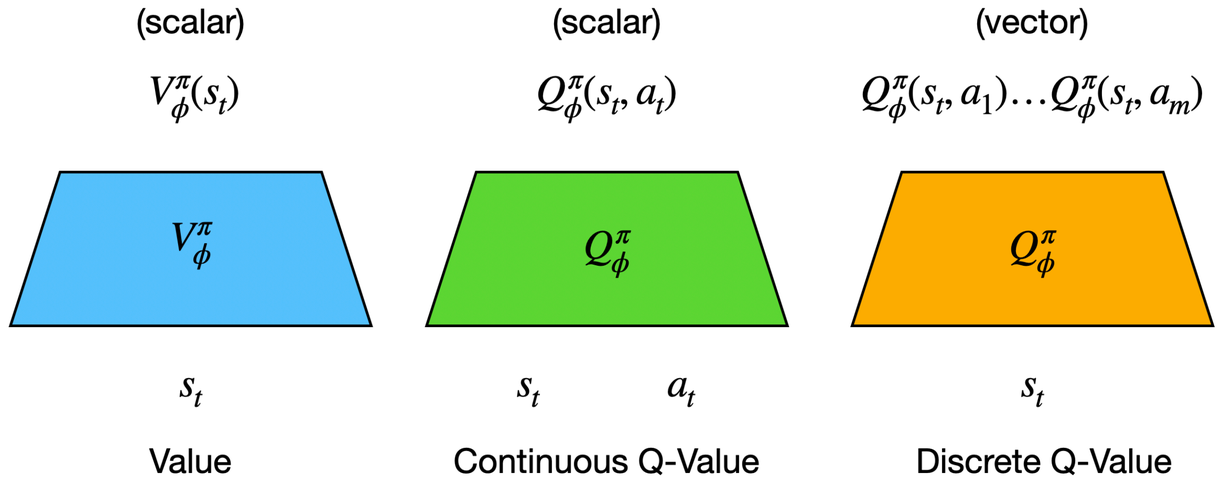
To play more with the Tabular Environment, checkout the colab

Pause for Questions.

Limitations of Tabular Environment

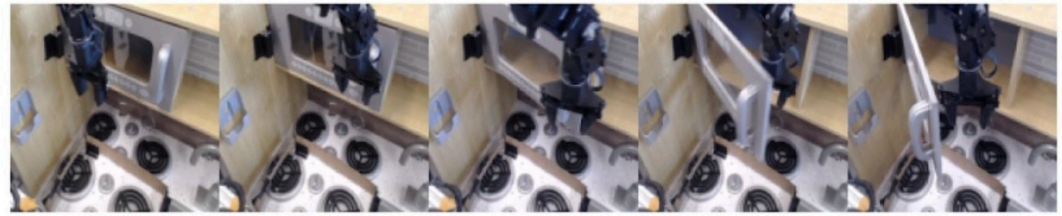
Moving towards parametric value functions

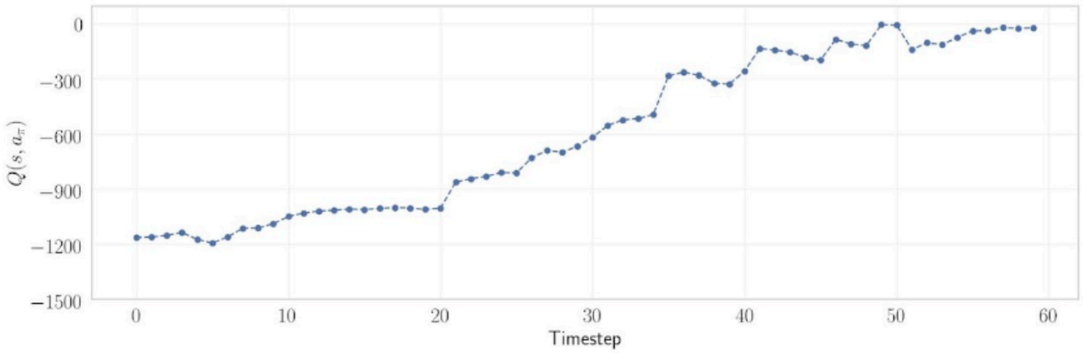
- A tabular value function intractable as state/action space grows!
- Learning a parametric value function can also allow for generalization
  - Related states have similar q-values (smoothness)
- Can use regression to learn value functions with a neural network

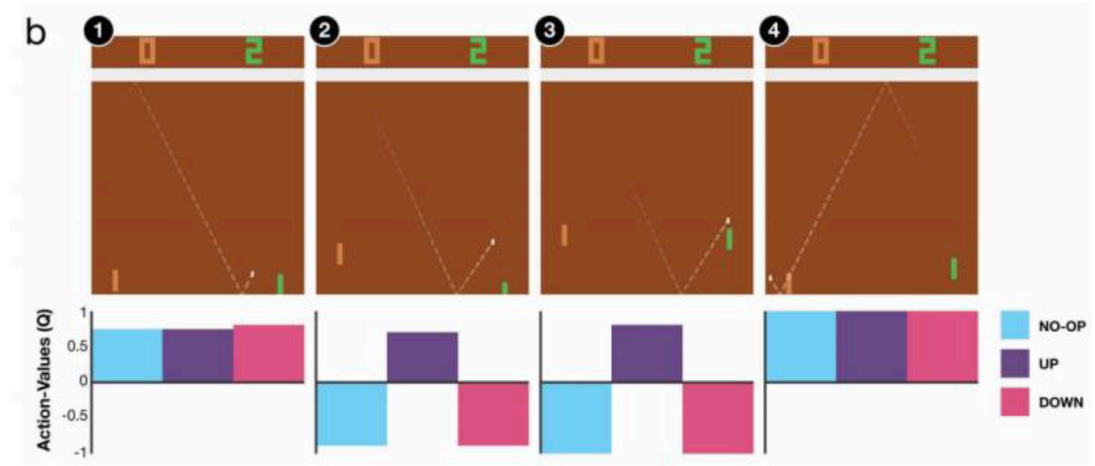


Discrete vs Continuous Q-values

- Different formulations for continuous and discrete action spaces:
  - In **continuous (a)**, typically condition on an action and output a scalar, where you sample actions from your policy for an expectation calculation
  - In **discrete (b)**, typically learn a value for each bin (output) and can enumerate over all actions for an expectation







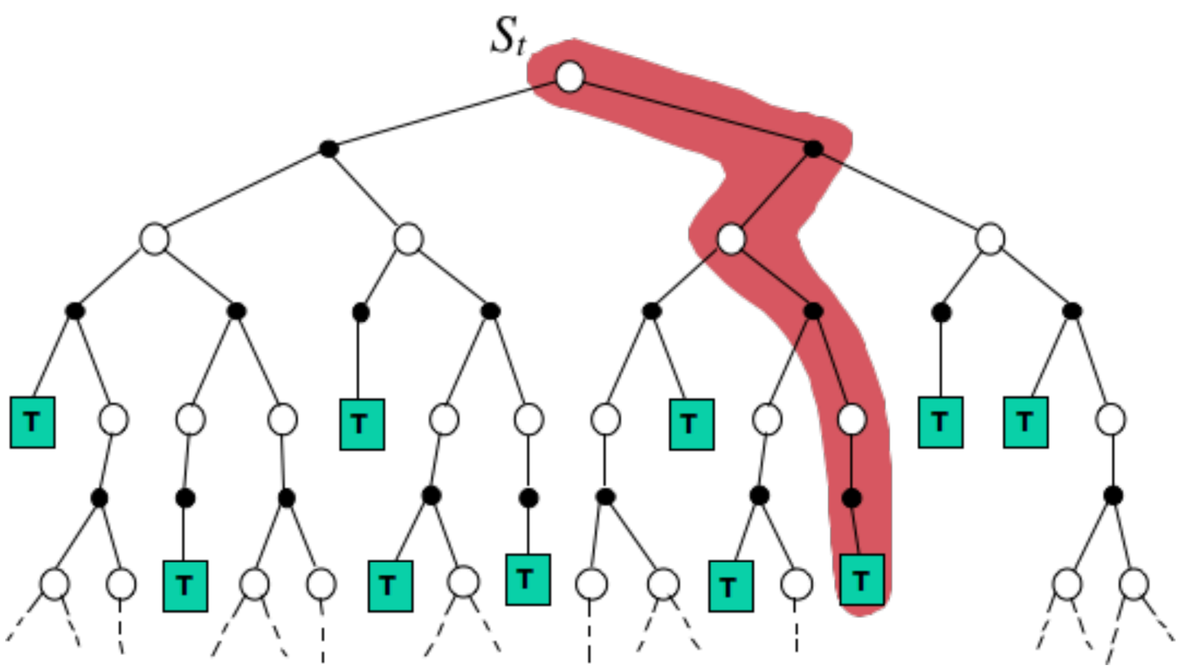
Q-Learning as a Regression Problem

Whiteboard

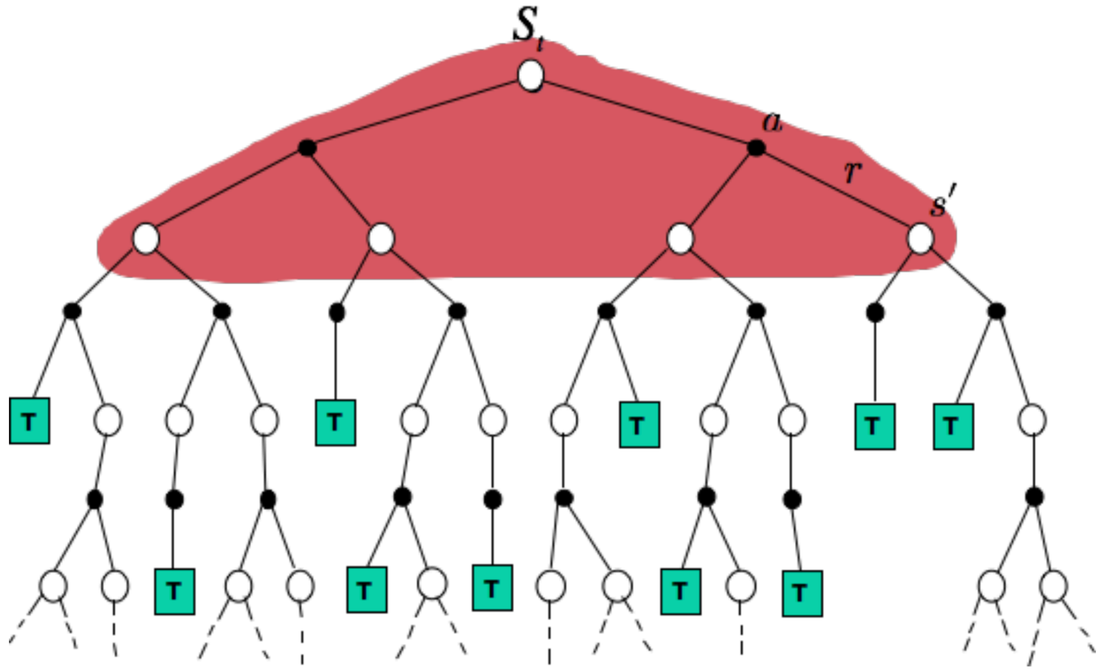
$$
\begin{aligned}
V^{\pi}(s) &= E_{\pi}\left[\mathcal{R}_t \mid S_t = s\right] && \text{(Definition)} \\
&= E_{\pi}\left[\sum_{k=0}^{\infty} \gamma^k \, r(s_{t+k}, a_{t+k}) \;\middle|\; S_t = s\right] && \text{(Monte–Carlo Rollout, MC)} \\
&= E_{\pi}\left[r(s_t, a_t) + \gamma \sum_{k=0}^{\infty} \gamma^k \, r(s_{t+k+1}, a_{t+k+1}) \;\middle|\; S_t = s\right] && \text{(Expansion)} \\
&= E_{\pi}\left[r(s_t, a_t) + \gamma \, V^{\pi}(s_{t+1}) \mid S_t = s\right] && \text{(Dynamic Programming, TD)} \\
&= E_{\pi}\left[r(s_t, a_t) + \gamma \, r(s_{t+1}, a_{t+1}) + \cdots + \gamma^{k+1} \, V^{\pi}(s_{t+k+1}) \;\middle|\; S_t = s\right] && \text{(N-Step)}
\end{aligned}
$$

Bias vs Variance Tradeoff!

MC vs TD Visualization



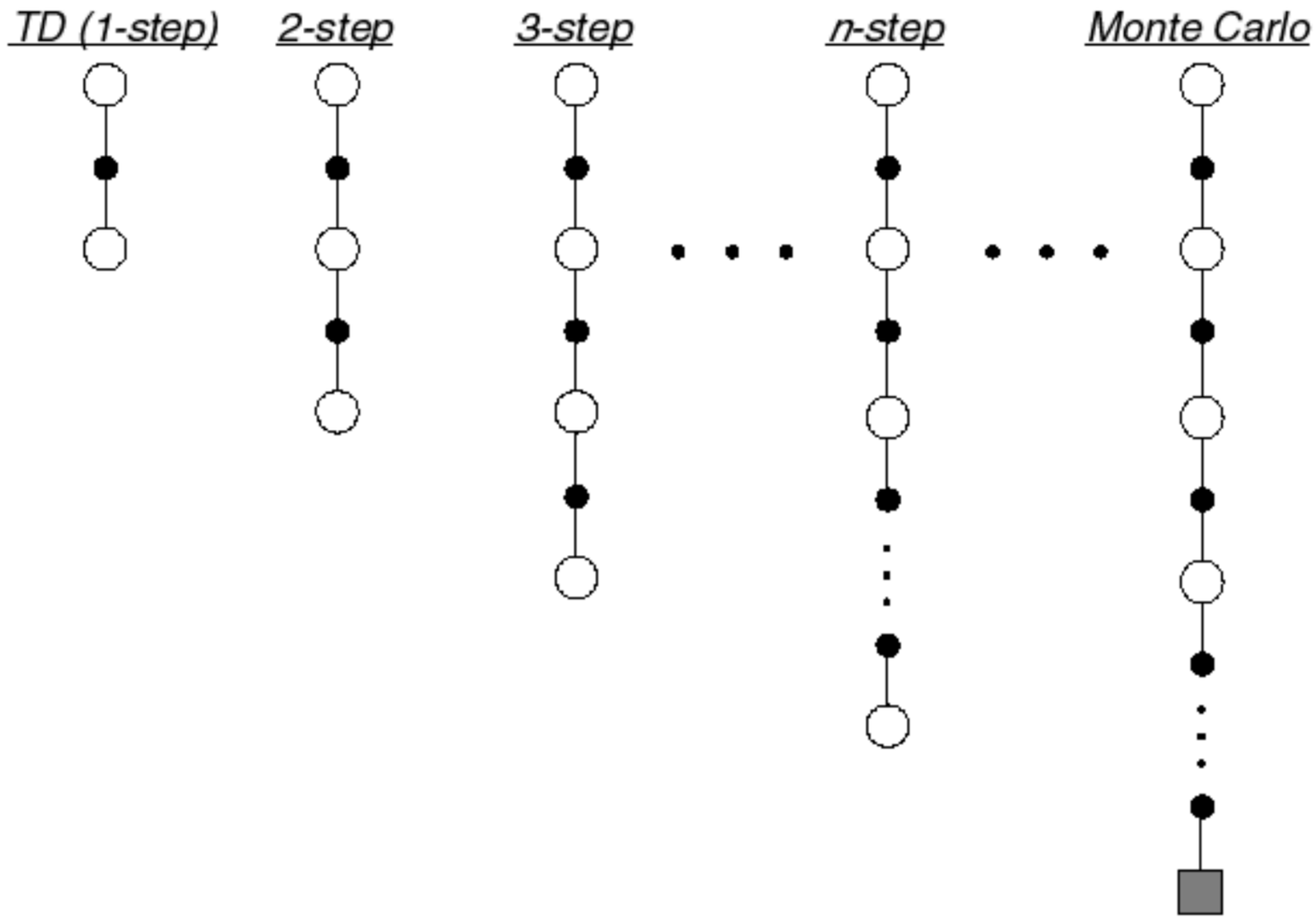
Monte-Carlo Rollouts
(Regression)



TD Backup
(Dynamic Programming)

N-Step Returns

Let TD target look *n* steps into the future



Bias vs Variance Tradeoff

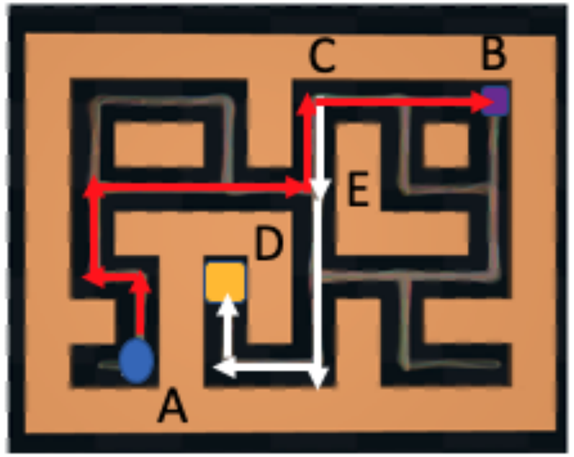
TD, MC, N-Step Return

Method	Bootstrap Depth	Bias	Variance
TD	1 Steps	High	Low
N-Step	N Steps	Medium	Medium
Monte Carlo	Full Episode	Low	High

Why Dynamic Programming?

Aside on Stitching

Our maze navigation task

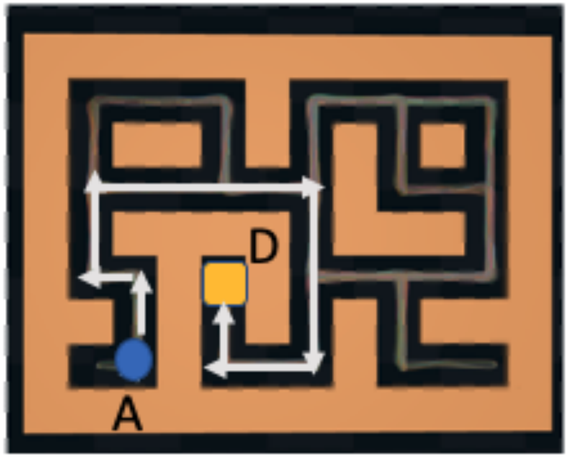


Start (A) Goal (D)

trajectory from A → B

trajectory from C → D

"Stitching" enables finding optimal behavior



Trajectory found by stitching segments of trajectories

Pause for Questions.

Practical Implementation Detail

Semi-Gradient and Target Network

Going back to our formulation:

$$Q(s,a) \leftarrow r(s,a) + \gamma \max_{\hat{a}} Q(s', \hat{a})$$

Semi-Gradient

$$Q_\theta(s,a) \leftarrow r(s,a) + \gamma \max_{\hat{a}} \text{stopgrad}\left(Q_\theta(s', \hat{a})\right)$$

Target Network

$$Q_\theta(s,a) \leftarrow r(s,a) + \gamma \max_{\hat{a}} \text{stopgrad}\left(Q_{\text{target}}(s', \hat{a})\right)$$

Update Types

$w' \leftarrow w$ when $\mod(n, N) = 0$

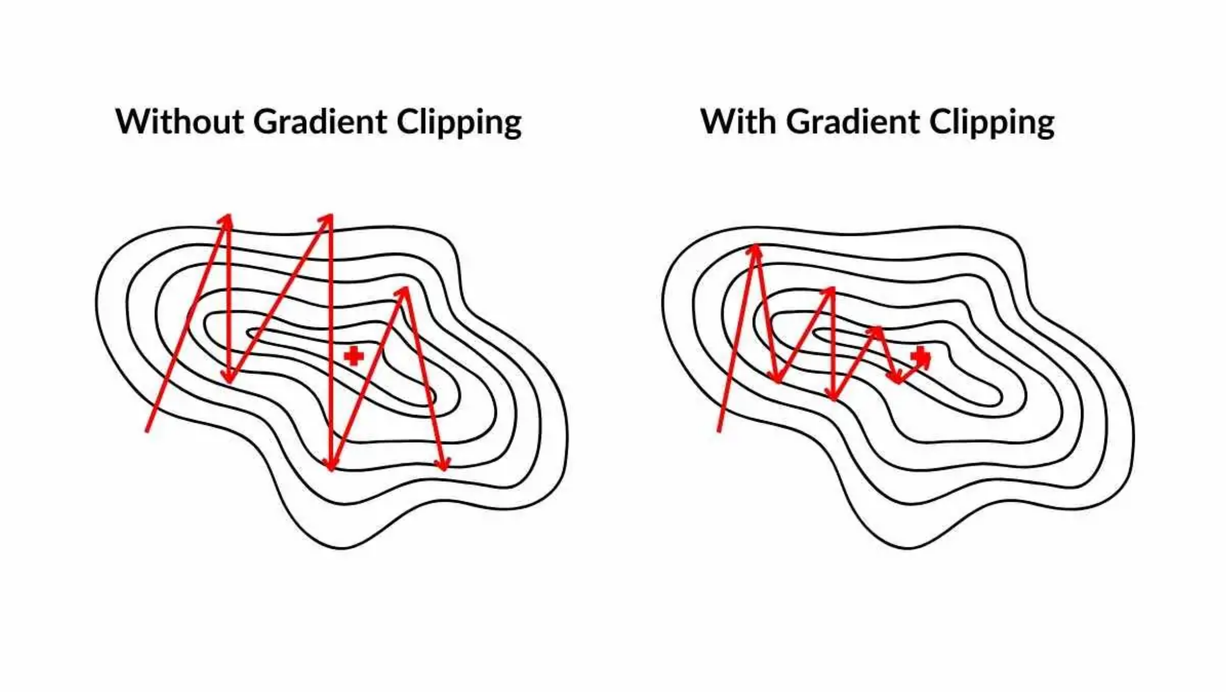
Hard Target Update

$w' \leftarrow \tau \cdot w + (1-\tau) \cdot w'$

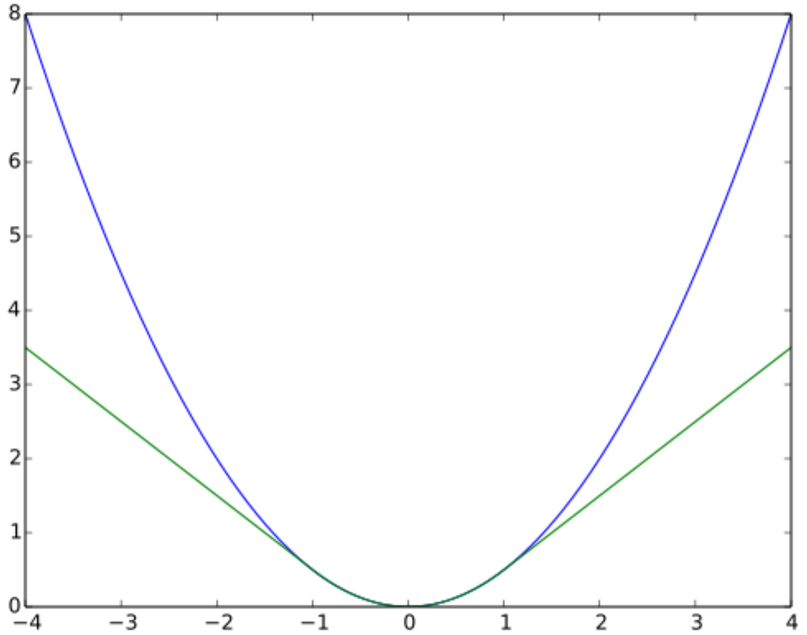
Soft Target Update (Polyak)

Practical Implementation Details

Dealing with TD Gradients



Gradient Clipping

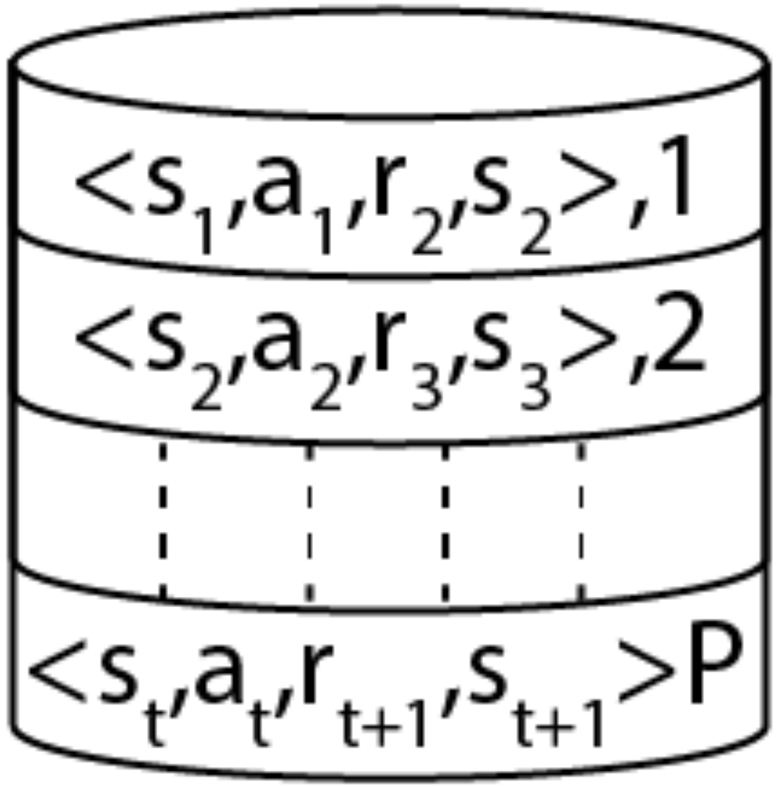


Huber Loss

Practical Implementation Details

Replay Buffers

- Replay Buffers are **memory structures** that store past experiences encountered by the agent, allowing the **experiences to be reused** instead of discarded after a single update
- They **break temporal correlations** between consecutive training samples and prevents recency bias
- **Improves sample efficiency** in Q-learning algorithms, allowing an agent to reuse its experience effectively



Q-Learning Overestimation

Q-Learning is typically formulated as follows:

$$Q(s,a) \leftarrow r(s,a) + \gamma \max_{\hat{a}} Q(s', \hat{a})$$

When parameterized by a function approximator like a NN, there is some inherent noise in the q-value estimates due to modeling errors

$$Q^{\text{approx}}(s', \hat{a}) = Q^{\text{target}}(s', \hat{a}) + Y_{s'}^{\hat{a}}$$

Due to the noise on the RHS ($Y_{s'}^{\hat{a}}$), there results in error on the LHS, expressed as:

$$Z_s := \gamma \left(\max_{\hat{a}} Q^{\text{approx}}(s', \hat{a}) - \max_{\hat{a}} Q^{\text{target}}(s', \hat{a}) \right)$$

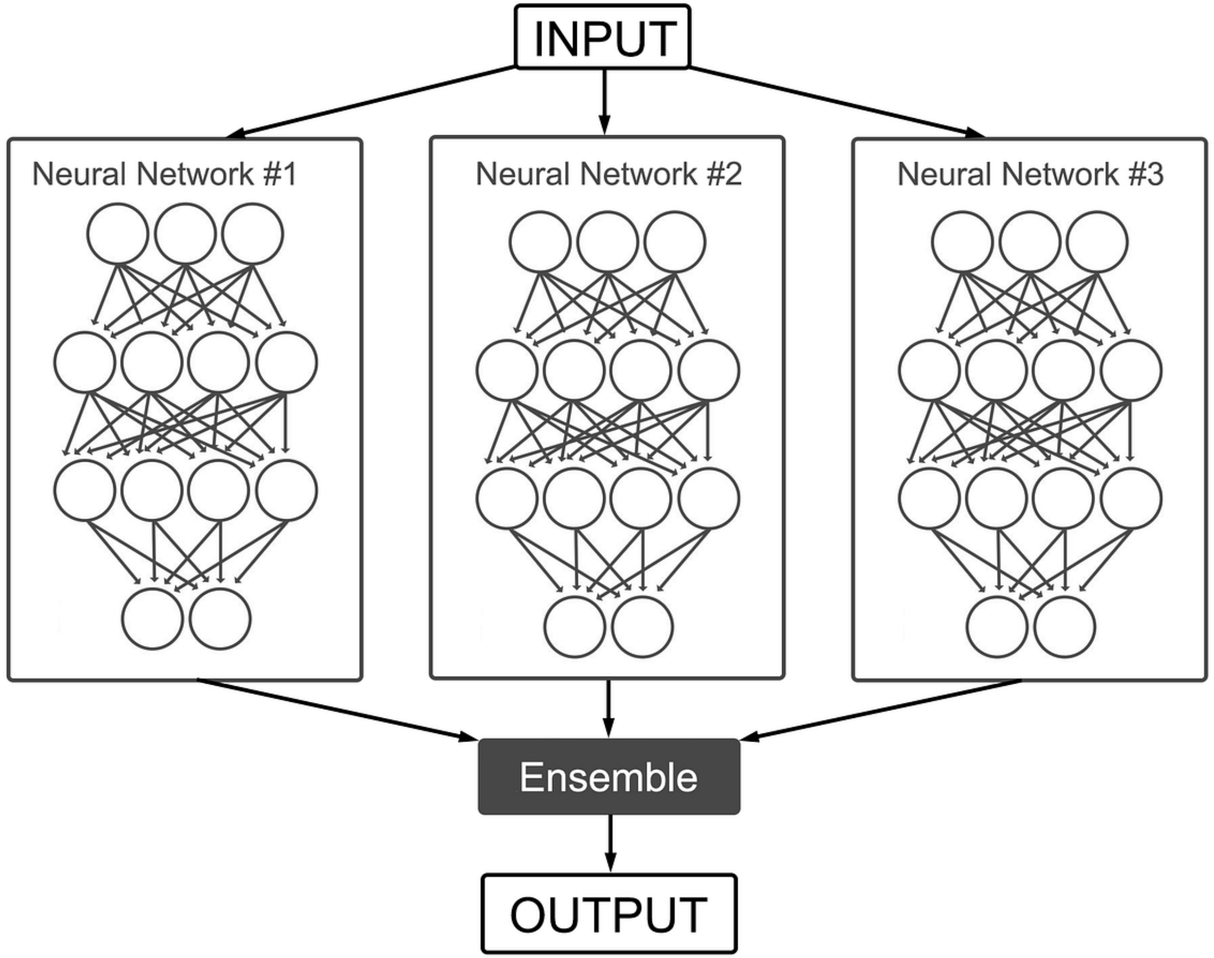
Even with zero mean ($Y_{s'}^{\hat{a}}$), Z_s may have positive mean:

$$\mathbb{E}\left[Y_{s'}^{\hat{a}}\right] = 0 \forall \hat{a} \implies \mathbb{E}\left[Z_s\right] > 0 \quad \text{(often)}$$

Practical Implementation Detail

Critic Ensembling

- One approach to tackle overestimation is to measure **epistemic uncertainty** with an ensemble of networks
- Since different model initializations may have **different modeling errors**, this can lead to ensembling with a **min reduction** to be effective
- Has been even shown to be effective in **offline RL** where distribution shift is a larger concern (Sac-N and EDAC)



Q-Learning vs Double Q-Learning

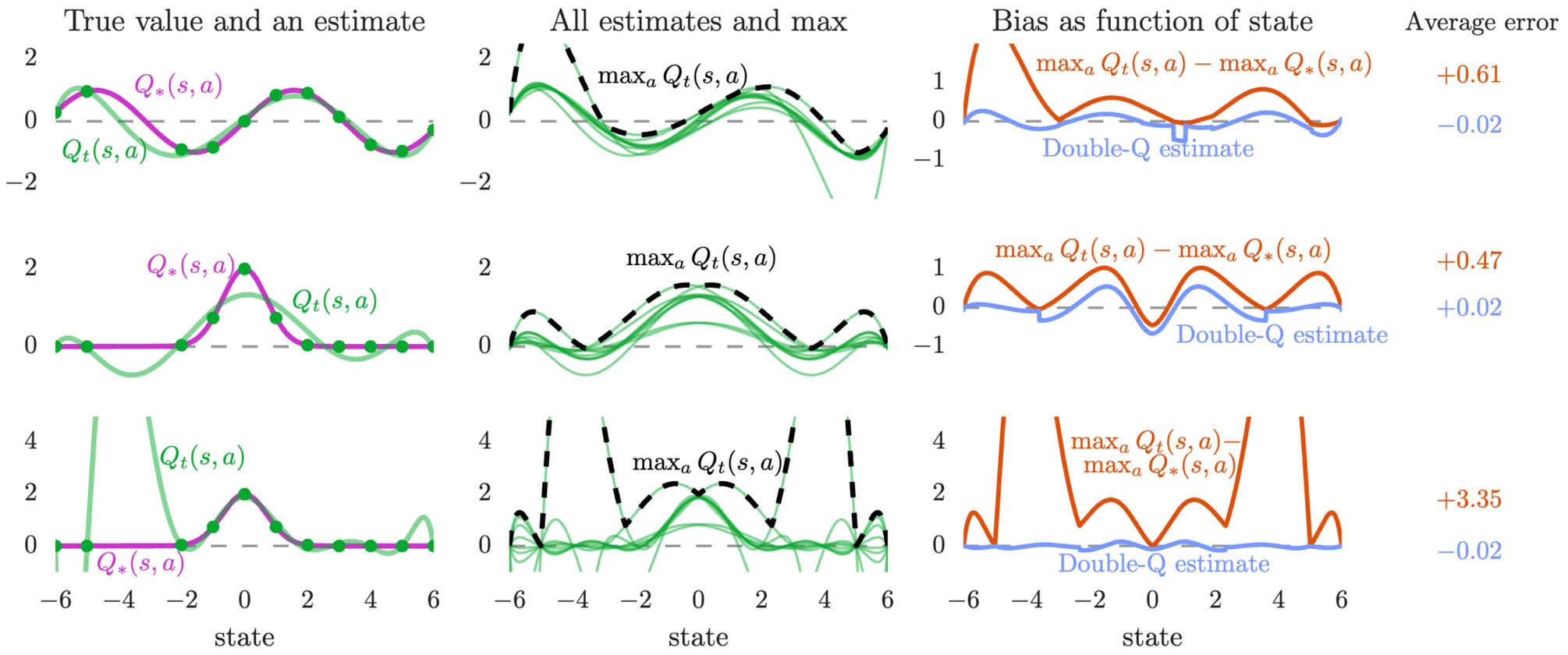


Figure 2: Illustration of overestimations during learning. In each state (x-axis), there are 10 actions. The **left column** shows the true values $V_*(s)$ (purple line). All true action values are defined by $Q_*(s,a) = V_*(s)$. The green line shows estimated values $Q(s,a)$ for one action as a function of state, fitted to the true value at several sampled states (green dots). The **middle column** plots show all the estimated values (green), and the maximum of these values (dashed black). The maximum is higher than the true value (purple, left plot) almost everywhere. The **right column** plots shows the difference in orange. The blue line in the right plots is the estimate used by Double Q-learning with a second set of samples for each state. The blue line is much closer to zero, indicating less bias. The three **rows** correspond to different true functions (left, purple) or capacities of the fitted function (left, green). (Details in the text)

Understanding HW Algorithms

Q-Learning : DQN

Whiteboard

Algorithm 1: DQN

Input: Replay buffer $\mathcal{D}$, Q-network Q_θ, target network $Q_{\theta'}$ with $\theta' \leftarrow \theta$, discount factor γ, batch size N, learning rate α, target update frequency C, exploration schedule ϵ

for *each interaction step* $t = 1, 2, \ldots$ **do**

Observe state s_t

Choose action a_t using ϵ-greedy:

$$a_t = \begin{cases} \text{random action}, & \text{with probability } \epsilon, \\ \arg\max_a Q_\theta(s_t, a), & \text{otherwise} \end{cases}$$

Execute a_t, observe (r_t, s_{t+1}), and store (s_t, a_t, r_t, s_{t+1}) in $\mathcal{D}$

`// Sample a mini-batch`

Sample $\{(s_i, a_i, r_i, s_{i+1})\}_{i=1}^{N} \sim \mathcal{D}$

`// Compute targets`

for $i = 1, \ldots, N$ **do**

$y_i \leftarrow r_i + \gamma \max_{a'} Q_{\theta'}(s_{i+1}, a')$

`// Q-network update`

$$\theta \leftarrow \theta - \alpha \nabla_\theta \frac{1}{N} \sum_{i=1}^{N} \left[Q_\theta(s_i, a_i) - y_i \right]^2$$

`// Periodic target update`

if $t \bmod C = 0$ **then**

$\theta' \leftarrow \theta$

Actor Critic: SAC

Whiteboard

Algorithm 1: Soft Actor-Critic (no entropy term)

Input: Replay buffer $\mathcal{D}$, critic nets $Q_{\theta_1}, Q_{\theta_2}$, actor net π_ϕ, target critics $\theta'_i \leftarrow \theta_i$, discount γ, smoothing τ, batch size N

for *each interaction step* $t = 1, 2, \ldots$ **do**

- Observe state s_t
- Sample action $a_t \sim \pi_\phi(\cdot \mid s_t)$
- Execute a_t, observe (r_t, s_{t+1}) and store in $\mathcal{D}$
- `// Sample a mini-batch`
- Sample $\{(s_i, a_i, r_i, s_{i+1})\}_{i=1}^{N} \sim \mathcal{D}$
- `// Compute targets (no entropy term)`
- **for** $i = 1, \ldots, N$ **do**
  - $a'_{i+1} \leftarrow \pi_\phi(s_{i+1})$;
  - $y_i \leftarrow r_i \; + \; \gamma \, \min_{j=1,2} Q_{\theta'_j}\left(s_{i+1}, a'_{i+1}\right)$
- `// Critic update`
- **for** $j = 1, 2$ **do**
  - $\theta_j \leftarrow \theta_j \; - \; \lambda_Q \, \nabla_{\theta_j} \frac{1}{N} \sum_i \left[Q_{\theta_j}(s_i, a_i) - y_i \right]^2$
- `// Actor update (maximize Q only)`
- $\phi \leftarrow \phi \; + \; \lambda_\pi \, \nabla_\phi \frac{1}{N} \sum_i Q_{\theta_1}\left(s_i, \pi_\phi(s_i)\right)$
- `// Target networks soft-update`
- **for** $j = 1, 2$ **do**
  - $\theta'_j \leftarrow \tau \, \theta_j + (1 - \tau) \, \theta'_j$

Thanks!