Documention of Python-based ViSoS simulator for REFORMula Challenge Innovation Competition

Balázs NÉMETH

The goal of the Virtual Solar Sail (ViSoS) simulation is to model the effects of solar radiation pressure and gravitational forces that determine the motion of the ViSoS within the Solar System. The code enables the simulation and testing of control mechanisms, aiming to minimize the distance between the ViSoS and Earth.

This documentation is an automated translation of the Hungarian version. The English version will be updated to provide a higher level of technical language.

1. Preliminary requirements for running the simulation

The Python simulator was developed using Python version 3.9.6 and executed in the associated IDLE Shell 3.9.6 editor.[1] Within these parameters, the algorithm proved to be functional, and we recommend the use of these versions. During the previous REFORMula Challenge Innovation Competition, we observed instances where the same Python code produced different (in some cases significantly different) results when executed in different versions. For comparison, we provide several execution results in the documentation; see Chapter a 6.

The successful execution of the code requires the use of several Python extensions, which must be downloaded in advance. This can be achieved by copying the following command into the Windows Command Prompt (cmd.exe), running it with administrator privileges:

```
pip install numpy scipy matplotlib xlwings
```

2. Effects influencing the motion of the ViSoS

The following section presents the physical laws that were taken into account to determine the motion of the ViSoS. In alignment with the nature of the competition, simplifications were applied during the formulation of the equations of motion.

[1] Under Windows 10 operating system, utilizing Intel i7 10$^{\text{th}}$ Gen processor.

2.1. Newton's law of universal gravitation

The gravitational interaction between the celestial bodies and the ViSoS is described by the following equation:

$$\mathbf{a}_g = -\frac{GM}{r^3}\mathbf{r}, \tag{1}$$

where:

- G - gravitational constant ($6.67430 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$),
- M - mass of the celestial body (e.g., Sun, planets, Moon - the mass attraction of the ViSoS is negligible in comparison),
- $\mathbf{r}$ - position vector between the solar sail and the gravitating body, with its magnitude: $r = \|\mathbf{r}\|$.

The positions of the planets are calculated from the data points stored in the '.mat'files. These data are sourced from the NASA Horizons System, where the predicted motion data of celestial bodies are available.[2] The planetary motions are interpreted in an X-Y coordinate system with the origin at the Sun. The motion data of the celestial bodies are determined by linear interpolation at the specific time of the simulation:

$$\mathbf{r}_i(t) = \mathrm{interp1d}(t, \text{data points}), \tag{2}$$

where i is the index of the celestial body (Sun, Earth, Moon, Venus, Mars), and t is the current simulation time [s]. Using interpolation, the position (x, y coordinates) of each celestial body can be calculated at each time step of the simulation. The position vector relative to the solar sail is calculated as:

$$\mathbf{r}_i(t) = \mathbf{r}_{\text{ViSoS}}(t) - \mathbf{r}_i(t), \tag{3}$$

where $\mathbf{r}_{\text{ViSoS}}$ is the current position of the ViSoS at time t, as shown in Figure 1.

In the simulation, for each celestial body, we individually calculate the gravitational acceleration on the solar sail by the respective body, based on (1):

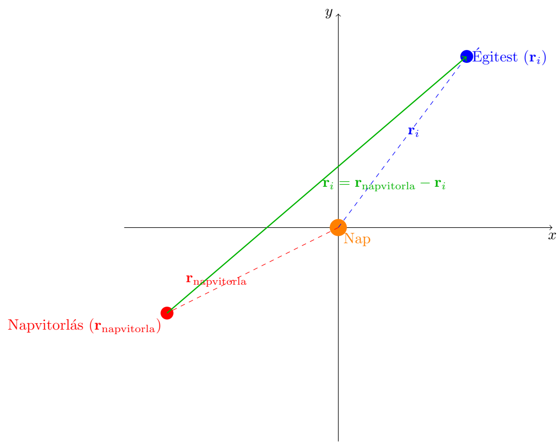
$$\mathbf{a}_i(t) = -\frac{GM_i}{r(t)^3}\mathbf{r}_i(t), \tag{4}$$

where i denotes the respective celestial body. The overall gravitational acceleration is obtained by summing the contributions as follows:

$$\mathbf{a}_{\text{gravity}} = \mathbf{a}_{\text{Sun}} + \mathbf{a}_{\text{Earth}} + \mathbf{a}_{\text{Moon}} + \mathbf{a}_{\text{Venus}} + \mathbf{a}_{\text{Mars}}. \tag{5}$$

The result is a single acceleration vector, which is used in the numerical simulation of the solar sail's motion.

[2] `https://ssd.jpl.nasa.gov/horizons/`



1. ábra. The geometric setup of the Sun, the celestial body, and the solar sail.

2.2. Effect of radiation pressure

The acceleration that the solar sail gains from the Sun's radiation pressure can be expressed by the following equation:

$$\mathbf{a}_r = \frac{PA\mathcal{O}\cos\alpha}{m}\mathbf{n}, \tag{6}$$

where:

- $\mathbf{a}_r$ - the acceleration acting on the solar sail, which originates from the Sun's radiation. The direction of the acceleration is determined by the normal vector ($\mathbf{n}$) of the solar sail's plane, and thus, the solar sail moves in the direction of the acceleration along this vector.

- P - the radiation pressure from the Sun, which characterizes the force exerted by sunlight on the solar sail. The magnitude of the radiation pressure ($4.563 \times 10^{-6}\,\mathrm{N/m^2}$) depends on the distance between the Sun and the ViSoS. In the context of the competition, it is assumed that the ViSoS does not significantly differ from Earth, so a constant value valid for Earth's environment is used.

- A - the area of the solar sail ($76\,\mathrm{m} \times 76\,\mathrm{m}$), which determines how much surface area is capable of reflecting the Sun's radiation. For the ViSoS considered in the competition, this is a fixed value – in reality, the larger the sail, the greater the force exerted on the solar sail.

- $\mathcal{O}$ - the degree of shading, see the detailed calculation of it in (12).

- m - the mass of the solar sail ($300\,\mathrm{kg}$), which determines how quickly the solar sail responds to the radiation pressure acting on it. A more massive (inertial) ViSoS responds more slowly, as acceleration is inversely proportional to mass.

- α - the control angle of the ViSoS, which represents the angle between the Sun's radiation and the normal vector of the ViSoS. The $\cos\alpha$ factor accounts for how much the ViSoS is actually aligned with the Sun's radiation. If $\alpha = 0$, the ViSoS is fully facing the Sun and can reflect the maximum amount of light.

- **n** - the normal vector of the ViSoS plane, which determines the direction in which the solar sail will move. The radiation pressure is always parallel to the normal vector of the ViSoS plane, so the ViSoS will move in the direction of the force along this vector.

The normal vector of the ViSoS ($\mathbf{n}$) is a unit vector perpendicular to the sail's plane, which is calculated based on the position relative to the Sun ($\mathbf{r}$) and the solar sail's control angle (α). The normal vector can be computed as follows:

1. The position vector between the Sun and the ViSoS is given by (3): $\mathbf{r} = \begin{bmatrix} r_x & r_y \end{bmatrix}^T$, where r_x and r_y are the current x and y coordinates of the solar sail (since the Sun is at the center of the coordinate system).

2. The plane of the ViSoS (and hence the normal vector) is rotated relative to the Sun by an angle α. The rotation is performed using a two-dimensional rotation matrix:

$$\mathbf{R}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}. \tag{7}$$

   The rotated vector, using the direction of the Sun's radiation ($\mathbf{r}$), is: A Nap sugárzásának irányából ($\mathbf{r}$) kiindulva az elforgatott vektor:

$$\mathbf{R}(\alpha) \cdot \mathbf{r} = \begin{bmatrix} \cos\alpha \cdot r_x - \sin\alpha \cdot r_y \\ \sin\alpha \cdot r_x + \cos\alpha \cdot r_y \end{bmatrix} \tag{8}$$

3. After the rotation, the $\mathbf{n}$ normal vector is normalized to ensure the stability of the calculations, so that it remains a unit vector. The reason for this is that the Sun-ViSoS distance is enormous, and the code does not need to work with such large vectors, while also making the direction more interpretable for the participants. The actual normal vector is:

$$\mathbf{n} = \begin{bmatrix} \cos\alpha \cdot r_x - \sin\alpha \cdot r_y \\ \sin\alpha \cdot r_x + \cos\alpha \cdot r_y \end{bmatrix} \cdot \frac{1}{r}. \tag{9}$$

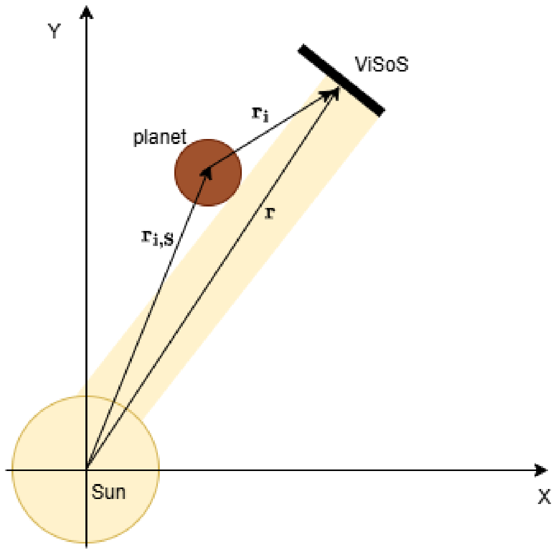
2.3. Calculating occlusion

When a celestial body is positioned between the Sun and the solar sail, it can partially block the radiation coming from the Sun. The degree of occlusion depends on the geometric arrangement (the relative positions of the Sun, the celestial body, and the solar sail) and the size of the celestial body, as illustrated in Figure 2. To account for the effect of occlusion, the following principles and calculations are applied.

In modeling occlusion, the following distances are determined: r: the distance between the Sun and the ViSoS, r_i: the distance between the i-th celestial body and the ViSoS, $r_{i,S}$: the distance between the Sun and the celestial body. It is assumed that the Sun, the celestial body, and the solar sail form a triangle, where the angles and distances determine whether the celestial body partially or completely occludes the Sun.

The degree of the Sun's occlusion depends on the relative size of the planet and the Sun, as well as the distance between the planet and the solar sail. The area factor expressing this is:

$$T_i = \max\left(0, \min\left(1, \frac{P_i^2}{R^2 \left(\frac{r_i}{r}\right)^2}\right)\right), \tag{10}$$



2. ábra. Geometric representation of the Sun and celestial body's occlusion.

where P_i is the radius of the celestial body, and R is the radius of the Sun.

Taking into account the angles of the triangle, the degree of occlusion by the celestial body is determined by the angles:

- $\beta = \arctan\left(\frac{P_i}{r_i}\right)$: the apparent angle of the celestial body,
- $\gamma = \beta + \arccos\left(\frac{r^2+r_i^2-r_{i,S}^2}{2rr_i}\right)$: the angle subtended by the celestial body, the Sun, and the solar sail,
- $\delta = \arctan\left(\frac{R}{r}\right)$: the apparent angle of the Sun.

The angle ratio can be calculated as follows (for simplicity):

$$\omega_i = \begin{cases} 0 & \text{ha } \gamma > \delta + 2\beta, \\ 1 & \text{ha } \gamma < \delta, \\ \text{interp}(\gamma, [\delta, \delta + 2\beta], [1, 0]) & \text{otherwise.} \end{cases} \tag{11}$$

The total occlusion for the i-th celestial body is the product of the area factor and the angle ratio: $T_i\omega_i$.

For all celestial bodies, if more than one body occludes the Sun, the largest occlusion is taken into account.

$$\mathcal{O} = 1 - \max\left(T_i\omega_i\right). \tag{12}$$

3. Equations of the motion

The equations of motion of the solar sail are solved using numerical integration, as the accelerations are not constant over time (e.g., due to gravitational effects, changes in radiation pressure, and variations in control angles). For this, the Euler method is applied, which is a simple, first-order numerical solution technique.

Based on the kinematic equations, the change in the position and velocity of the body can be derived from the acceleration. The fundamental equations are:

$$\mathbf{v}(t+\Delta t) = \mathbf{v}(t) + \mathbf{a}(t)\cdot\Delta t, \tag{13a}$$

$$\mathbf{r}(t+\Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\cdot\Delta t + \frac{1}{2}\mathbf{a}(t)\cdot(\Delta t)^2, \tag{13b}$$

where Δt is the time step length [s]. In the Python code, the time step is 3600 s, which corresponds to 1 hour.

The Euler method assumes that the acceleration ($\mathbf{a}(t)$) is constant between time steps. Therefore, the acceleration is calculated only at the current time point (t), and the equations are solved using this value. This simplification provides accurate results for small time steps (Δt), when the change in acceleration is small. For celestial bodies in the solar system and the ViSoS, which experiences slow changes, 1 hour is a suitable choice.

The steps for the numerical solution in the simulation are as follows:

1. At each time step (t), the accelerations acting on the solar sail (gravitational and radiation pressure) are calculated:

$$\mathbf{a}_t = \mathbf{a}_{\text{gravity}} + \mathbf{a}_{\text{radiation}}. \tag{14}$$

2. The velocity of the solar sail is updated using the current acceleration:

$$\mathbf{v}(t+\Delta t) = \mathbf{v}(t) + \mathbf{a}(t)\cdot\Delta t. \tag{15}$$

3. The new position of the solar sail is calculated based on the kinematic equations:

$$\mathbf{r}(t+\Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\cdot\Delta t + \frac{1}{2}\mathbf{a}(t)\cdot(\Delta t)^2. \tag{16}$$

This method provides a simple yet effective solution for simulating the motion of the solar sail, especially when the changes in acceleration over time are slow.

4. Control and rotation

The trajectory of the solar sail is determined by the control angle (α), which must be calculated based on the angle between the ViSoS normal vector and the solar radiation. The control strategy aims for the optimal trajectory (reaching the minimum distance between the ViSoS and the Earth on May 30, 2025), but the motion planning must take into account the (simplified) rotational limitations of the ViSoS. The angle for the ViSoS should be provided in radians.

4.1. ViSoS beforgatása a Nap felé

The program always rotates the ViSoS towards the Sun, and this rotation angle (θ) depends on the position of the solar sail. The solar sail continuously follows the Sun, and thus the control must be planned relative to this rotation. The degree of rotation is given by the following equation:

$$\theta = \arctan\left(\frac{r_y}{r_x}\right), \tag{17}$$

where r_x and r_y are the x and y coordinates of the solar sail in the orbital plane.

The control angle (α) is always defined relative to this θ angle, and this value must be calculated by the teams during the simulation.

4.2. Constraints on rotation

The rotation limitations of the solar sail play an important role in the physical applicability. Two basic constraints are applied:

- During one simulation step (1 hour), a maximum of 1° rotation change can be achieved relative to the previous α value.
- Additionally, the control angle (α) cannot exceed the ±90° limit to prevent the solar sail from over-rotating to the maximum adjustable angle and facing away from the Sun.

In the Python code, we handle this as follows:

$$\alpha = \text{np.clip}(\text{inputszog}, \alpha - \text{smooth}, \alpha + \text{smooth}),$$

where the smooth variable represents the maximum angle deviation (1°), and:

$$\alpha = \text{np.clip}(\alpha, -\pi/2, \pi/2),$$

which guarantees that the control angle always remains between ±90°.

4.3. Overall rotation angle

The total rotation angle of the ViSoS, which is determined by the control angle and the current orientation of the solar sail, is calculated as follows:

$$\Theta = \theta + \alpha. \tag{18}$$

This total rotation angle determines the new direction of the ViSoS at each time step.

4.4. Remarks

In terms of ViSoS control, it is important to consider that noise with a small magnitude and normal distribution is introduced into the system at two points:

- First, we add this noise (a random number) to the α angle sent to the ViSoS, as calculated by the teams. The reason for this is that in reality, the ViSoS would not be able to implement the desired angle precisely.
- Second, when calculating the resulting $\mathbf{a_r}$ — in terms of its magnitude, as seen in (6) — we also consider a noise source with a normal distribution. The reason for this is that, in terms of light reflection, we have used an ideal model, but in reality, the actual force may differ due to factors like deformation of the sail surface caused by the flexibility of the lightweight structure.

The teams need to write the visos_control function at the beginning of the Python code, for which they can choose any solution until January 20. They can either work directly in the Python code (with the setting Pyt = 1) or use the control_excel.xlsx Excel sheet (with the setting Pyt = 0). [3] Based on the control algorithm submitted on January 20, we will create the motion profile for the period up to January 31, which will be integrated into the online simulator. The control algorithm itself will not be included directly in the online simulator (teams can input it after January 31), but only the motion characteristics (ViSoS trajectory, velocity, α, Θ, etc.) will be included.

5. Visualization

At the end of the simulation, the code generates several graphical plots that illustrate various aspects of the solar sail's movement. These plots assist in analyzing the solar sail's trajectory and in fine-tuning the α angle as a control signal.

The first figure illustrates the trajectory angle of the solar sail (Θ) and the control angle (α) as a function of time. The latter is the control signal that determines the solar sail's rotation towards the Sun, while the Θ angle represents the solar sail's total angular deviation along its path, as described by equation (18). This figure aids in the evaluation of control strategies. Determining the optimal α angle is crucial, as it directly affects the solar sail's acceleration due to radiation pressure and its trajectory.

The second figure shows the trajectories of the ViSoS $\mathbf{r}(t)$ and Earth $\mathbf{r}_{\text{Earth}}(t)$ in a two-dimensional space, on the x, y plane. The solar sail's path (blue line) demonstrates the route taken by the ViSoS during the simulation, while the Earth's path (green line) serves as a reference. This figure helps visually assess how closely the ViSoS approaches the Earth and whether the control strategy has successfully minimized the distance between the ViSoS and Earth.

The third figure shows the distance between the solar sail and Earth ($d(t)$) as a function of time:

$$d(t) = \|\mathbf{r}(t) - \mathbf{r}_{\text{Earth}}(t)\|.$$

This figure illustrates how effective the control strategy was in ensuring a trajectory that stays close to Earth. The goal is for the distance to be minimized by the end of the simulation (though this is not a requirement for the January submission, as the simulation will continue on the online platform until the end of the competition).

The fourth figure shows the solar sail and Earth trajectories in three-dimensional space. The addition of the time dimension helps to understand how the motion of the solar sail evolves over time. Based on the second figure, it might appear that the ViSoS is close to Earth; however, the 3D representation can reveal that, although their paths are near each other, they may not be at the same point at the same time.

The four graphical representations assist in determining the α angle, as analyzing the $\alpha(t)$ and $\theta(t)$ curves allows us to understand how the control strategy influences the trajectory. Based on the evolution of the ViSoS trajectory and the distance to Earth, we can fine-tune the temporal changes in $\alpha(t)$. Additionally, the results provide feedback on which α angles lead to a smaller distance from Earth.

Using the graphs, the control strategy can be iteratively improved, allowing the solar sail to reach the Earth's orbit more efficiently. Naturally, additional graphs can be

[3] If written directly in Python, the simulation runs significantly faster.

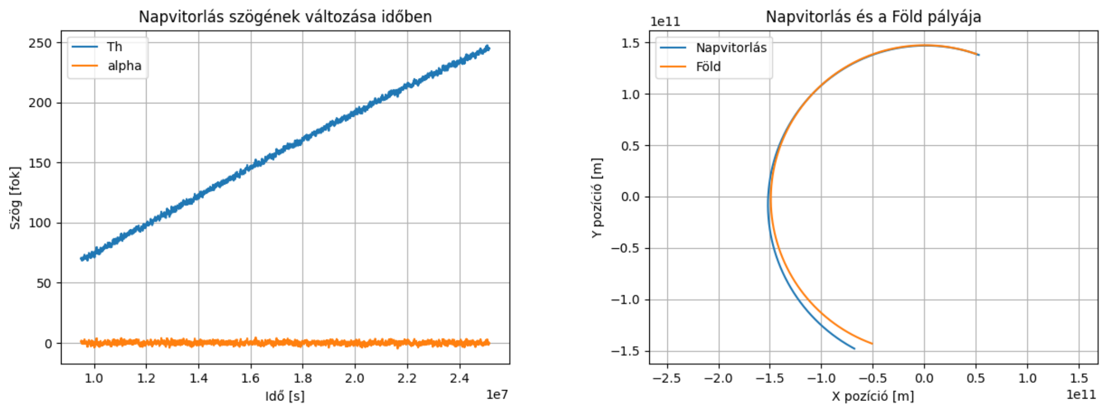
freely created by the teams; the four presented above only show the most important characteristics.

Additionally, the Python code generates a fifth graph, which is essentially the same as the second one. The only difference is that this one is an animation, where we can observe the Earth and the ViSoS in accelerated motion.

6. Exemplary scenarios

Here are two examples that can help verify the results of the simulation running on your own machine. The simulations were run with the selection Pyt = 1.

In the first simulation example, we gave the ViSoS an $\alpha = 0°$ control signal, meaning that it faces the Sun with its entire surface. This represents the realization of the maximum available thrust, and the solar sail begins to move away from the Sun with the highest possible acceleration. Figure 3 illustrates this case: in this scenario, the ViSoS continuously turns along its trajectory and moves away from the Sun (and also from the Earth, heading outward).



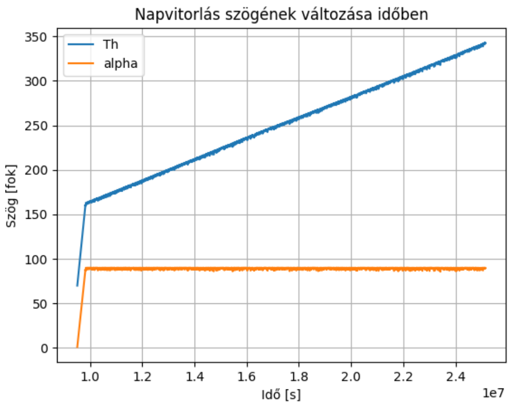
(a) ViSoS angles.

(b) ViSoS orbit.

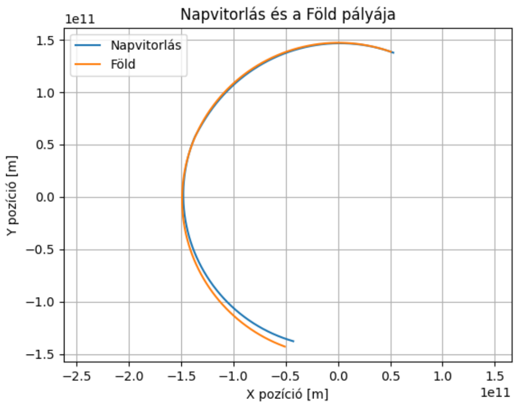
3. ábra. ViSoS motion at $\alpha = 0°$ control signal.

Figure 4 shows the case when we give the ViSoS a control signal of $\alpha = 90°$. Since the simulation always starts with $\alpha = 0°$, it takes 90 hours for the system to actually achieve this desired value. The $\alpha = 90°$ control signal means that the Sun is hitting the ViSoS from the side, which results in no thrust being generated after the adjustment. In this case, only gravitational forces move the solar sail (and to a very small extent, the thrust from solar radiation due to simulated noise). As a result, the ViSoS will move toward the Sun, meaning it will end up closer to the Earth by the end of the simulation.

During the development process of the simulator, we learned that the initial phase of the ViSoS trajectory is crucial in determining how it will ultimately finish. Therefore, during the early (offline) phase of the competition, it is important to fine-tune the ViSoS movement as much as possible to avoid being too far outside or too close to the Earth. If the trajectory starts too far from or too close to the Earth, it becomes difficult to correct the movement of the ViSoS at the end of its path.



(a) ViSoS angles.



(b) ViSoS orbit.

4. ábra. ViSoS motion at $\alpha = 90°$ control signal.