Bluetooth Indoor Positioning using RSSI and Least Square Estimation

Yapeng Wang[1], Shusheng Shi[2], Xu Yang[1] and Athen Ma[1]
MPI-QMUL Information Systems Research Centre
Macao Polytechnic Institute
Macao S.A.R., China
yapeng@mpi-qmul.org
International School
Beijing University of Post and Telecommunications
Beijing, China
ee06b362@buptis.cn

Abstract **– Bluetooth is a widely used short-distance wireless communication technology. Using Bluetooth as an indoor positioning technique can be very useful in many scenarios. This paper studied wireless positioning technology and proposed a positioning method using linear least square method to fit the relations between distance and signal strength by measuring RSSI (Received Signal Strength Indicator). We have tested our method inside a square room fitted with four access points and one mobile device. LSE (Least Square Estimation) is used to calculate the position of the mobile device. Results show that it is a practical method to locate Bluetooth devices.**

Index Terms – Wireless Indoor Positioning, RSSI, Bluetooth, Least Square Estimation.

I. INTRODUCTION

Bluetooth technology is a short distance wireless technology that is widely used. It provides low cost, short wireless communications, which can be used to construct fixed and mobile communication sets, making a variety of information devices within close range. Bluetooth technology works in the global generic 2.4 GHz ISM (Industrial, Scientific and Medical) Frequency band, thereby eliminating the cost of spectrum licenses and barriers of country borders. Bluetooth is widely used in various electronic devices such as laptop computers, mobile phones, cameras, printers etc.

Bluetooth technology supports point-to-point and point to multipoint communications. The basic network structure of Bluetooth is Piconet, which is a personal network that interconnects devices within a small region. Piconet is mainly structured by one master and several slave devices, they are using the same frequency hopping sequence in order to communicate with each other. Bluetooth technology is independent of the operation system and different communication protocols, and can be transplanted to many application areas. The popular applications include data, images, sounds and other short-distance communications situations, such as multi-function headset, data exchange (e.g. business cards and messages between PC, cellphones, PDAs), cable replacement (serial and printing cables).

Wireless positioning using Bluetooth has been studied in some paper. However, most of the study is just on theoretical and lack of real implementation. In this paper, we analysed the possible Bluetooth positioning solution and proposed a practical system backed by experiments.

II. WIRELESS INDOOR POSITIONING

Wireless positioning can be categorized and outdoor and indoor technologies. Outdoor positioning mainly uses satellite based technology such as the well known GPS (Global Positioning System) system [1]. However, satellite based positioning system need the tracking device to be able to receive line-of-sight signals from satellites so that is cannot be used indoors. Indoor wireless positioning systems were proposed using different techniques. Some popular methods includes:

A. Angle of arrival

Angle of arrival (AOA) [2, 3] is a geographic positioning method, also called the triangulation method. The received arrival orientation of signal determines the location of mobile devices. The receiver uses a directional antenna or antenna array to measure the received orientation of the target signal sender. Two such AOA measurements will be able to lock the target's location (shown in Fig. 1).

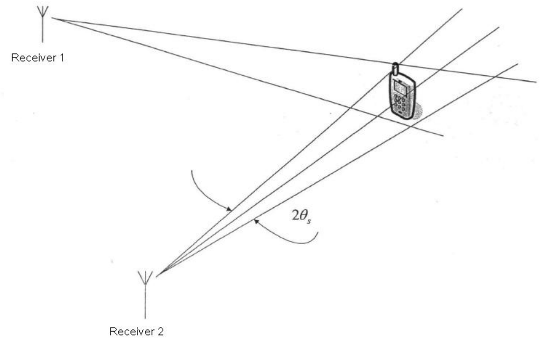

Fig. 1 AOA geographic positioning technology

The accuracy of position estimation depends on the sender relative to the receiver position. If the sender is precisely in a straight line between the two receivers, the AOA measurement would not lock the target's location. Thus, usually more than two receivers will provide to positioning accuracy. In indoor environment, because the signal path will be blocked by objects and walls all around, AOA technology is not applicable to indoor positioning system. In addition, AOA needs to put up expensive antenna array at receivers to track signal arrival orientation, in general, it is not a good solution for low-cost indoor applications.

B. Distanced based triangulation

The distance between mobile device and the receiver can be estimated by RSS (Received Signal Strength), TOA (time of arrival), received signal phases and other techniques. In order to estimate the location of mobile device in two-dimensional space, at least three measurements will be needed. Fig. 2 shows the three measurements of positioning.

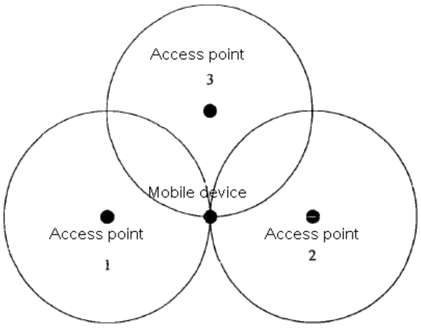

Fig. 2 Magnetization as a function of applied field.

If the distance estimation value between receiver and mobile device is d, mobile device can be positioned to a circle and the receiver as the centred, d as the radius of the circle. The second measure reduces the ambiguity of positioning, and the mobile device positioned in two crossing circular arc. The third measure locks the position of mobile device.

C. *Fingerprint based positioning*

Recently, the signal fingerprint technology is adopted as a new positioning technology. Because received signal is dependent to its terrain and transmission obstacles, showing very strong site specificity. Therefore for each location, the multipath channel structure of each location is unique, if the same RF signal is launched from that location, this multi-path characteristic can be considered as the fingerprint of the location or characteristic signature. Such a scheme is also applicable for indoor applications. However, it is not mature and complex, and we will put attention to it on further studies.

In this paper, it will use the distance positioning technology based on received signal strength for Bluetooth devices. The distance and location can be produce by the measurement value of RSS according to an algorithm describing the relation estimation between receiver and sender.

III. POSITIONING ALGORITHM

A. *Free space transmission*

According to observations, in most simple environments, wireless signal strength is decreased with the α power of distance. α is called the distance power gradient or path loss gradient. If the transmission power is *Pt* and the distance is d meters, the signal strength will be the proportion with $P_t d^{-\alpha}$. In the simplest case, Signal strength in free space is lost with the square of the distance (α = 2). When antenna transmits signal, the signal will be transmitted in all directions. In a sphere with radius d, the density of signal intensity equals to the total signal divided by the sphere area, i.e. $4\pi d^2$. Taking into account the electromagnetic wave frequency and additional losses, in free space the relation between transmission power Pt and received power Pr is shown below:

$$\frac{P_t}{P_r} = G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \tag{1}$$

Here G_t and G_r are transmitted antenna gain and received antenna gain from transmitter and receiver respectively; d is the distance between transmitter and receiver, the carrier wavelength is $\lambda = c / f$, where c is the speed of light in free space and f is carrier frequency. If let $P_0 = P_t G_t G_r \left(\frac{\lambda}{4\pi} \right)^2$ as the received signal strength at first meter (d=1m), the equation can be shown as:

$$P_r = \frac{P_0}{d^2} \tag{2}$$

The simplest method to describe the relations between the power and distance of received signal is to consider the received signal power P_r is proportional to the distance between receiver and transmitter with the power α, which is called distance power gradient:

$$P_r = P_0 d^{-\alpha} \tag{3}$$

In free space, $\alpha = 2$ so we get (2). In the simplified two way model of urban wireless signal channel $\alpha = 4$. In indoor situations, α can change in different situations. E.g. in the corridors and indoor open space α is less than 2, while in the metal building the value can reach to 6.

B. *Linear least square method*

Least square method [4, 5] is mathematical method which is widely used in data processing, error estimation and other studying field. Nowadays, Least square method become a very important tools when doing parameters estimation, data processing, regression analysis and experimental formula fitting, etc.

We know that the mean value of arithmetic is the most reliable value of true value, that is:

$$\bar{x} = A + \frac{1}{n} \sum_{i=1}^{n} \Delta x_i \tag{4}$$

Here A is the measured true value. So letting $\bar{x}$ replaces A must show the minimum error. Fo rthe normal distribution, the probability of error Δx_1 in the range of $d(\Delta x_1)$ will be:

$$P_1 = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp\left(-\frac{(\Delta x_1)^2}{2\sigma_1^2} \right) d(\Delta x_1) \tag{5}$$

This apply similarly the probabilities of error $\Delta x_2, \ldots \Delta x_n$. Because each measurement is the independent incident, the probabilities that each error $\Delta x_1, \ldots \Delta x_n$ will shows in the same time will be multiply of each probabilities:

$$P = P_1 P_2 \cdots P_n$$
$$= \frac{1}{\sigma_1 \cdots \sigma_n (\sqrt{2\pi})^n} \exp\left(-\frac{1}{2\sigma^2}\right)\left[(\Delta x_1)^2 + \cdots + (\Delta x_n)^2\right] d(\Delta x_1)\cdots d(\Delta x_n)$$

The minimum of errors means the maximum of probability P, i.e. minimum of $\sum_{i=1}^{n}(\Delta x_i / \sigma_i)^2$,deducing Bessel formula has shown, $\sum_{i=1}^{n} v_i^2 = \frac{n-1}{n}\sum_{i=1}^{n}(\Delta x_i)^2$, so $\sum_{i=1}^{n}(\Delta x_i / \sigma_i)^2$ will be minimum, and this is the basic meaning of Least Square Method. Introducing the weight parameter P, least square method can be expressed as:

$$\sum_{i=1}^{n} P_i v_i^2 = \min \tag{6}$$

In equal-precision measurement, $\sigma_1 = \sigma_2 = \cdots = \sigma_n$, $P_1 = P_2 = \cdots = P_n$, so the least square method can be shown as:

$$\sum_{i=1}^{n} v_i^2 = \min \tag{7}$$

The term of the least square method may be understood as, using x instead of the true value A obtains the error which was "minimal", and “square" means squared error.

In general, the least square method can be used for the processing of linear parameters, can also be used for the processing of nonlinear parameters. A large number of practical measure problems are linear, and nonlinear parameters by means of series expansion method can be approximated into a linear form in the region. Therefore, the linear least square method is the primary content. In this paper, we use the linear square method to estimate linear parameters.

C. Parameter estimation

Suppose that there are M arbitrary specifically functions of the linear combinations of x, for instance, the functions of x can be $1,\ x,\ x^2,\ x^M$, in this situation, their common linear combination is,

$$y(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_M x^M \tag{8}$$

If $x_1, x_2 \cdots x_t$ are known variables with number of t, $Y_1, Y_2 \cdots Y_n$ are direct measured values with number of n, where $n > t$, to get the unknown variables with number of t, the expression is:

$$\begin{cases} Y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1t}x_t \\ Y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2t}x_t \\ \cdots\cdots \\ Y_n = a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nt}x_t \end{cases} \tag{9}$$

As the number of equations is bigger than the number of unknowns, which is overdetermined equation, it cannot be solved by algebraic method. Clearly, to make full use of information provided of the n measurement results, it must be given an appropriate approach to overcome the above problems, where the least square method is basic method to solve these problems.

Suppose that the measured value of Y_i is l_i , and assume there is not existed system errors and rough errors, with random errors, the residual equation will be given:

$$\begin{cases} v_1 = l_1 - (a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1t}x_t) \\ v_2 = l_2 - (a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2t}x_t) \\ \cdots\cdots \\ v_n = l_n - (a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nt}x_t) \end{cases} \tag{10}$$

Letting $\sum_{i=1}^{n} v_1^2 = \sum_{i=1}^{n}\left[l_i - \sum_{j=1}^{t} a_{ij}x_j \right]^2 = \min$, if we solve it by matrix, getting derivative on both side. Here giving the matrix form of least square method, setting column vectors:

$$L = \begin{bmatrix} l_1 \\ l_2 \\ \cdots \\ l_n \end{bmatrix} \text{ X=} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \text{ V=} \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix} \text{ and n*t matrix A=} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nt} \end{bmatrix} \tag{11}$$

So the residual equation with the linear parameters will be:

$$\begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ \cdots \\ l_n \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nt} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \tag{12}$$

i.e. in equal-precision measurement, the matrix form of least square of $\vec{v}$ will be:

$$\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix} = \min \tag{13}$$

i.e.

$$V^T V = \min \tag{14}$$

Using the derivative of matrix and its properties, letting $\frac{\partial}{\partial x}(V^T V) = 0$, the matrix form of the normal equations will be given:

$$A^T A x = A^T y \tag{15}$$

Letting $A^T A = C$, we have $Cx = A^T y$, so the solution expression of the normal equations in matrix form will be:

$$x = C^{-1} A^T y \tag{16}$$

IV. Implementation

We use Java Bluetooth API called avetanaBluetooth to make the measurement of Bluetooth RSSI. It use the command which accessing the HCI layer and use HCI_read_RSSI of the Bluetooth specification. In this study, we use a notebook computer of type Thinkpad T61 (where the program runs) and a mobile phone with Bluetooth (Nokia N79).

The fitting solving of RSSI and distance relations is developed by least square method. To fit the relations it needs to get different values of RSSI in different distance, and fitting these two groups by using least square method to estimate the parameters. Formula (8) is involved. In practice, considering the precision and result of the calculation, (8) can be simplified as:

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \tag{17}$$

In this case, letting y corresponds to the practice measured values of distances, x corresponds to the measured value of RSSI, depending different values of x and y, established equations groups and solving by least square method, to get the fitting values of parameters a(i).

We have setup a test environment of a square room of 6*8 metres. Three sides of the structure are walls and on side is a glass window. The laptop computer is the base station and the phone is the mobile device. For different distance between base station and mobile device, we measure several different values of RSSI. Letting 1 metre as the interval of the distance increasing, in each distance, from 1 to 10 metre, we measure 10 times of RSSI and getting average of these as the final measured value. Letting the distance corresponds to y, and the average RSSI reading corresponds to x, according to (17), we can solve it by least square method.

Suppose N access points, their coordinates: $P_k = (x_k, y_k), k \in 1, \cdots, N$. r_i is the distance between the access point and mobile device. Here we have:

$$\begin{cases} r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ r_j = \sqrt{(x - x_j)^2 + (y - y_j)^2} \end{cases} \tag{18}$$

Obtain by the above formula,

$$r_i^2 - r_j^2 = (x - x_i)^2 + (y - y_i)^2 - (x - x_j)^2 - (y - y_j)^2 \tag{19}$$

When $\tilde{r}_i$ and $\tilde{r}_j$ are measured values, so the critical conditions posed by these values and the coordinates are, $\tilde{r}_i^2 - \tilde{r}_j^2 = (x - x_i)^2 + (y - y_i)^2 - (x - x_j)^2 - (y - y_j)^2$, then we have:

$$2(x_j - x_i)x + 2(y_j - y_i)y = \tilde{r}_i^2 - \tilde{r}_j^2 + x_j^2 - x_i^2 + y_j^2 - y_i^2 \tag{20}$$

Suppose that $m = (x, y)^T$, then we have,

$$H \cdot m = C \tag{21}$$

Here,

$$H = \begin{bmatrix} h_{x(2,1)} & h_{y(2,1)} \\ \cdots & \cdots \\ h_{x(N,1)} & h_{y(N,1)} \\ \cdots & \cdots \\ h_{x(N,N-1)} & h_{y(N,N-1)} \end{bmatrix} \quad C = \begin{bmatrix} C_{2,1} \\ \cdots \\ C_{N,1} \\ \cdots \\ C_{N,N-1} \end{bmatrix} \quad m = \begin{bmatrix} x \\ y \end{bmatrix} \tag{22}$$

And

$$\begin{aligned} h_{x(i,j)} &= 2(x_j - x_i) \\ h_{y(i,j)} &= 2(y_j - y_i) \\ c_{i,j} &= \tilde{r}_i^2 - \tilde{r}_j^2 + x_j^2 - x_i^2 + y_j^2 - y_i^2 \end{aligned} \tag{23}$$

Therefore, the coordinates of the mobile deice can be obtained by the following formula:

$$m = \left(H^T H\right)^{-1} H^T C \tag{24}$$

We set 4 access points and 1 mobile device in the room, provide the coordinates of each access point's location shown on Fig. 3.

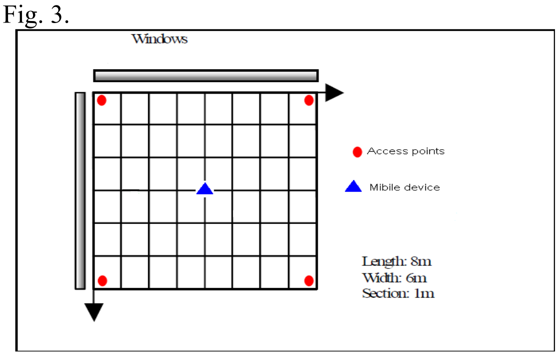

Fig. 3 Structure of indoor positioning experiments

After calculating measured RSSI values, we can compare the estimated distance vs. the real distance. As shown in Fig. 4

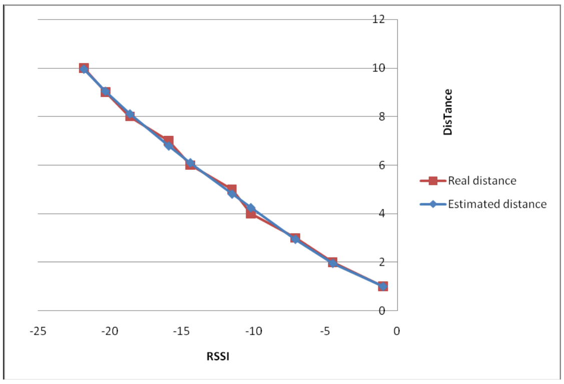

Fig. 4 Real distance VS estimated distance

The result shows they are very close to each other. However, it is noticed that the real distance is based on the line of sight signal between base station and mobile, i.e. there is no obstacles between them.

Further using LSE algorithm mentioned (18) to (24), we can further calculate the mobile coordinates. E.g. in Fig. 5 in one estimation, the real coordinates (mobile carrier) are located at (3, 4). From this figure above, we can get the estimated coordinates are (2.78, 4.21) in meter. Comparing to real coordinates (3.00, 4.00), the eccentric distance, namely the distance between (2.78, 4.21) and (3.00, 4.00), is 0.3041 meter, so the positioning precision is allowed in centimetre degree, which is basically followed the positioning requirement.

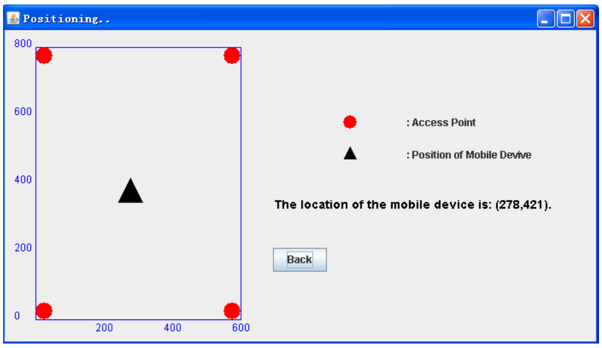

Fig. 5 Positioning result implemented by JAVA

Error analyses

We have tested the robustness of the algorithm by introducing obstacles between them. In practice, we use human body. And also move the mobile devices in different places. Then we do positioning to get different value of estimated locations. Calculate the eccentric distance of each point. The we got the error distribution of eccentric distance in this indoor environment.

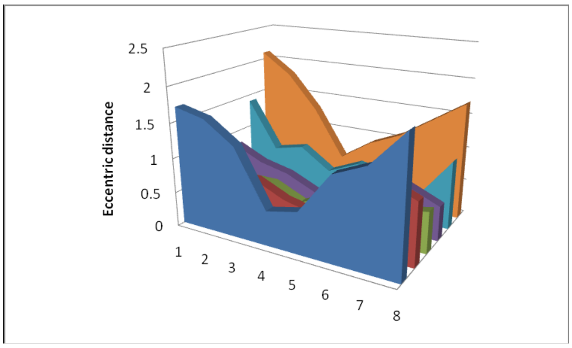

Fig. 6 The eccentric distance of each location.

From the figure, we can see the biggest eccentric distance is 2.1m, and the smallest eccentric distance is 0.3m. The error becomes distinctly bigger when approaching the corners in the indoor structure. And the error around windows is bigger than the opposite side. The reason of it may be:

(1) The mobile device carrier influences the result. When approaching to access points, the error is bigger. So human body will affect the estimated value.

(2) The special structure of the corners, existing more powerful signal reflection and refraction. And the special of window also affect the estimated value, according to this figure, the value near to window is bigger than the opposite side.

V CONCLUSION

In this paper, we have studied the Bluetooth positioning and proposed a practical positioning technique using linear least square method. A Java program was produce to measure the RSSI and then calculate the estimated position of the mobile. The result show that using the proposed method, we can create a practical positioning system using Bluetooth technology.

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