HyperAgent: Advancing Scalable Exploration through Fast Uncertainty Estimation in RL

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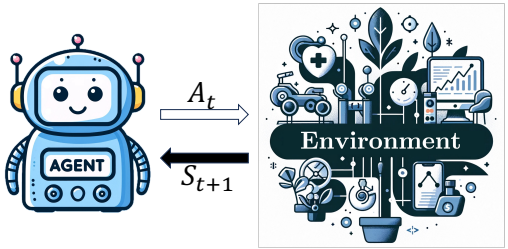
July 25, 2024

Outline

Reinforcement Learning Problem



Agent-Environment Interface.

- Interactive Experience:

$$A_0, S_1, A_1, S_2, \ldots, A_t, S_{t+1}, \ldots$$

Environment $M = (\mathcal{S}, \mathcal{A}, P)$

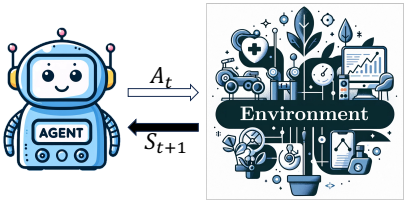
- State $S_{t+1} \sim P(\cdot \mid S_t, A_t)$ for $t = 0, 1, \ldots$.

$\text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t) \to \pi_t$ max long-term rewards

- **Reward** $R_{t+1} = r(S_t, A_t, S_{t+1})$ preference
- **Data** $\mathcal{D}_t = \mathcal{D}_{t-1} \cup \{A_{t-1}, S_t\}$ **accumulated**.
- **Policy** $\pi_t = \text{Agent}(\mathcal{S}, \mathcal{A}, r, \mathcal{D}_t)$.
- **Action** $A_t \sim \pi_t(\cdot \mid S_t)$;
- **Objective** $\pi_{\text{agent}} = (\pi_0, \pi_1, \ldots)$ to maximize

$$\mathbb{E}[\sum_{t=0}^{T-1} R_{t+1} \mid \pi_{\text{agent}}, M]\,. \tag{1}$$

Challenges for Deploying RL in Real-world



Agent-Environment Interface.

- Interactive Experience:

$$\underbrace{A_0, S_1, A_1, S_2, \ldots, A_t, S_{t+1}, \ldots}_{\mathcal{D}}$$

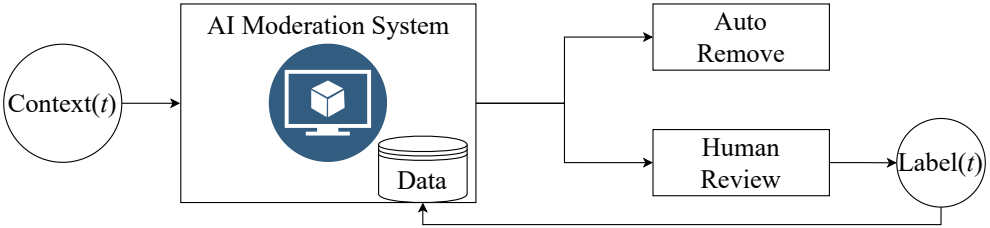
Complex Environment:

- **Large state space**: language, vision & audios. $|\mathcal{S}| \approx 10^{100}$.
- **Data accumulates** as interacting. $|\mathcal{D}| \uparrow$

Resource Constraints for Agent:

- **Bounded** Per-step Computation & Memory
- **Limited** Data Collection Budgets

Example: Human-AI Collaboration in Automated Content Moderation



The Human-AI agile collaboration pipeline for risk oversight in an **online** production environment

- **Challenge** 1: Natural language input in post context. "Cold start" problem.
  Large foundation models (GPTs) are used in Moderation system.
- **Challenge** 2: Real-time safety-critical decision-making.
  Huge amount of posts are generated every second. **Filter out harmful** post, **aligning human value**.
- **Challenge** 3: Limited human reviewer
  can only provide feedback and moderate on a small portion of posts.

Research Question

Can we design **scalable** and **data-efficient** RL algorithms under **resource constraints**?

- **Scalability**: **Bounded per-step computation and memory complexity** even when large foundation model (e.g., GPT) is involved in the online decision-making process.
- **Data-efficiency**: Handle limited data collection budgets. **Sublinear regret**.

Outline

Development of RL Algorithms: A History of "Scale up!"

R. Bellman,
R. A. Howard,
D. Blackwell
Dynamic Programming
(Markov) Decision Processes

50s-60s

R.S. Sutton & A. Barto
C. J.C.H. Watkins
D. Bertsekas & J. Tsitsiklis
Temporal Difference, Q-Learning
Approximate (Neuro) DPs
Parallel and Distributed Computation

70s-90s

DeepMind
OpenAI
Etc.
Deep RL (Advances in Algorithms and Computations)
AlphaGo & GPTs (Foundation Models)

Scale up!

2010s --

Computation First

- **Per-step** $O(\text{poly}(|\mathcal{S}|))$ $\Rightarrow$ **Function Approximation** (FA), e.g. Neural Networks; (70s – 90s)
- **Per-step** $O(\text{poly}(|\mathcal{D}|))$ $\Rightarrow$ **Incremental Update** with SGD, Replay Buffer and/or Target Network.
- **FA + Incremental Update** $\Rightarrow$ **Bounded** $\tilde{O}(1)$ **Per-step Complexity** $\Rightarrow$ **Scalable** algorithm. (10s –)

Practical Advancements for "Efficient" Deep RL

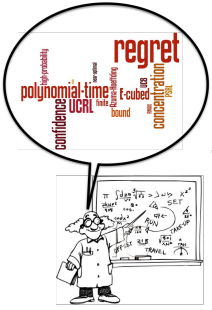
Algorithm	Components
DDQN (16)	Incremental SGD with experience replay (finite buffer) and target network
Rainbow (18)	(**DDQN**) + Prioritized replay, Dueling networks, Distributional RL, Noisy Nets.
BBF (23)	(**DDQN**) + Prioritized replay, Dueling networks, Distributional RL, Self-Prediction, Harder resets, Larger network, Annealing hyper-parameters.

Table: Components in **STOA algorithms**, e.g. DDQN [VHGS16], Rainbow [HMVH$^+$18], BBF [SCC$^+$23].

- ✓ **Scalable**: e.g. DDQN use incremental SGD with experience replay and target network.
- ✗ **Deployment inefficient**: **Complicated** components and **many** heuristics. **Hard to tune**.
- ✗ **Data inefficient**: e.g. BBF use ϵ-greedy **exploration strategy** which suffer **linear regret** in some environment, **provably** [Kak03, Str07, OVRRW19, DMM$^+$22]. **In practice**, deep RL **data hungury** .

Principled Approaches for Data Efficiency

Goal: **Sequential decision-making** under **uncertainty** with **sublinear regret**.



R.A. Fisher, William R. Thompson (30s)
T. L. Lai and H. Robbins (80s)
Sequential design and allocations (MAB)

E3/PSRL/UCRL ...
Tabular RL

Eluder dimension/Information Ratio
Bellman Rank/Bilinear Structure
Decoupling coefficient/DEC/GEC ...
Structural assumption and Algorithms for RL with function approximation

Data First

Data Efficient Exploration: Uncertainty Estimation Matters

Posterior Sampling Reinforcement Learning (PSRL): **data-efficient exploration** strategy

- **Require**: Prior distribution $\mathbb{P}(M \in \cdot)$ for underlying model M.
- **For each episode** ℓ, denote t_ℓ the beginning time step
  - Sample $\hat{M}_\ell \sim \mathbb{P}\left(M \in \cdot \mid \mathcal{D}_{t_\ell}\right)$.
  - **Return** the optimal policy $\pi_\ell = \pi^{\hat{M}_\ell}$ under $\hat{M}_\ell$.

- Require **conjugacy** for **tractable posterior update** **(uncertainty estimation)**.
- **Only feasible** in simple environments:
  - Tabular MDP with dirichlet prior [Str00, OVR17] $\tilde{O}(H^2\sqrt{SAK})$ regret **sublinear in** K **episodes**.
  - Linear-Gaussian bandit [RVR16, RVRK$^+$18] $O(d\sqrt{T \log A})$ regret **subliear in** T **time steps.**

Data Efficient Exploration needs Scalable Uncertainty Estimation

✗ **Intractable Computation** in **Posterior Sampling**:

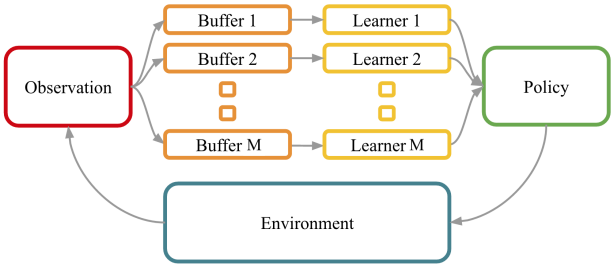
- **Model-based**: **No conjugacy** for exact Bayesian inference for posterior over transition models.
  - **[LL24] (AISTATS)**: **First prior-dependent** bound under FA and **improved prior-free** bound in the context of **linear mixture MDPs**.
- **Model-free**: Beyond conjugacy, sample from **intricate distribution over value functions** [Zha22, DMZZ21, ZXZ+22]

✗ **Unbounded Per-step Complexity** $\mathrm{poly}(|\mathcal{D}|)$ in **Approximate Posterior Sampling**:

- **Store entire history and retrain for each episode**, e.g. RLSVI [OVRRW19], LSVI-PHE [ICN+21].
- **Langevin Monte-Carlo (LMC)** based methods [XZM+22, ILX+24]

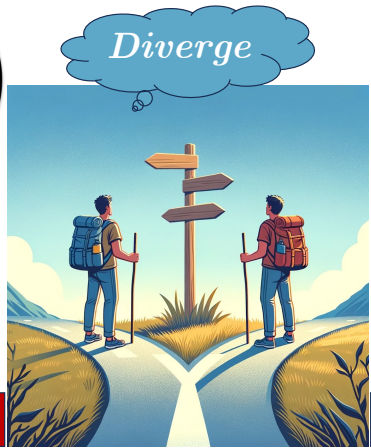
- **Same issues for OFU**: (1) ✗ **Intractability** [JKA+17, JLM21, DKL+21, FKQR21, LLX+23]; (2) ✗ **Unbounded resource demands** as data accumulates [WSY20, AJZ23].

Ensemble Sampling for Approximate Posterior Sampling



- **Ensemble Sampling (ES)**: approximate the posterior distribution by **uniformly sampling from a set of ensemble models**. E.g., BootstrapDQN [OBPVR16], **Ensemble+** [OAC18, OVRRW19].
- ✓ Each ensemble perform incremental update, **no retraining**.
- ✗ **Computationally expensive** in practice: say, update > 100 neural networks for each time step.
- ✗ **No rigorous** understanding in terms of statistical and computational complexity.

Theory! Data First



Scale up! Computation First

Outline

Our HyperAgent [LXHL24] aims to ...

Preview of Contributions - For Practitioners

Algorithm	Components
DDQN (16)	Incremental SGD with experience replay (finite buffer) and target network
Rainbow (18)	(**DDQN**) + Prioritized replay, Dueling networks, Distributional RL, Noisy Nets.
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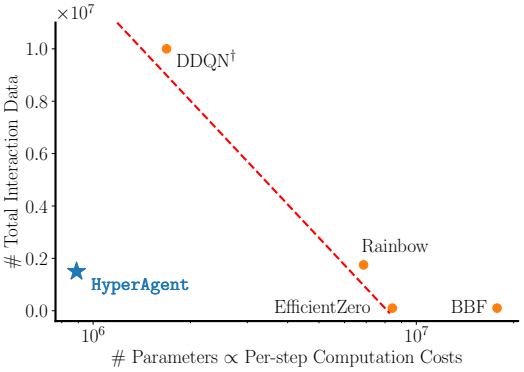
HyperAgent	(**DDQN**) + **Hypermodel**

Table: Techniques used in different algorithms, e.g. DDQN [VHGS16], Rainbow [HMVH$^+$18], BBF [SCC$^+$23] and **our HyperAgent**.

- ✓ **Simple**: **Only one** additional component, **hypermodel**, compatiable with all feedforward DNN.
  $\Rightarrow$ **Easy to deploy** empirically.
- ✓ **Scalable**: **Incremental** SGD under DNN function approximation, same as DDQN;
  $\Rightarrow$ **bounded per-step computation**.

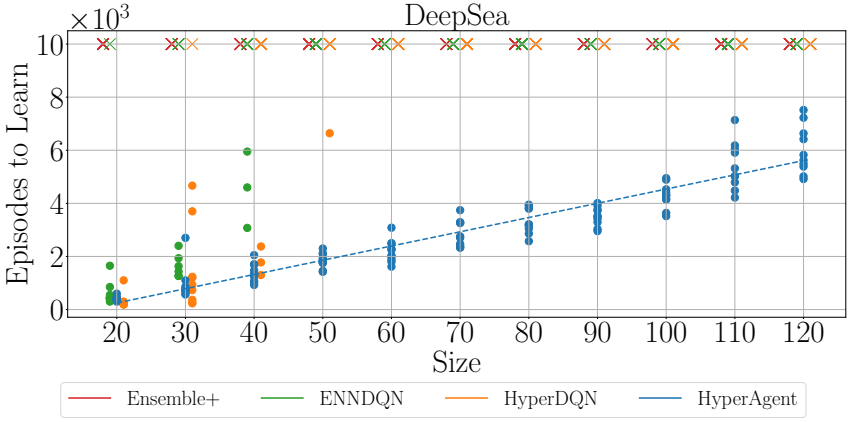
Preview of Contributions - For Practitioners



How much data and parameters to achieve **Human-level performance (1 IQM)** in Atari suite?

- ✓ **Data efficient**: only 15% data consumption of DDQN[VHGS16] by **DeepMind**. (1.5**M interactions**)
- ✓ **Computation efficient**: only 5% model parameters of BBF[SCC+23] by **DeepMind**.
- **Ensemble+** [OAC18, OVRRW19] achieves a **mere** 0.22 **IQM score under** 1.5**M** interactions but necessitates **double the parameters of** `HyperAgent`.

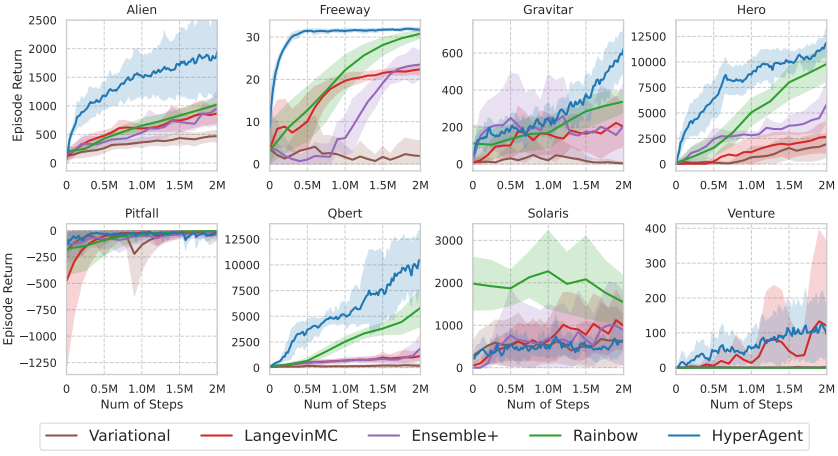
HyperAgent: Data ffficiency in DeepSea benchmarks



Comparison with **Ensemble+** [OAC18, OVRRW19], **HyperDQN** [LLZ$^+$22], **ENN-DQN**[OWA$^+$23b].

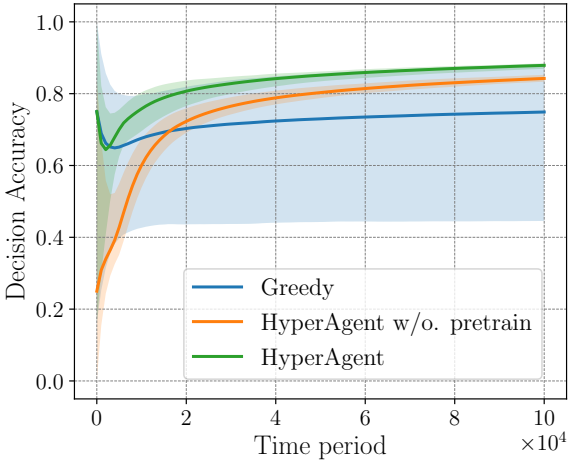
- ✓ **Scalable** as size $N \uparrow$. State representation: one-hot vector in high-dimension $\mathbb{R}^N$.
- ✓ **Data efficient**: `HyperAgent` the only and first achieving **optimal episode complexity** $\Theta(N)$.

HyperAgent for hardest exploration in Atari



Comparison on approximate posterior sampling methods: **Variational approximation** (SANE [AL21]), **Langevin Monte-Carlo** (AdamLMCDQN [ILX$^+$24]) and **Ensemble+** [OAC18, OVRRW19]

HyperAgent for Online Automated Content Moderation



The human-AI collaboration pipeline for online automated content moderation, focusing on hate speech detection.

- **10x labeling effort reduction**,
- **Higher detection accuracy**.
- **Pretrained foundation model helps**: augment pretrained GPT-2 backbone with hypermodel.

Preview of Contributions - For Theoretists (RL)

	Practice in Deep RL			**Theory** in Tabular RL	
Algorithm	**Tractable**	**Incremental**	**Efficient**	**Regret**	**Per-step Computation**
PSRL	✗	✗	✗	$\tilde{O}(H^2\sqrt{SAK})$	$O(S^2A)$
RLSVI	✓	✗	✗	$\tilde{O}(H^2\sqrt{SAK})$	$O(S^2A)$
Ensemble+	✓	✓	●	N/A	N/A
`HyperAgent`	✓	✓	✓	$\tilde{O}(H^2\sqrt{SAK})$	$\tilde{O}(\log(K)SA + S^2A)$

- **HyperAgent** not only demonstrates superior empirical performance in deep RL benchmarks
- but also achieves **theoretical milestones**, i.e., the first method to achieve $\tilde{O}(\log K)$ **per-step computation** & **near-optimal regret** in **tabular** K**-episodic RL** among practically scalable algorithms.

Preview of Contributions - For Theoretists (Contextual Bandit)

Decision Sets Lower Bound	Invariant & Compact $\Omega(d\sqrt{T})$	Variant & Compact $\Omega(d\sqrt{T\log T})$	Invariant & Finite $\Omega(\sqrt{dT\log\lvert\mathcal{A}\rvert})$	Variant & Finite $\Omega(\sqrt{dT\log\lvert\mathcal{A}\rvert\log T})$
TS	$O(d^{\frac{3}{2}}\sqrt{T}\log T)$	$O(d^{\frac{3}{2}}\sqrt{T}\log T)$	$O(d\sqrt{T\log\lvert\mathcal{A}\rvert}\log T)$	$O(d\sqrt{T\log\lvert\mathcal{A}\rvert}\log T)$
ES[Qin]	N/A	N/A	$O(\sqrt{dT\log\lvert\mathcal{A}\rvert\log(\lvert\mathcal{A}\rvert T/d)})$	N/A
LMC[Xu]	$O((d\log T)^{\frac{3}{2}}\sqrt{T})$	$O((d\log T)^{\frac{3}{2}}\sqrt{T})$	N/A	N/A
ES[Janz]	$O((d\log T)^{\frac{5}{2}}\sqrt{T})$	$O((d\log T)^{\frac{5}{2}}\sqrt{T})$	N/A	N/A
(Ours)	$O(d^{\frac{3}{2}}\sqrt{T}(\log T)^{\frac{3}{2}})$	$O(d^{\frac{3}{2}}\sqrt{T}(\log T)^{\frac{3}{2}})$	$O(d\sqrt{T\log\lvert\mathcal{A}\rvert}\log T)$	$O(d\sqrt{T\log\lvert\mathcal{A}\rvert}\log T)$

Table: Regret lower and upper bounds under **various decision set setups in linear contextual bandit**. The per-step computation complexity is $O(d^2 + d|\mathcal{A}|T)$ for ES [QWLVR22], $O(d^2T)$ for LMC [XZM+22], $O(d^3\log T)$ for ES [JLS24], and $O(d^3\log T)$ for our HyperAgent.

- **Logarithmic** per-step computation complexity in total time periods T.
- **HyperAgent** matches exact TS, closing a gap in theory for scalable exploration.

Preview of Contributions - For Practitioners & Theoretists

Algorithmic Mechanism

- Value-based **approximate posterior sampling** via **hypermodel** and **index sampling** schemes.
- $\Rightarrow$ **Near-optimal regret bound** $\Rightarrow$ **Data-efficient** Exploration

$\Uparrow$

Key Lemma

- **Incremental approximation of posteriors** over value function **without conjugacy**.
- $\Rightarrow$ **Logarithmic** per-step **computation complexity** $\Rightarrow$ **Scalable** Uncertainty Estimation.

$\Uparrow$

- **Fundamental Tools** for **dynamic (non-i.i.d.) data**: **First Probability Tool** for **Sequential Random Projection** – a **non-trivial martingale extension** of Johnson-Lindenstrauss (JL). [Li24a]
- **Fundamental Tools** for **static data**: **Simple, Unified JL analysis** that covers existing and new JL construction that **traditional analysis cannot handle**. [Li24b]

Outline

HyperAgent: Introducing Hypermodel

Paramatric function — Reference distribution

- **Hypermodel**: (f_θ , P_ξ) s.t.

  Index Sampling: $f_\theta(x, \xi)$ is an (approximate) **posterior predictive sample** on data x.

  Index sample $\xi \sim P_\xi$

- [DLI+20, LLZ+22, OWA+23a]

Example: predictive sampling from Linear-Gaussian model

- Suppose $\theta^* \sim N(\mu, \Sigma)$ where Σ represent the **model uncertainty**.
- **Box-Muller Transform**: $P_\xi = N(0, I_M)$, $\theta = (A \in \mathbb{R}^{d \times M}, \mu \in \mathbb{R}^d)$

$$\xi \sim P_\xi \Rightarrow f_\theta(x, \xi) := \langle x, \mu + A\xi \rangle \sim N(x^\top \mu, x^\top A A^\top x)$$

- **Uncertain Estimation**: **Find** A s.t. $AA^\top = \Sigma$. $\Rightarrow f_\theta(x, \xi) \sim \langle \theta^*, x \rangle$.

HyperAgent: Introducing Hypermodel

Paramatric function — Reference distribution

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$$\xi \sim P_\xi \Rightarrow f_\theta(x, \xi) := \langle x, \mu + \boldsymbol{A}\xi \rangle \sim N(x^\top \mu, x^\top \boldsymbol{A}\boldsymbol{A}^\top x)$$

- **Uncertain Estimation**: **Find** $\boldsymbol{A}$ s.t. $\boldsymbol{A}\boldsymbol{A}^\top = \boldsymbol{\Sigma}$. $\Rightarrow f_\theta(x, \xi) \sim \langle \theta^*, x \rangle$.

Ensemble Sampling as Special Case of Hypermodel Index Sampling

Sampling from Linear-Gaussian model $N(x^\top \mu, x^\top \Sigma x)$, we could perform

Ensemble Sampling (# of models M)

$P_\xi = \mathcal{U}\{e_1, \ldots, e_M\}$ and $\theta = A = [\tilde{\theta}_1, \ldots, \tilde{\theta}_M] \in \mathbb{R}^{d \times M}$, s.t. $\tilde{\theta}_m \sim N(\mu, \Sigma)$.

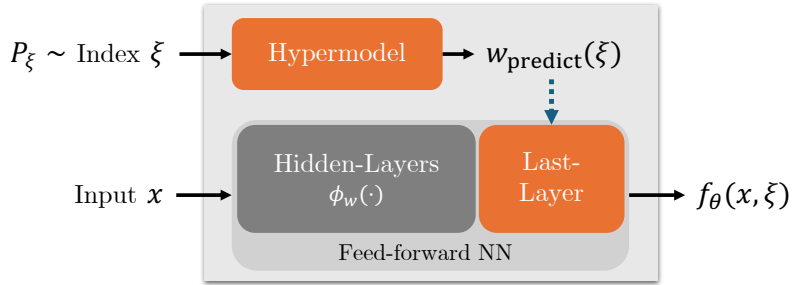
$$\xi \sim P_\xi \Rightarrow f_\theta(x, \xi) := \langle x, A\xi \rangle \quad \text{where} \quad A\xi \rangle \sim \mathcal{U}\{\tilde{\theta}_1, \ldots, \tilde{\theta}_M\}$$

Histogram approximation:

- $\tilde{\mu} = \mathbb{E}[A\xi \mid A] = \frac{1}{M} \sum_{i=1}^M \tilde{\theta}_i \to \mu$ as $M \uparrow$.
- $\mathrm{Cov}[A\xi \mid A] = \frac{1}{M} \sum_{i=1}^M (\tilde{\theta}_i - \tilde{\mu})(\tilde{\theta}_i - \tilde{\mu})^\top \to \Sigma$ as $M \uparrow$.
- **Problem**: $M \uparrow$ leads to **unbounded computation**.

HyperAgent: Hypermodel for Feedforward Deep Networks

- **Base model**: DNN $\langle \phi_w(\cdot), w_{\text{predict}} \rangle$



- **Hypermodel**: [LXHL24] chooses $f_\theta(x, \xi) = \langle \phi_w(x), w_{\text{predict}}(\xi) \rangle$ with $w_{\text{predict}}(\xi) = A\xi + b$

$$f_\theta(x, \xi) = \underbrace{\langle \phi_w(x), b \rangle}_{\text{'mean' } \mu_\theta(x)} + \underbrace{\langle \phi_w(x), A\xi \rangle}_{\text{'variance' } \sigma_\theta(x,\xi)}$$

The degree of uncertainty

HyperAgent: Hypermodel for Deep RL

- **Base model** for **DQN-type value function**

$$f_\theta(s,a) = \langle\, \phi_w(s)\,, \theta^{(a)} \rangle$$

with parameters $\theta = \{w,\ (\theta^{(a)} \in \mathbb{R}^d) : a \in \mathcal{A}\}$

Action-specific parameters for discrete action set $\mathcal{A}$

- **Hypermodel** for **randomized value function** depends on (s,a) and a **random** index $\xi \sim P_\xi$:

$$f_\theta(s,a,\xi) = \langle\, \phi_w(s)\,, \underbrace{A^{(a)}\xi + b^{(a)}}_{\theta^{(a)}(\xi)} \rangle$$

Random index $\xi \sim P_\xi$

with parameters $\theta = \{w,\ (A^{(a)} \in \mathbb{R}^{d\times M}, b^{(a)}) : a \in \mathcal{A}\}$.

Action-specific parameters

- **Tabular** representation: $\phi_w(s)$ is fixed one-hot vector in $\mathbb{R}^{|\mathcal{S}|}$ where $d = |S|$. (**Unification!**)

HyperAgent: Seemless Integration to DDQN

Algorithm `HyperAgent` Framework

Input: Initial parameter θ_{init}, hypermodel f_θ with reference dist. P_ξ and perturbation dist. $P_{\mathbf{z}}$.
Init. $\theta = \theta^- = \theta_{\text{init}}$, train step $j = 0$ and buffer D
for each episode $k = 1, 2, \ldots$ **do**
  Sample index mapping $\boldsymbol{\xi}_k \sim P_\xi$
  Set $t = 0$ and **Observe** $S_{k,0} \sim \rho$
  repeat
    Select $A_{k,t} = \arg\max_{a \in \mathcal{A}} f_\theta(S_{k,t}, a, \boldsymbol{\xi}_k(S_{k,t}))$
    Observe $S_{k,t+1}$ **from environment** and $R_{k,t+1} = r(S_{k,t}, A_{k,t}, S_{k,t+1})$.
    Sample perturbation random vector $\mathbf{z}_{k,t+1} \sim P_{\mathbf{z}}$
    D.**add**$((S_{k,t}, A_{k,t}, R_{k,t+1}, S_{k,t+1}, \mathbf{z}_{k,t+1}))$
    Increment step counter $t \leftarrow t + 1$
    $\theta, \theta^-, j \leftarrow \texttt{update}(D, \theta, \theta^-, \boldsymbol{\xi}^- = \boldsymbol{\xi}_k, t, j)$
  until $S_{k,t} = s_{\text{terminal}}$
end for

HyperAgent: Objective for Generic Hypermodel (f_θ, P_ξ)

- For a transition tuple $d = (s, a, r, s', \mathbf{z}) \in D$ and given index ξ, the temporal difference (TD) error:

perturbation random vector

discounted factor

$$\ell^{\gamma,\sigma}(\theta; \theta^-, \boldsymbol{\xi}^-, \xi, d) = \left(f_\theta(s, a, \xi) - (r + \sigma\, \xi^\top \mathbf{z} + \gamma \max_{a' \in \mathcal{A}} f_{\theta^-}(s', a', \boldsymbol{\xi}^-(s'))) \right)^2 \quad (2)$$

control the std of injected noise

target parameters, fixed here and updated in an outer loop

main parameters, optimization variables

- $\boldsymbol{\xi}^-$: the target index mapping s.t. $\boldsymbol{\xi}^-(s)$ one-to-one maps each state $s \in \mathcal{S}$ to a random vector from P_ξ, all of which are **independent** with ξ.

HyperAgent: Objective and Training

- Integrate ξ over Equation (2) yields objective $L^{\gamma,\sigma,\beta}$ where $\beta \geqslant 0$ is for the prior regularization

$$L^{\gamma,\sigma,\beta}(\theta;\theta^-,\boldsymbol{\xi}^-,D) = \mathbb{E}_{\tilde{\xi}\sim P_\xi}[\sum_{d\in D}\frac{1}{|D|}\ell^{\gamma,\sigma}(\theta;\theta^-,\boldsymbol{\xi}^-,\xi,d)] + \frac{\beta}{|D|}\|\theta\|^2 \tag{3}$$

- Optimize **main objective Equation (3)** using **mini-batch SGD (default Adam)**, i.e., sampled loss

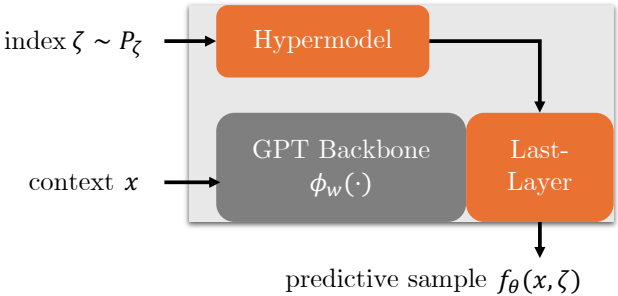
$$\tilde{L}(\theta;\theta^-,\boldsymbol{\xi}^-,\tilde{D}) = \frac{1}{|\tilde{\Xi}|}\sum_{\xi\in\tilde{\Xi}}\left(\sum_{d\in\tilde{D}}\frac{1}{|\tilde{D}|}\ell^{\gamma,\sigma}(\theta;\theta^-,\boldsymbol{\xi}^-,\xi,d)\right) + \frac{\beta}{|D|}\|\theta\|^2 \tag{4}$$

a batch of data $\tilde{D}$ sampled from D

a batch of indices $\tilde{\Xi}$ sampled from P_ξ

- **Update** the main parameters θ in each step according to Equation (4), and updates the target parameters θ^- periodically with less frequency. $\Rightarrow$ **Bounded per-step computation**.

HyperAgent for Contextual Bandits with GPT involved



- Online learning for **content moderation** with **GPT** involved can be formulated as a **contextual bandit** problem.
- **HyperAgent** can be applied to the problem with **GPT** as the base model.
- Remove the **target hypermodel** in the **HyperAgent** framework for contextual bandit.

Outline

Theoretical Understanding via Unified Representation and Algorithm

- **Tabular** representation: $\phi_w(s)$ is fixed one-hot vector in $\mathbb{R}^{|\mathcal{S}|}$ where $d = |S|$. (**Unification!**)
- **Tabular** `HyperAgent`: short notations

$$
\begin{aligned}
f_\theta(s, a, \xi) &= \langle \phi_w(s), A^{(a)}\xi + b^{(a)} \rangle \\
&= \langle \underbrace{(A^{(a)})^\top \phi_w(s)}_{\tilde{m}_{sa}} + \underbrace{(b^{(a)})^\top \phi_w(s)}_{\mu_{sa}}, \xi \rangle
\end{aligned}
$$

- Parameters in k-th episode $\theta_k = (\mu_{k,sa}, \tilde{m}_{k,sa} \in \mathbb{R}^M, \forall (s,a) \in \mathcal{S} \times \mathcal{A})$.
- $\phi_w(s)$ **fixed mapping**, e.g. tabular and linear FA.
- $\Rightarrow$ Equation (3) of **HyperAgent permits closed-form solution.**
  - HyperDQN [LLZ+22] & ENN-DQN[OWA+23b] can **not** derive closed-form solution.

Insights from closed-form solution

- **Incremental update** with computation complexity $O(M)$:

$$\tilde{m}_{k,sa} = \frac{(N_{k-1,sa}+\beta)\,\tilde{m}_{k-1,sa} + \sum_{t\in E_{k-1,sa}} \sigma\,\mathbf{z}_{\ell,t+1}}{(N_{k,sa}+\beta)} \in \mathbb{R}^M \tag{5}$$

Perturbation random vector

Visitation counts of (s,a) up to episode k

Set of timesteps encoutering (s,a) in episode $k-1$

Lemma 1 (Sequential posterior approximation via incremental update).

For $\tilde{m}_k$ recursively defined in Equation (5) with $\mathbf{z} \sim \mathcal{U}(\mathbb{S}^{M-1})$. For any $k \geqslant 1$, define the good event of ε-approximation

$$\mathcal{G}_{k,sa}(\varepsilon) := \left\{ \| \tilde{m}_{k,sa} \|^2 \in \left((1-\varepsilon)\frac{\sigma^2}{N_{k,sa}+\beta}, (1+\varepsilon)\frac{\sigma^2}{N_{k,sa}+\beta} \right) \right\}.$$

*The **joint event** $\cap_{(s,a)\in\mathcal{S}\times\mathcal{A}} \cap_{k=1}^{K} \mathcal{G}_{k,sa}(\varepsilon)$ holds w.p. at least $1-\delta$ if $M \simeq \varepsilon^{-2}\log(SAHK/\delta)$.*

Insights from closed-form solution

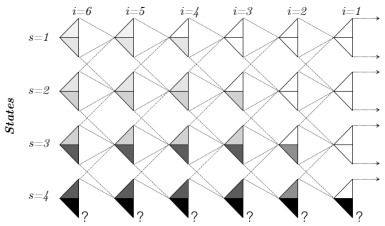
Stochastic Bellman Operator F_k^γ induced by Equation (3) w. $\theta = \theta_k^{(i+1)}, \theta^- = \theta_k^{(i)}$ **iteratively**:

$$f_{\theta_k^{(i+1)}, \boldsymbol{\xi}_k} = F_k^\gamma f_{\theta_k^{(i)}, \boldsymbol{\xi}_k} \approx (r_{sa} + \gamma \langle V_{f_{\theta_k^{(i)}, \boldsymbol{\xi}_k}}, \hat{P}_{k,sa} \rangle) + \tilde{m}_{k,sa}^\top \boldsymbol{\xi}_k(s), \tag{6}$$

Empirical transition

"Randomized bonus" $\propto \sqrt{\frac{1}{N_{k,sa}}}$

where $f_{\theta, \xi^-}(s,a) = f_\theta(s,a,\xi^-(s))$ and $V_Q(s) := \max_a Q(s,a), \forall s$ is the greedy value w.r.t. Q.



Setup: $N_{k,(4,\searrow)} = 1$. Other (s,a) almost **infinite data**.

(1) Propagation of uncertainty from later time period to earlier time period due to **iterative applying** F_k^γ.

(2) Darker shade indicates **higher** degree of uncertainty.

(3) Incentivize deep exploration.

Outline

Step 1: Rewrite incremental update on $\tilde{m}_{k,sa}$

- E_ℓ: the collection of time steps in episode ℓ.
- $E_{\ell,sa}$: the collection of time steps in episode ℓ encountering state-action pair (s,a).
- Define a sequence of indicator variables $x_{\ell,t} = \mathbb{1}_{t \in E_{\ell,sa}}$. Note

$$\sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t}^2 = N_{k,sa}$$

- Define short notations $\mathbf{z}_0 = \mathbf{z}_{0,sa}$ and $x_0 = \sqrt{\beta}$. Let $\beta = \sigma^2/\sigma_0^2$. Equation (5) now becomes

$$\frac{(N_{k,sa} + \beta)}{\sigma} \tilde{m}_{k,sa} = x_0 \mathbf{z}_0 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t} \mathbf{z}_{\ell,t+1} \tag{7}$$

- **Lemma 1 $\Rightarrow$ w.h.p. Equation (8) holds for all $(s,a) \in \mathcal{S} \times \mathcal{A}$ and $k \in [K]$ simultaneously**:

$$(1-\varepsilon)(x_0^2 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{t,\ell}^2) \leqslant \|x_0 \mathbf{z}_0 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t} \mathbf{z}_{\ell,t+1}\|^2 \leqslant (1+\varepsilon)(x_0^2 + \sum_{\ell=1}^{k-1} \sum_{t \in E_\ell} x_{\ell,t}^2) \tag{8}$$

Classical JL for random projection

- Try to relate Equation (8) to the classical **Johnson–Lindenstrauss (JL)** lemma:

$$\text{Consider } \Pi = (\mathbf{z}_1, \ldots, \mathbf{z}_d) \in \mathbb{R}^{M \times d}, \quad x = (x_1, \ldots, x_d)^\top \in \mathbb{R}^d, \quad \text{then } \Pi x = \sum_{i=1}^{d} x_i \mathbf{z}_i$$

Lemma 2 (Distributional JL lemma [JL84]).

For any $0 < \varepsilon, \delta \leqslant 1/2$ and $d \geqslant 1$ there exists a distribution $\mathcal{D}_{\varepsilon,\delta}$ on $\mathbb{R}^{M \times d}$ for $M = O\left(\varepsilon^{-2} \log(1/\delta)\right)$ such that for any $x \in \mathbb{R}^d$

$$\mathbb{P}_{\Pi \sim \mathcal{D}_{\varepsilon,\delta}} \left(\|\Pi x\|_2^2 \notin \left[(1-\varepsilon)\|x\|_2^2, (1+\varepsilon)\|x\|_2^2 \right] \right) < \delta$$

- **Existing JL analysis** based on the **assumption**: x fixed non-random or the projection matrix Π is generated **independently** with the data x, i.e.

$$\Pi := (\mathbf{z}_1, \ldots, \mathbf{z}_d) \perp\!\!\!\perp x := (x_1, \ldots, x_d).$$

Step 2: Indentify dependence structure

Sequential dependence structure in `HyperAgent` when interacting with environment is that

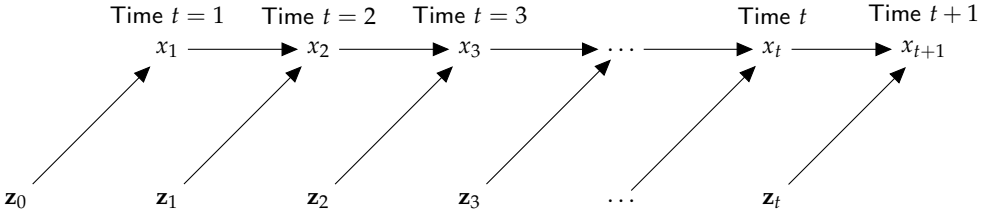
- E_ℓ: the collection of time steps in episode ℓ.
- $E_{\ell,sa}$: the collection of time steps in episode ℓ
- $x_{\ell,t} = \mathbb{1}_{t \in E_{\ell,sa}}$ is **dependent** on the environmental and algorithmic randomness in all previous time steps:

$$\mathbf{z}_0, (x_{1,t'}, \mathbf{z}_{1,t'+1})_{t' \in E_1}, (x_{2,t'}, \mathbf{z}_{2,t'+1})_{t' \in E_2}, \ldots, (x_{\ell,t'}, \mathbf{z}_{\ell,t'+1})_{t' < t};$$

- $\mathbf{z}_{\ell,t+1}$ is **independent** of the environmental and algorithmic randomness in all previous time steps:

$$\mathbf{z}_0, (x_{1,t'}, \mathbf{z}_{1,t'+1})_{t' \in E_1}, (x_{2,t'}, \mathbf{z}_{2,t'+1})_{t' \in E_2} \ldots, (x_{\ell,t'+1}, \mathbf{z}_{\ell,t'+1})_{t' < t}, x_{\ell,t},$$

Difficulty and Novelty in the Mathemtical Analysis: No Prior Art



Sequential dependence of **high-dimensional** R.V. due to the adaptive nature of sequential decision-making.

Difficulty: **(1)** Conditioned on x_t, $(\mathbf{z}_s)_{s<t}$ **loss** their independence; **(2) No** characterization on $P_{(\mathbf{z}_s)_{s<t}|x_t}$.
$\Rightarrow$ **Traditional analysis of random projection cannot handle sequential dependence** [Li24a].

First probability tool for **sequential random projection**. [Li24a]

- A **non-trivial martingale extension** of the **Johnson–Lindenstrauss (JL)**.
- **Technical novelty**: a careful construction of **stopped process** with non-trivial application of **'method of mixtures'** in self-normalized martingale.

Theorem 1 (Sequential random projection in adaptive processes [Li24a]).

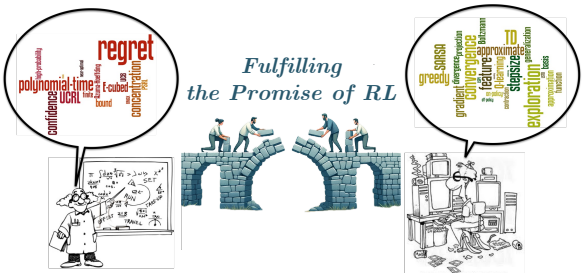
– Let $\varepsilon \in (0,1)$ be fixed and $(\mathcal{F}_t)_{t\geqslant 0}$ be a filtration. Let $\mathbf{z}_0 \in \mathbb{R}^M$ be an $\mathcal{F}_0$-measurable random vector satisfies $\mathbb{E}[\|\mathbf{z}_0\|^2] = 1$ and $|\|\mathbf{z}_0\|^2 - 1| \leqslant (\varepsilon/2)$.
– Let $(\mathbf{z}_t)_{t\geqslant 1} \subset \mathbb{R}^M$ be a stochastic process adapted to filtration $(\mathcal{F}_t)_{t\geqslant 1}$ such that it is $\sqrt{c_0/M}$-sub-Gaussian and each $\mathbf{z}_t$ is unit-norm. – Let $(x_t)_{t\geqslant 1} \subset \mathbb{R}$ be a stochastic process adapted to filtration $(\mathcal{F}_{t-1})_{t\geqslant 1}$ such that it is c_x-bounded. Here, c_0 and c_x are absolute constants.
– For any fixed $x_0 \in \mathbb{R}$, if the following condition is satisfied

$$M \geqslant \frac{16c_0(1+\varepsilon)}{\varepsilon^2}\left(\log\left(\frac{1}{\delta}\right) + \log\left(1 + \frac{c_x T}{x_0^2}\right)\right),$$

we have, with probability at least $1-\delta$

$$\forall t \in \{0,1,\ldots,T\}, \quad (1-\varepsilon)(\sum_{i=0}^{t} x_i^2) \leqslant \|\sum_{i=0}^{t} x_i \mathbf{z}_i\|^2 \leqslant (1+\varepsilon)(\sum_{i=0}^{t} x_i^2).$$

Simple, Efficient, Scalable: Bridging Theory and Practice



HyperAgent

- **Simple, Efficient** and **Scalable**;
- **Practically** useful for safety-critical decision-making in Human-AI interplay.
- Bridging theory and practice. **No prior art**.

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