

HyperAgent: Advancing Scalable Exploration through Fast Uncertainty Estimation in RL

Yingru Li

<https://richardli.xyz/>

The Chinese University of Hong Kong, Shenzhen, China

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Motivation: RL under Resource Constraints

Existing solution and their limitations

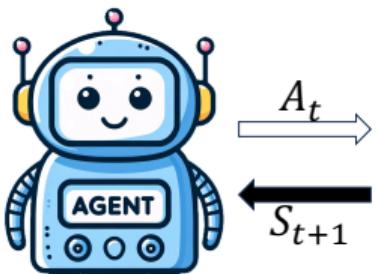
Our Contributions

HyperAgent: Scalable Uncertainty and Exploration

Theoretical insights with tabular representation

Reduce sequential posterior approx. to sequential random projection

Reinforcement Learning Problem



Agent-Environment Interface.

- ▶ Interactive Experience:

$A_0, S_1, A_1, S_2, \dots, A_t, S_{t+1}, \dots$

Environment $M = (\mathcal{S}, \mathcal{A}, P)$

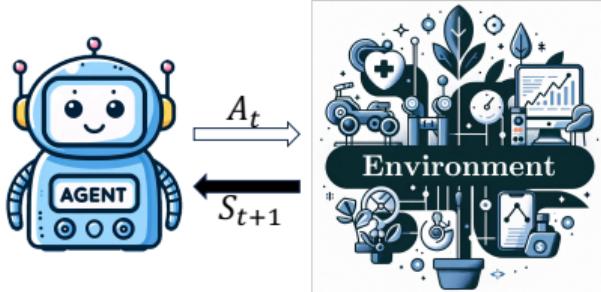
- ▶ State $S_{t+1} \sim P(\cdot | S_t, A_t)$ for $t = 0, 1, \dots$

Agent($\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t$) $\rightarrow \pi_t \max$ long-term rewards

- ▶ Reward $R_{t+1} = r(S_t, A_t, S_{t+1})$ preference
- ▶ Data $\mathcal{D}_t = \mathcal{D}_{t-1} \cup \{A_{t-1}, S_t\}$ accumulated.
- ▶ Policy $\pi_t = \text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t)$.
- ▶ Action $A_t \sim \pi_t(\cdot | S_t)$;
- ▶ Objective $\pi_{\text{agent}} = (\pi_0, \pi_1, \dots)$ to maximize

$$\mathbb{E}\left[\sum_{t=0}^{T-1} R_{t+1} \mid \pi_{\text{agent}}, M\right]. \quad (1)$$

Challenges for Deploying RL in Real-world



Agent-Environment Interface.

- ▶ Interactive Experience:

$$\underbrace{A_0, S_1, A_1, S_2, \dots, A_t, S_{t+1}, \dots}_{\mathcal{D}}$$

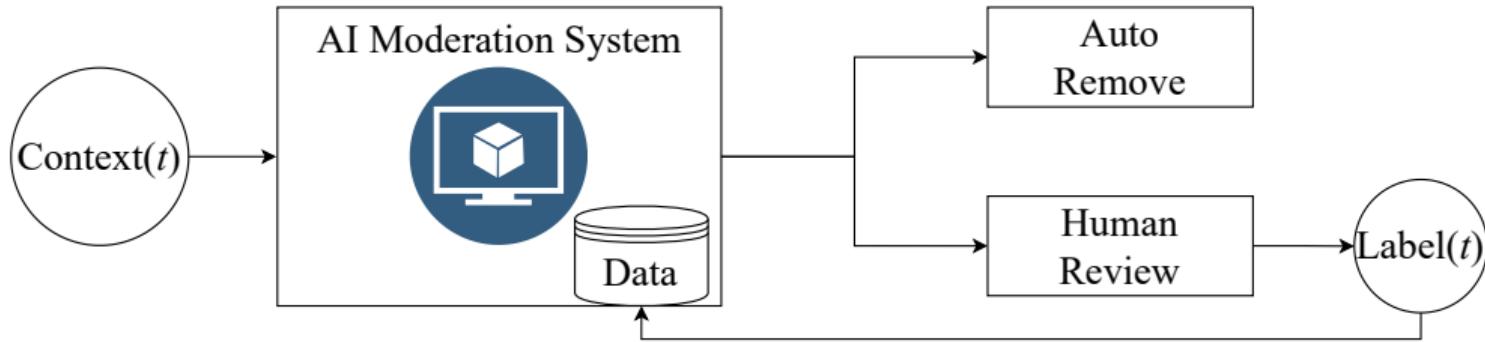
Complex Environment:

- ▶ **Large state space:** language, vision & audios.
 $|S| \approx 10^{100}$.
- ▶ **Data accumulates** as interacting. $|\mathcal{D}| \uparrow$

Resource Constraints for Agent:

- ▶ **Bounded** Per-step Computation & Memory
- ▶ **Limited** Data Collection Budgets

Example: Human-AI Collaboration in Automated Content Moderation



The Human-AI agile collaboration pipeline for risk oversight in an **online** production environment

- ▶ **Challenge 1:** Natural language input in post context. "Cold start" problem.
Large foundation models (GPTs) are used in Moderation system.
- ▶ **Challenge 2:** Real-time safety-critical decision-making.
Huge amount of posts are generated every second. **Filter out harmful post, aligning human value.**
- ▶ **Challenge 3:** Limited human reviewer
can only provide feedback and moderate on a small portion of posts.

Research Question

Can we design **scalable** and **data-efficient** RL algorithms under **resource constraints**?

- ▶ **Scalability:** **Bounded per-step computation and memory complexity** even when large foundation model (e.g., GPT) is involved in the online decision-making process.
- ▶ **Data-efficiency:** Handle limited data collection budgets. **Sublinear regret.**

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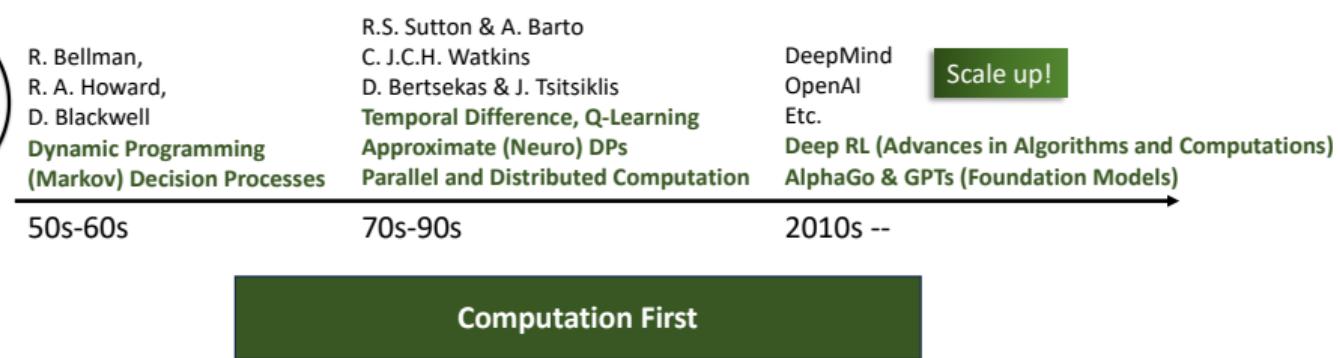
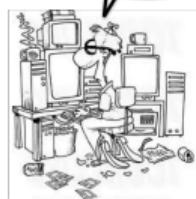
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Development of RL Algorithms: A History of "Scale up!"



Scale up!

- ▶ **Per-step $O(\text{poly}(|\mathcal{S}|)) \Rightarrow \text{Function Approximation}$ (FA), e.g. Neural Networks; (70s – 90s)**
- ▶ **Per-step $O(\text{poly}(|\mathcal{D}|)) \Rightarrow \text{Incremental Update}$ with SGD, Replay Buffer and/or Target Network.**
- ▶ **FA + Incremental Update $\Rightarrow \text{Bounded } \tilde{O}(1) \text{ Per-step Complexity} \Rightarrow \text{Scalable}$ algorithm. (10s –)**

Practical Advancements for “Efficient” Deep RL

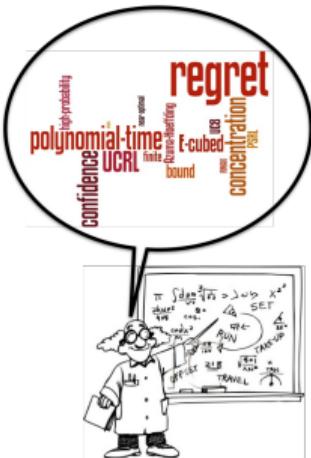
Algorithm	Components
DDQN (16)	Incremental SGD with experience replay (finite buffer) and target network
Rainbow (18)	(DDQN) + Prioritized replay, Dueling networks, Distributional RL, Noisy Nets.
BBF (23)	(DDQN) + Prioritized replay, Dueling networks, Distributional RL, Self-Prediction, Harder resets, Larger network, Annealing hyper-parameters.

Table: Components in **STOA algorithms**, e.g. DDQN [VHGS16], Rainbow [HMVH⁺18], BBF [SCC⁺23].

- ▶ ✓ **Scalable**: e.g. DDQN use incremental SGD with experience replay and target network.
- ▶ ✗ **Deployment inefficient**: Complicated components and many heuristics. Hard to tune.
- ▶ ✗ **Data inefficient**: e.g. BBF use ϵ -greedy exploration strategy which suffer linear regret in some environment, provably [Kak03, Str07, OVRWW19, DMM⁺22]. In practice, deep RL data hungry .

Principled Approaches for Data Efficiency

Goal: Sequential decision-making under uncertainty with **sublinear regret**.



Posterior Sampling Reinforcement Learning (PSRL): **data-efficient exploration** strategy

- ▶ **Require:** Prior distribution $\mathbb{P}(M \in \cdot)$ for underlying model M .
- ▶ **For each episode ℓ ,** denote t_ℓ the beginning time step
 - Sample $\hat{M}_\ell \sim \mathbb{P}(M \in \cdot | \mathcal{D}_{t_\ell})$.
 - **Return** the optimal policy $\pi_\ell = \pi^{\hat{M}_\ell}$ under \hat{M}_ℓ .
- ▶ **Require conjugacy** for **tractable posterior update** (**uncertainty estimation**).
- ▶ **Only feasible** in simple environments:
 - Tabular MDP with dirichlet prior [Str00, OVR17] $\tilde{O}(H^2 \sqrt{SAK})$ regret **sublinear in K episodes**.
 - Linear-Gaussian bandit [RVR16, RVRK⁺18] $O(d\sqrt{T \log A})$ regret **sublinear in T time steps**.

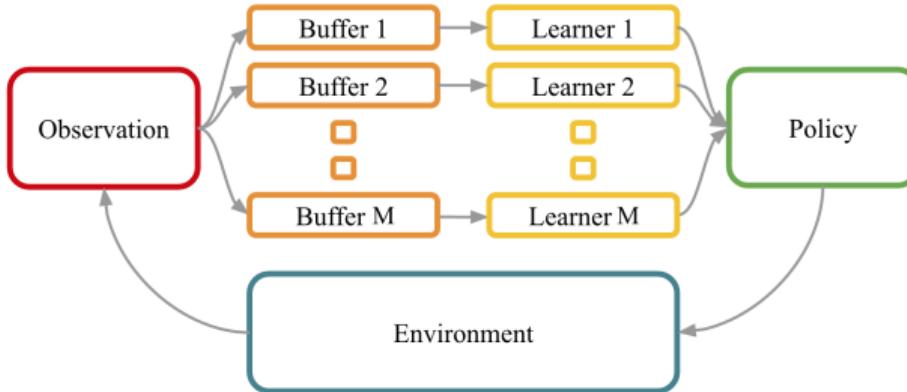
X Intractable Computation in Posterior Sampling:

- ▶ **Model-based:** No conjugacy for exact Bayesian inference for posterior over transition models.
 - [LL24] (AISTATS): First prior-dependent bound under FA and improved prior-free bound in the context of linear mixture MDPs.
- ▶ **Model-free:** Beyond conjugacy, sample from intricate distribution over value functions [Zha22, DMZZ21, ZXZ⁺22]

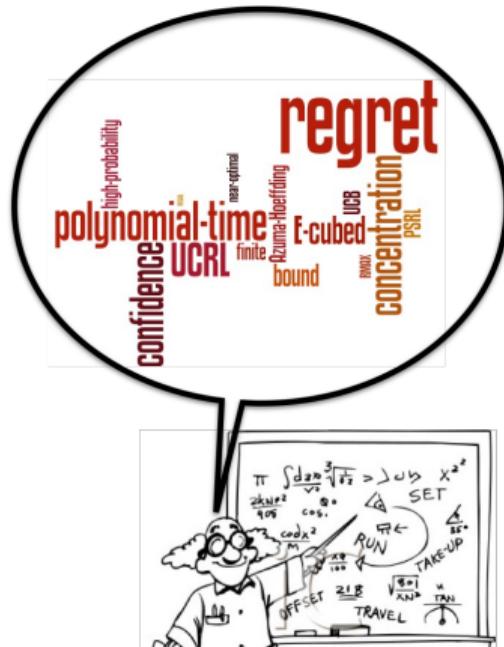
X Unbounded Per-step Complexity $\text{poly}(|\mathcal{D}|)$ in Approximate Posterior Sampling:

- ▶ Store entire history and retrain for each episode, e.g. RLSVI [OVRRW19], LSVI-PHE [ICN⁺21].
- ▶ Langevin Monte-Carlo (LMC) based methods [XZM⁺22, ILX⁺24]
- ▶ Same issues for OFU: (1) **X Intractability** [JKA⁺17, JLM21, DKL⁺21, FKQR21, LLX⁺23];
(2) **X Unbounded resource demands** as data accumulates [WSY20, AJZ23].

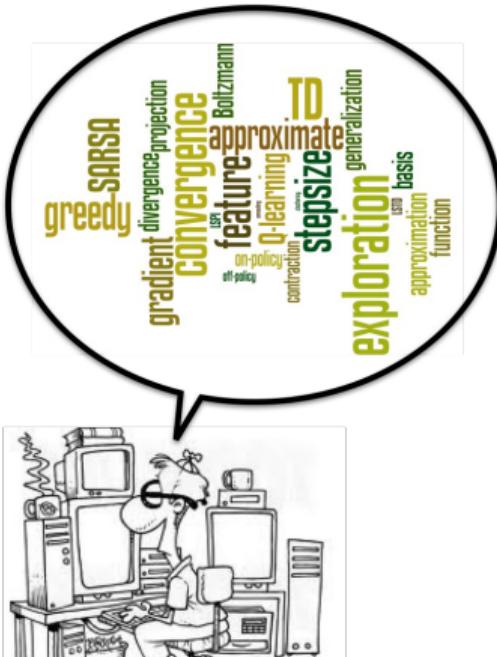
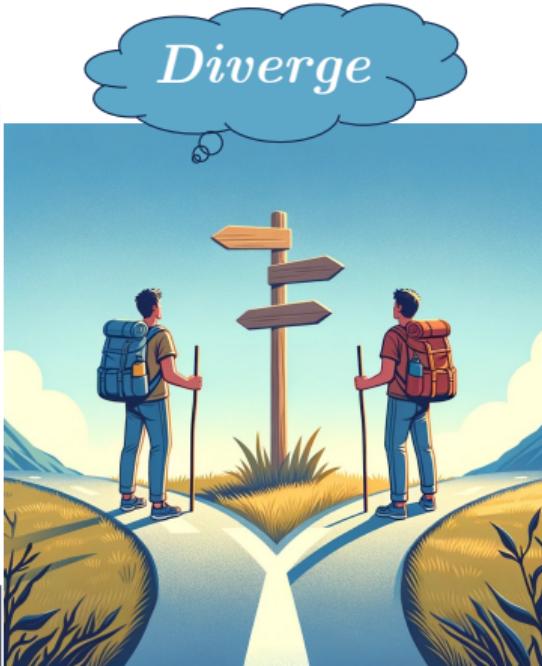
Ensemble Sampling for Approximate Posterior Sampling



- ▶ **Ensemble Sampling (ES)**: approximate the posterior distribution by **uniformly sampling from a set of ensemble models**. E.g., BootstrapDQN [OBPVR16], **Ensemble+** [OAC18, OVRRW19].
- ▶ ✓ Each ensemble perform incremental update, **no retraining**.
- ▶ ✗ **Computationally expensive** in practice: say, update > 100 neural networks for each time step.
- ▶ ✗ **No rigorous** understanding in terms of **statistical** and **computational complexity**.



Theory! Data First



Scale up! Computation First

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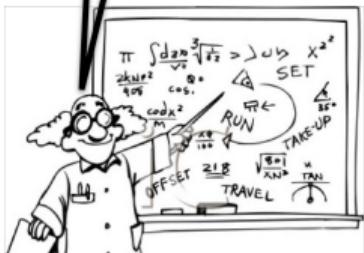
Our Contributions

HyperAgent: Scalable Uncertainty and Exploration

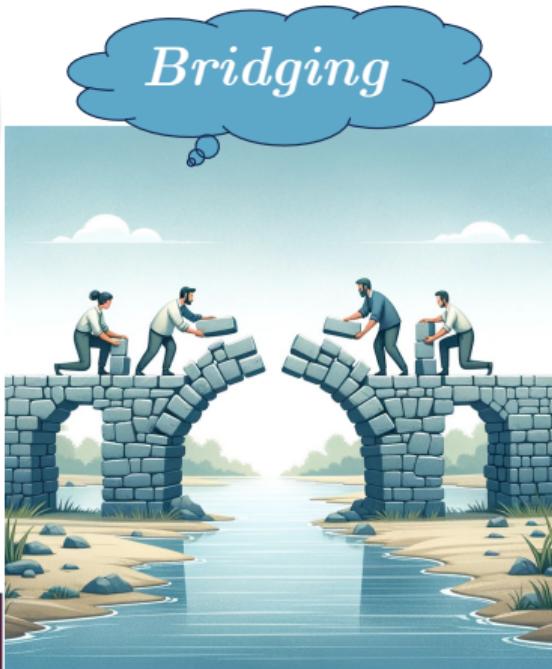
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Reduce sequential posterior approx. to sequential random projection

Our HyperAgent [LXHL24] aims to ...



Theory! Data First



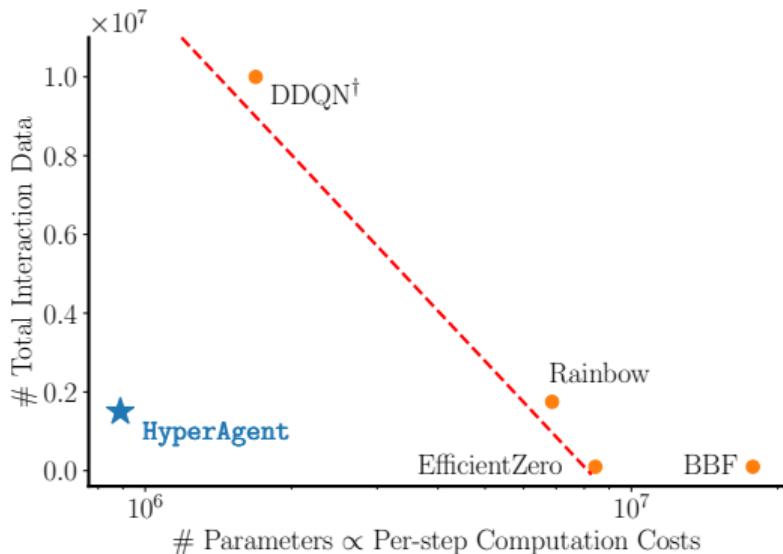
Preview of Contributions - For Practitioners

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HyperAgent	(DDQN) + Hypermodel

Table: Techniques used in different algorithms, e.g. DDQN [VHGS16], Rainbow [HMVH⁺18], BBF [SCC⁺23] and our HyperAgent.

- ▶ ✓ **Simple:** Only one additional component, **hypermodel**, compatible with all feedforward DNN.
⇒ Easy to deploy empirically.
- ▶ ✓ **Scalable:** Incremental SGD under DNN function approximation, same as DDQN;
⇒ bounded per-step computation.

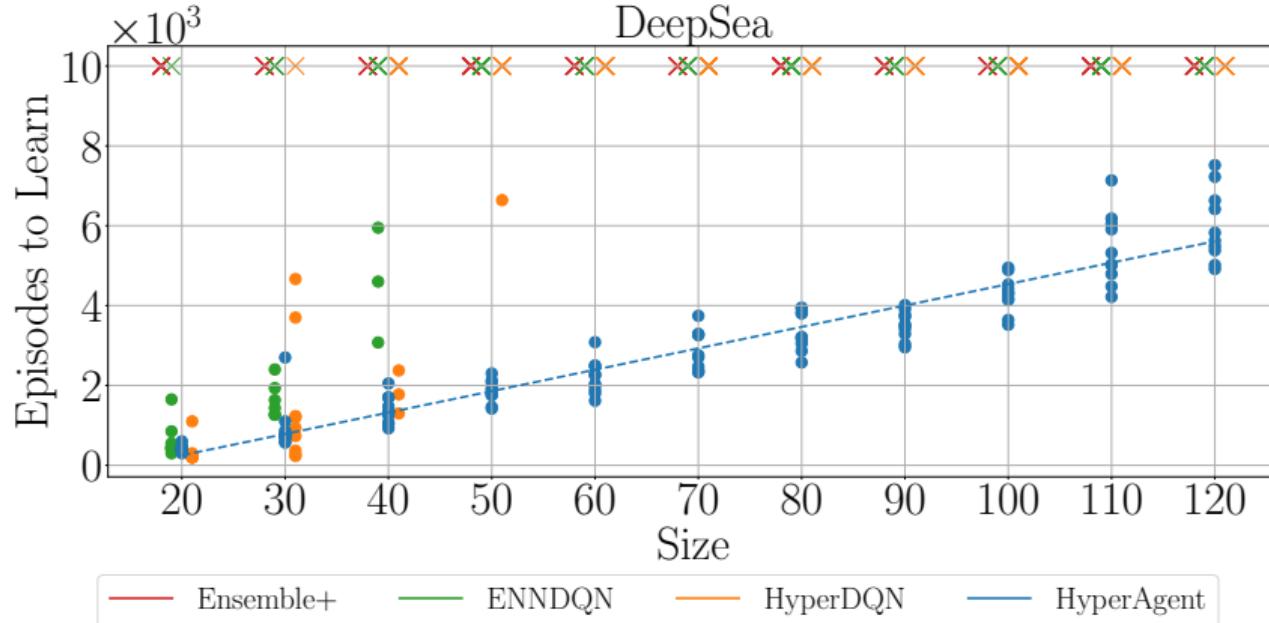
Preview of Contributions - For Practitioners



How much data and parameters to achieve **Human-level performance (1 IQM)** in Atari suite?

- ▶ ✓ **Data efficient:** only 15% data consumption of DDQN[VHGS16] by **DeepMind**. (1.5M interactions)
- ▶ ✓ **Computation efficient:** only 5% model parameters of BBF[SCC⁺23] by **DeepMind**.
- ▶ **Ensemble+** [OAC18, OVRRW19] achieves a mere 0.22 IQM score under 1.5M interactions but necessitates **double the parameters** of HyperAgent.

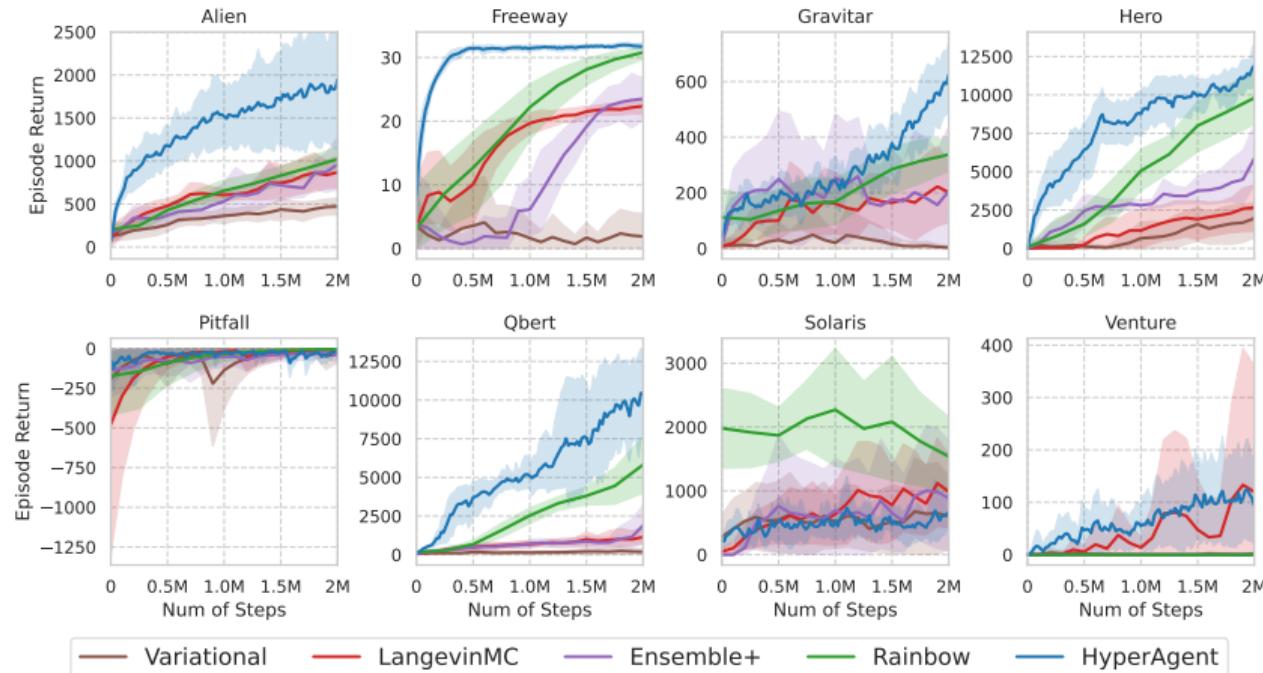
HyperAgent: Data efficiency in DeepSea benchmarks (Deep Exploration)



Comparison with **Ensemble+** [OAC18, OVRRW19], **HyperDQN** [LLZ⁺22], **ENN-DQN**[OWA⁺23b].

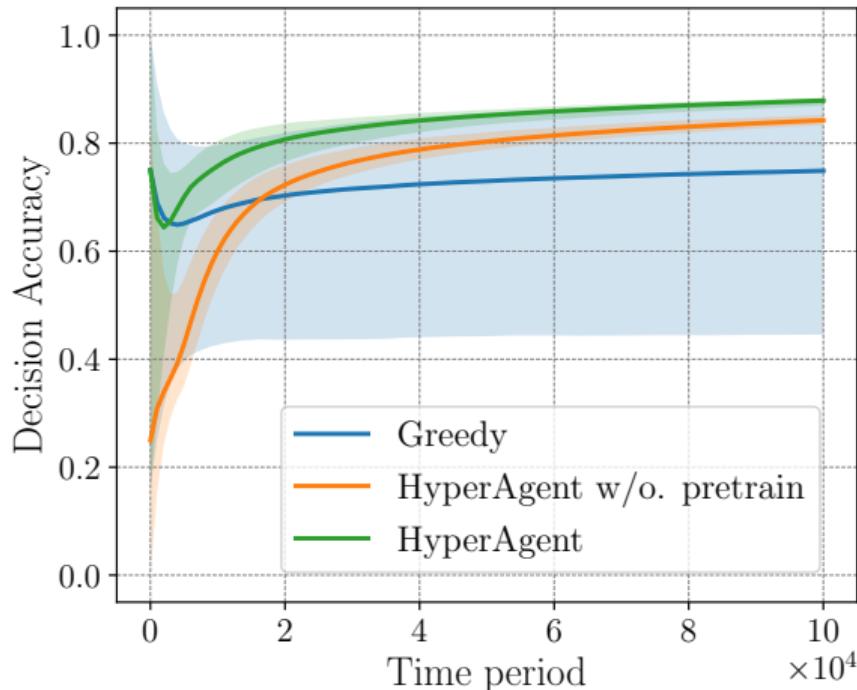
- ▶ ✓ **Scalable** as size $N \uparrow$. State representation: one-hot vector in high-dimension \mathbb{R}^N .
- ▶ ✓ **Data efficient**: HyperAgent the only and first achieving **optimal episode complexity** $\Theta(N)$.

HyperAgent for hardest exploration in Atari



Comparison on approximate posterior sampling methods: **Variational approximation** (SANE [AL21]), **Langevin Monte-Carlo** (AdamLMCDQN [ILX⁺24]) and **Ensemble+** [OAC18, OVRRW19]

HyperAgent for Online Automated Content Moderation



- ▶ 10x labeling effort reduction,
- ▶ Higher detection accuracy.
- ▶ Pretrained foundation model helps: augment pretrained GPT-2 backbone with hypermodel.

The human-AI collaboration pipeline for online automated content moderation, focusing on hate speech detection.

Preview of Contributions - For Theoreticians (RL)

	Practice in Deep RL			Theory in Tabular RL	
Algorithm	Tractable	Incremental	Efficient	Regret	Per-step Computation
PSRL	✗	✗	✗	$\tilde{O}(H^2 \sqrt{SAK})$	$O(S^2 A)$
RLSVI	✓	✗	✗	$\tilde{O}(H^2 \sqrt{SAK})$	$O(S^2 A)$
Ensemble+	✓	✓	🟡	N/A	N/A
HyperAgent	✓	✓	✓	$\tilde{O}(H^2 \sqrt{SAK})$	$\tilde{O}(\log(K)SA + S^2A)$

- ▶ **HyperAgent** not only demonstrates superior empirical performance in deep RL benchmarks
- ▶ but also achieves **theoretical milestones**, i.e., the first method to achieve $\tilde{O}(\log K)$ **per-step computation** & **near-optimal regret** in tabular K -episodic RL among practically scalable algorithms.

Preview of Contributions - For Theoretists (Contextual Bandit)

Decision Sets	Invariant & Compact	Variant & Compact	Invariant & Finite	Variant & Finite
Lower Bound	$\Omega(d\sqrt{T})$	$\Omega(d\sqrt{T \log T})$	$\Omega(\sqrt{dT \log \mathcal{A} })$	$\Omega(\sqrt{dT \log \mathcal{A} \log T})$
TS	$O(d^{\frac{3}{2}}\sqrt{T} \log T)$	$O(d^{\frac{3}{2}}\sqrt{T} \log T)$	$O(d\sqrt{T \log \mathcal{A} \log T})$	$O(d\sqrt{T \log \mathcal{A} \log T})$
ES[Qin]	N/A	N/A	$O(\sqrt{dT \log \mathcal{A} \log(\mathcal{A} T/d)})$	N/A
LMC[Xu]	$O((d \log T)^{\frac{3}{2}}\sqrt{T})$	$O((d \log T)^{\frac{3}{2}}\sqrt{T})$	N/A	N/A
ES[Janz]	$O((d \log T)^{\frac{5}{2}}\sqrt{T})$	$O((d \log T)^{\frac{5}{2}}\sqrt{T})$	N/A	N/A
(Ours)	$O(d^{\frac{3}{2}}\sqrt{T}(\log T)^{\frac{3}{2}})$	$O(d^{\frac{3}{2}}\sqrt{T}(\log T)^{\frac{3}{2}})$	$O(d\sqrt{T \log \mathcal{A} \log T})$	$O(d\sqrt{T \log \mathcal{A} \log T})$

Table: Regret lower and upper bounds under **various decision set setups in linear contextual bandit**. The per-step computation complexity is $O(d^2 + d|\mathcal{A}|T)$ for ES [QWLVR22], $O(d^2T)$ for LMC [XZM⁺22], $O(d^3 \log T)$ for ES [JLS24], and $O(d^3 \log T)$ for our HyperAgent.

- ▶ **Logarithmic** per-step computation complexity in total time periods T .
- ▶ **HyperAgent** matches exact TS, closing a gap in theory for scalable exploration.

Algorithmic Mechanism

- ▶ Value-based **approximate posterior sampling** via **hypermodel** and **index sampling** schemes.
- ▶ ⇒ **Near-optimal regret bound** ⇒ **Data-efficient Exploration**

↑

Key Lemma

- ▶ **Incremental approximation** of posteriors over value function **without** conjugacy.
- ▶ ⇒ **Logarithmic per-step computation complexity** ⇒ **Scalable Uncertainty Estimation**.

↑

- ▶ **Fundamental Tools** for dynamic (non-i.i.d.) data: **First Probability Tool** for **Sequential Random Projection** – a non-trivial martingale extension of Johnson-Lindenstrauss (JL). [Li24a]
- ▶ **Fundamental Tools** for static data: **Simple, Unified JL analysis** that covers existing and new JL construction that **traditional analysis cannot handle**. [Li24b]

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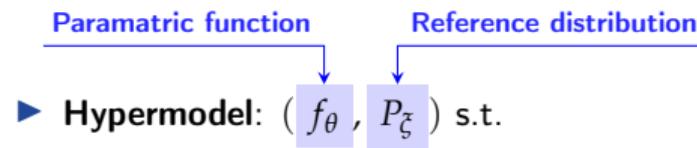
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HyperAgent: Introducing Hypermodel



Index Sampling: $f_\theta(x, \xi)$ is an (approximate) posterior predictive sample on data x .

↑ Index sample $\xi \sim P_\xi$

- [DLI⁺20, LLZ⁺22, OWA⁺23a]

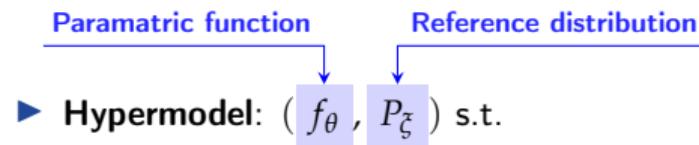
Example: predictive sampling from Linear-Gaussian model

- Suppose $\theta^* \sim N(\mu, \Sigma)$ where Σ represent the **model uncertainty**.
- Box-Muller Transform: $P_\xi = N(0, I_M)$, $\theta = (A \in \mathbb{R}^{d \times M}, \mu \in \mathbb{R}^d)$

$$\xi \sim P_\xi \Rightarrow f_\theta(x, \xi) := \langle x, \mu + A\xi \rangle \sim N(x^\top \mu, x^\top A A^\top x)$$

- Uncertain Estimation: Find A s.t. $AA^\top = \Sigma \Rightarrow f_\theta(x, \xi) \sim \langle \theta^*, x \rangle$.

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- Uncertain Estimation:** Find A s.t. $A A^\top = \Sigma \Rightarrow f_\theta(x, \xi) \sim \langle \theta^*, x \rangle$.

Sampling from Linear-Gaussian model $N(x^\top \mu, x^\top \Sigma x)$, we could perform

Ensemble Sampling (# of models M)

$P_\xi = \mathcal{U}\{e_1, \dots, e_M\}$ and $\theta = A = [\tilde{\theta}_1, \dots, \tilde{\theta}_M] \in \mathbb{R}^{d \times M}$, s.t. $\tilde{\theta}_m \sim N(\mu, \Sigma)$.

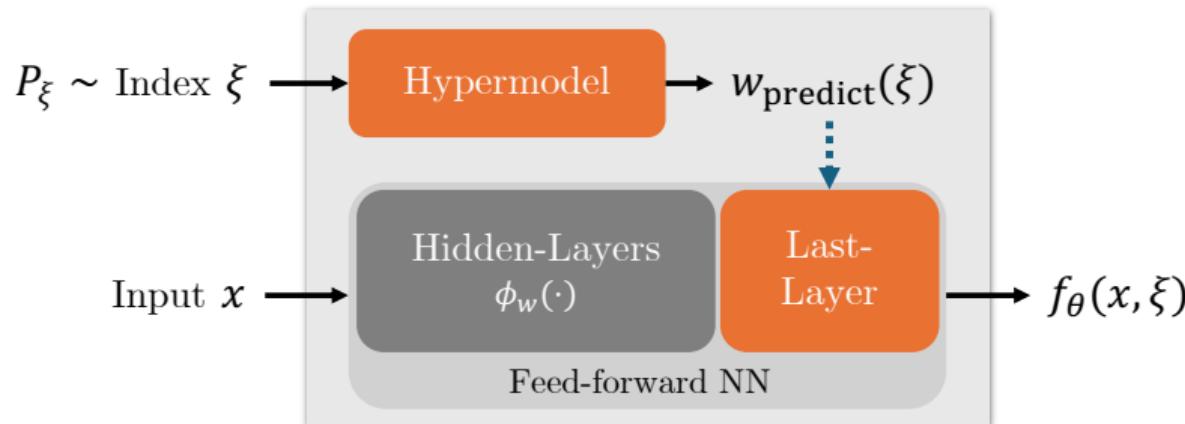
$$\xi \sim P_\xi \Rightarrow f_\theta(x, \xi) := \langle x, A\xi \rangle \text{ where } \langle A\xi \rangle \sim \mathcal{U}\{\tilde{\theta}_1, \dots, \tilde{\theta}_M\}$$

Histogram approximation:

- ▶ $\tilde{\mu} = \mathbb{E}[A\xi | A] = \frac{1}{M} \sum_{i=1}^M \tilde{\theta}_i \rightarrow \mu$ as $M \uparrow$.
- ▶ $\text{Cov}[A\xi | A] = \frac{1}{M} \sum_{i=1}^M (\tilde{\theta}_i - \tilde{\mu})(\tilde{\theta}_i - \tilde{\mu})^\top \rightarrow \Sigma$ as $M \uparrow$.
- ▶ **Problem:** $M \uparrow$ leads to **unbounded computation**.

HyperAgent: Hypermodel for Feedforward Deep Networks

- **Base model:** DNN $\langle \phi_w(\cdot), w_{\text{predict}} \rangle$



- **Hypermodel:** [LXHL24] chooses $f_\theta(x, \xi) = \langle \phi_w(x), w_{\text{predict}}(\xi) \rangle$ with $w_{\text{predict}}(\xi) = A\xi + b$

$$f_\theta(x, \xi) = \underbrace{\langle \phi_w(x), b \rangle}_{\text{'mean'} \mu_\theta(x)} + \underbrace{\langle \phi_w(x), A\xi \rangle}_{\text{'variance'} \sigma_\theta(x, \xi)}$$

↑ The degree of uncertainty

- ▶ Base model for DQN-type value function

$$f_{\theta}(s, a) = \langle \phi_w(s), \theta^{(a)} \rangle$$

with parameters $\theta = \{w, (\theta^{(a)} \in \mathbb{R}^d) : a \in \mathcal{A}\}$

\uparrow Action-specific parameters for discrete action set \mathcal{A}

- ▶ Hypermodel for randomized value function depends on (s, a) and a random index $\xi \sim P_{\xi}$:

$$f_{\theta}(s, a, \xi) = \langle \phi_w(s), \underbrace{A^{(a)} \xi + b^{(a)}}_{\theta^{(a)}(\xi)} \rangle$$

\uparrow Random index $\xi \sim P_{\xi}$

with parameters $\theta = \{w, (A^{(a)} \in \mathbb{R}^{d \times M}, b^{(a)}) : a \in \mathcal{A}\}$.

\uparrow Action-specific parameters

- ▶ Tabular representation: $\phi_w(s)$ is fixed one-hot vector in $\mathbb{R}^{|S|}$ where $d = |S|$. (**Unification!**)

HyperAgent: Seemless Integration to DDQN

Algorithm HyperAgent Framework

```
1: Input: Initial parameter  $\theta_{\text{init}}$ , hypermodel  $f_\theta$  with reference dist.  $P_\xi$  and perturbation dist.  $P_z$  .
2: Init.  $\theta = \theta^- = \theta_{\text{init}}$ , train step  $j = 0$  and buffer  $D$ 
3: for each episode  $k = 1, 2, \dots$  do
4:   Sample index mapping  $\xi_k \sim P_\xi$ 
5:   Set  $t = 0$  and Observe  $S_{k,0} \sim \rho$ 
6:   repeat
7:     Select  $A_{k,t} = \arg \max_{a \in \mathcal{A}} f_\theta(S_{k,t}, a, \xi_k(S_{k,t}))$ 
8:     Observe  $S_{k,t+1}$  from environment and  $R_{k,t+1} = r(S_{k,t}, A_{k,t}, S_{k,t+1})$ .
9:     Sample perturbation random vector  $z_{k,t+1} \sim P_z$ 
10:     $D.\text{add}((S_{k,t}, A_{k,t}, R_{k,t+1}, S_{k,t+1}, z_{k,t+1}))$ 
11:    Increment step counter  $t \leftarrow t + 1$ 
12:     $\theta, \theta^-, j \leftarrow \text{update}(D, \theta, \theta^-, \xi^- = \xi_k, t, j)$ 
13:   until  $S_{k,t} = s_{\text{terminal}}$ 
14: end for
```

HyperAgent: Objective for Generic Hypermodel (f_θ, P_ξ)

- For a transition tuple $d = (s, a, r, s', \mathbf{z}) \in D$ and given index ξ , the temporal difference (TD) error:

$$\ell^{\gamma, \sigma}(\theta; \theta^-, \xi^-, \xi, d) = \left(f_\theta(s, a, \xi) - (r + \sigma \xi^\top \mathbf{z} + \gamma \max_{a' \in \mathcal{A}} f_{\theta^-}(s', a', \xi^-(s'))) \right)^2 \quad (2)$$

target parameters, fixed here and updated in an outer loop
main parameters, optimization variables

- ξ^- : the target index mapping s.t. $\xi^-(s)$ one-to-one maps each state $s \in \mathcal{S}$ to a random vector from P_ξ , all of which are **independent** with ξ .

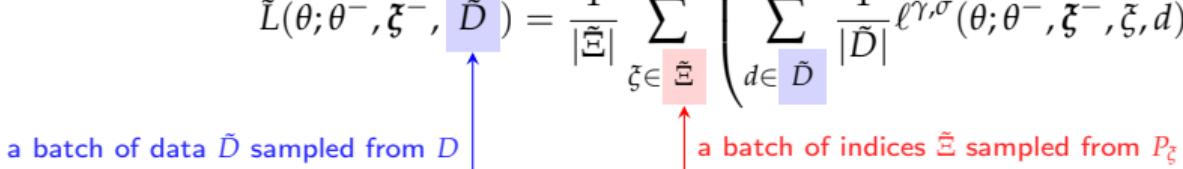
HyperAgent: Objective and Training

- ▶ Integrate ξ over Equation (2) yields objective $L^{\gamma,\sigma,\beta}$ where $\beta \geq 0$ is for the prior regularization

$$L^{\gamma,\sigma,\beta}(\theta; \theta^-, \xi^-, D) = \mathbb{E}_{\xi \sim P_\xi} \left[\sum_{d \in D} \frac{1}{|D|} \ell^{\gamma,\sigma}(\theta; \theta^-, \xi^-, \xi, d) \right] + \frac{\beta}{|D|} \|\theta\|^2 \quad (3)$$

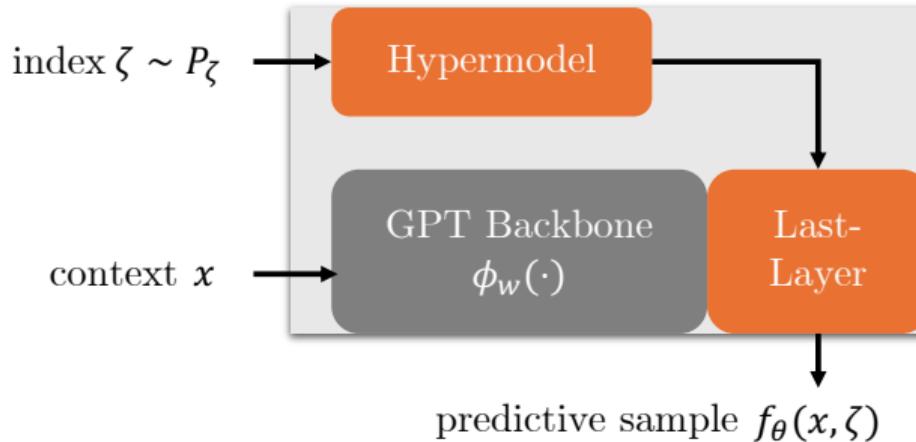
- ▶ Optimize **main objective Equation (3)** using **mini-batch SGD (default Adam)**, i.e., sampled loss

$$\tilde{L}(\theta; \theta^-, \xi^-, \tilde{D}) = \frac{1}{|\tilde{\Xi}|} \sum_{\xi \in \tilde{\Xi}} \left(\sum_{d \in \tilde{D}} \frac{1}{|\tilde{D}|} \ell^{\gamma,\sigma}(\theta; \theta^-, \xi^-, \xi, d) \right) + \frac{\beta}{|D|} \|\theta\|^2 \quad (4)$$



- ▶ Update the main parameters θ in each step according to Equation (4), and updates the target parameters θ^- periodically with less frequency. \Rightarrow **Bounded per-step computation.**

HyperAgent for Contextual Bandits with GPT involved



- ▶ Online learning for **content moderation** with **GPT** involved can be formulated as a **contextual bandit** problem.
- ▶ **HyperAgent** can be applied to the problem with **GPT** as the base model.
- ▶ Remove the **target hypermodel** in the **HyperAgent** framework for contextual bandit.

Outline

Motivation: RL under Resource Constraints

Existing solution and their limitations

Our Contributions

HyperAgent: Scalable Uncertainty and Exploration

Theoretical insights with tabular representation

Reduce sequential posterior approx. to sequential random projection

- ▶ **Tabular representation:** $\phi_w(s)$ is fixed one-hot vector in $\mathbb{R}^{|S|}$ where $d = |S|$. (**Unification!**)
- ▶ **Tabular HyperAgent:** short notations

$$\begin{aligned} f_\theta(s, a, \xi) &= \langle \phi_w(s), A^{(a)}\xi + b^{(a)} \rangle \\ &= \underbrace{\langle (A^{(a)})^\top \phi_w(s) + (b^{(a)})^\top \phi_w(s), \xi \rangle}_{\tilde{m}_{sa}} \end{aligned}$$

- ▶ Parameters in k -th episode $\theta_k = (\mu_{k,sa}, \tilde{m}_{k,sa} \in \mathbb{R}^M, \forall (s, a) \in \mathcal{S} \times \mathcal{A})$.
- ▶ $\phi_w(s)$ **fixed mapping**, e.g. tabular and linear FA.
- ▶ ⇒ Equation (3) of **HyperAgent permits closed-form solution**.
 - HyperDQN [LLZ⁺22] & ENN-DQN[OWA⁺23b] can **not** derive closed-form solution.

Insights from closed-form solution

► Incremental update with computation complexity $O(M)$:

$$\tilde{m}_{k,sa} = \frac{(N_{k-1,sa} + \beta) \tilde{m}_{k-1,sa} + \sum_{t \in E_{k-1,sa}} \sigma \mathbf{z}_{\ell,t+1}}{(N_{k,sa} + \beta)} \in \mathbb{R}^M \quad (5)$$

Perturbation random vector

Set of timesteps encountering (s,a) in episode $k-1$

Visitation counts of (s,a) up to episode k

Lemma 1 (Sequential posterior approximation via incremental update).

For \tilde{m}_k recursively defined in Equation (5) with $\mathbf{z} \sim \mathcal{U}(\mathbb{S}^{M-1})$. For any $k \geq 1$, define the good event of ε -approximation

$$\mathcal{G}_{k,sa}(\varepsilon) := \left\{ \|\tilde{m}_{k,sa}\|^2 \in \left((1-\varepsilon) \frac{\sigma^2}{N_{k,sa} + \beta}, (1+\varepsilon) \frac{\sigma^2}{N_{k,sa} + \beta} \right) \right\}.$$

The **joint event** $\cap_{(s,a) \in \mathcal{S} \times \mathcal{A}} \cap_{k=1}^K \mathcal{G}_{k,sa}(\varepsilon)$ holds w.p. at least $1 - \delta$ if $M \simeq \varepsilon^{-2} \log(SAHK/\delta)$.

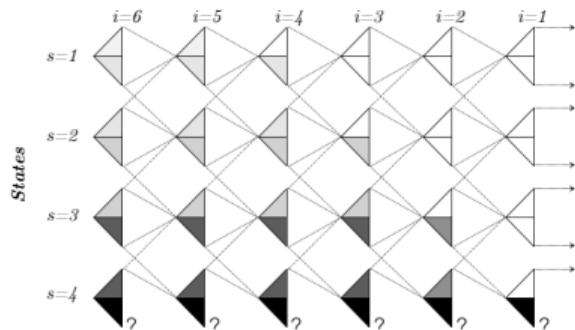
Insights from closed-form solution

Stochastic Bellman Operator F_k^γ induced by Equation (3) w. $\theta = \theta_k^{(i+1)}, \theta^- = \theta_k^{(i)}$ **iteratively**:

$$f_{\theta_k^{(i+1)}, \xi_k} = F_k^\gamma f_{\theta_k^{(i)}, \xi_k} \approx (r_{sa} + \gamma \langle V_{f_{\theta_k^{(i)}, \xi_k}}, \hat{P}_{k,sa} \rangle) + \tilde{m}_{k,sa}^\top \xi_k(s), \quad (6)$$

↑ Empirical transition
↑ "Randomized bonus" $\propto \sqrt{\frac{1}{N_{k,sa}}}$

where $f_{\theta, \xi^-}(s, a) = f_\theta(s, a, \xi^-(s))$ and $V_Q(s) := \max_a Q(s, a)$, $\forall s$ is the greedy value w.r.t. Q .



Setup: $N_{k,(4,\searrow)} = 1$. Other (s, a) almost **infinite data**.

- (1) **Propagation of uncertainty** from later time period to earlier time period due to **iterative applying** F_k^γ .
- (2) **Darker shade indicates higher** degree of uncertainty.
- (3) **Incentivize deep exploration.**

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Reduce **sequential posterior approx.** to sequential random projection

Step 1: Rewrite incremental update on $\tilde{m}_{k,sa}$

- ▶ E_ℓ : the collection of time steps in episode ℓ .
- ▶ $E_{\ell,sa}$: the collection of time steps in episode ℓ encountering state-action pair (s, a) .
- ▶ Define a sequence of indicator variables $x_{\ell,t} = \mathbb{1}_{t \in E_{\ell,sa}}$. Note

$$\sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t}^2 = N_{k,sa}$$

- ▶ Define short notations $\mathbf{z}_0 = \mathbf{z}_{0,sa}$ and $x_0 = \sqrt{\beta}$. Let $\beta = \sigma^2 / \sigma_0^2$. Equation (5) now becomes

$$\frac{(N_{k,sa} + \beta)}{\sigma} \tilde{m}_{k,sa} = \mathbf{x}_0 \mathbf{z}_0 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t} \mathbf{z}_{\ell,t+1} \quad (7)$$

- ▶ Lemma 1 ⇒ w.h.p. Equation (8) holds for all $(s, a) \in \mathcal{S} \times \mathcal{A}$ and $k \in [K]$ simultaneously:

$$(1 - \varepsilon) (\mathbf{x}_0^2 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t}^2) \leq \| \mathbf{x}_0 \mathbf{z}_0 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t} \mathbf{z}_{\ell,t+1} \|^2 \leq (1 + \varepsilon) (\mathbf{x}_0^2 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t}^2) \quad (8)$$

Classical JL for random projection

- ▶ Try to relate Equation (8) to the classical **Johnson–Lindenstrauss (JL)** lemma:

Consider $\Pi = (\mathbf{z}_1, \dots, \mathbf{z}_d) \in \mathbb{R}^{M \times d}$, $x = (x_1, \dots, x_d)^\top \in \mathbb{R}^d$, then $\Pi x = \sum_{i=1}^d x_i \mathbf{z}_i$

Lemma 2 (Distributional JL lemma [JL84]).

For any $0 < \varepsilon, \delta \leq 1/2$ and $d \geq 1$ there exists a distribution $\mathcal{D}_{\varepsilon, \delta}$ on $\mathbb{R}^{M \times d}$ for $M = O(\varepsilon^{-2} \log(1/\delta))$ such that for any $x \in \mathbb{R}^d$

$$\mathbb{P}_{\Pi \sim \mathcal{D}_{\varepsilon, \delta}} \left(\|\Pi x\|_2^2 \notin \left[(1 - \varepsilon) \|x\|_2^2, (1 + \varepsilon) \|x\|_2^2 \right] \right) < \delta$$

- ▶ Existing JL analysis based on the **assumption**: x fixed non-random or the projection matrix Π is generated **independently** with the data x , i.e.

$$\Pi := (\mathbf{z}_1, \dots, \mathbf{z}_d) \perp x := (x_1, \dots, x_d).$$

Step 2: Identify dependence structure

Sequential dependence structure in HyperAgent when interacting with environment is that

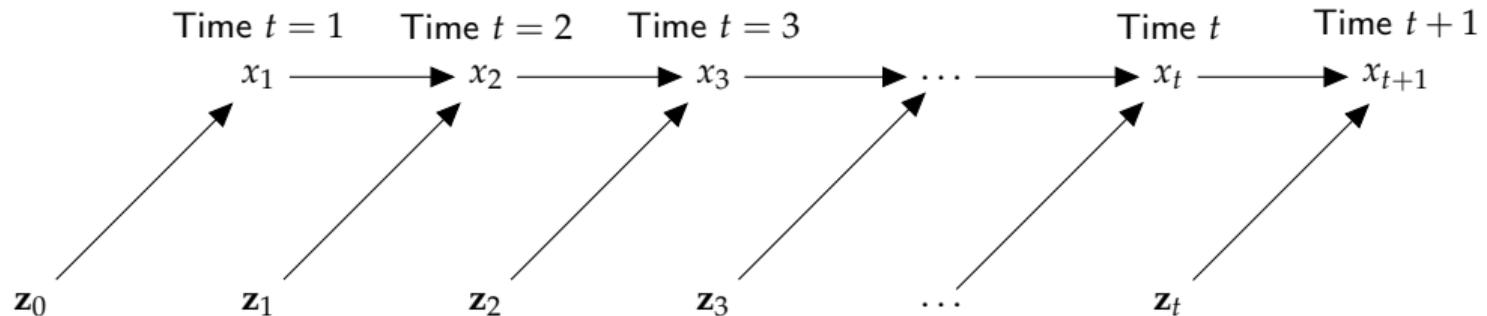
- ▶ E_ℓ : the collection of time steps in episode ℓ .
- ▶ $E_{\ell,sa}$: the collection of time steps in episode ℓ
- ▶ $x_{\ell,t} = \mathbb{1}_{t \in E_{\ell,sa}}$ is **dependent** on the environmental and algorithmic randomness in all previous time steps:

$$\mathbf{z}_0, (x_{1,t'}, \mathbf{z}_{1,t'+1})_{t' \in E_1}, (x_{2,t'}, \mathbf{z}_{2,t'+1})_{t' \in E_2}, \dots, (x_{\ell,t'}, \mathbf{z}_{\ell,t'+1})_{t' < t};$$

- ▶ $\mathbf{z}_{\ell,t+1}$ is **independent** of the environmental and algorithmic randomness in all previous time steps:

$$\mathbf{z}_0, (x_{1,t'}, \mathbf{z}_{1,t'+1})_{t' \in E_1}, (x_{2,t'}, \mathbf{z}_{2,t'+1})_{t' \in E_2}, \dots, (x_{\ell,t'+1}, \mathbf{z}_{\ell,t'+1})_{t' < t}, x_{\ell,t},$$

Difficulty and Novelty in the Mathematical Analysis: No Prior Art



Sequential dependence of high-dimensional R.V. due to the adaptive nature of sequential decision-making.

Difficulty: (1) Conditioned on x_t , $(z_s)_{s < t}$ loss their independence; (2) No characterization on $P_{(z_s)_{s < t} | x_t}$.
⇒ Traditional analysis of random projection **cannot handle sequential dependence** [Li24a].

First probability tool for sequential random projection. [Li24a]

- ▶ A non-trivial martingale extension of the Johnson–Lindenstrauss (JL).
- ▶ Technical novelty: a careful construction of stopped process with non-trivial application of ‘method of mixtures’ in self-normalized martingale.

Sequential Random Projection

Theorem 1 (Sequential random projection in adaptive processes [Li24a]).

- Let $\varepsilon \in (0, 1)$ be fixed and $(\mathcal{F}_t)_{t \geq 0}$ be a filtration. Let $\mathbf{z}_0 \in \mathbb{R}^M$ be an \mathcal{F}_0 -measurable random vector satisfies $\mathbb{E}[\|\mathbf{z}_0\|^2] = 1$ and $|\|\mathbf{z}_0\|^2 - 1| \leq (\varepsilon/2)$.
- Let $(\mathbf{z}_t)_{t \geq 1} \subset \mathbb{R}^M$ be a stochastic process adapted to filtration $(\mathcal{F}_t)_{t \geq 1}$ such that it is $\sqrt{c_0/M}$ -sub-Gaussian and each \mathbf{z}_t is unit-norm.
- Let $(x_t)_{t \geq 1} \subset \mathbb{R}$ be a stochastic process adapted to filtration $(\mathcal{F}_{t-1})_{t \geq 1}$ such that it is c_x -bounded. Here, c_0 and c_x are absolute constants.
- For any fixed $x_0 \in \mathbb{R}$, if the following condition is satisfied

$$M \geq \frac{16c_0(1 + \varepsilon)}{\varepsilon^2} \left(\log\left(\frac{1}{\delta}\right) + \log\left(1 + \frac{c_x T}{x_0^2}\right) \right),$$

we have, with probability at least $1 - \delta$

$$\forall t \in \{0, 1, \dots, T\}, \quad (1 - \varepsilon)\left(\sum_{i=0}^t x_i^2\right) \leq \left\|\sum_{i=0}^t x_i \mathbf{z}_i\right\|^2 \leq (1 + \varepsilon)\left(\sum_{i=0}^t x_i^2\right).$$

Simple, Efficient, Scalable: Bridging Theory and Practice



HyperAgent

- ▶ Simple, Efficient and Scalable;
- ▶ Practically useful for safety-critical decision-making in Human-AI interplay.
- ▶ Bridging theory and practice. No prior art.



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