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A functional programming language based on dependent type theory

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Thesis Registration Form

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Thesis Title: A functional programming language based on dependent type theory

Topic of the Thesis:
(Upon consulting with your supervisor, give a 150-300-word-long synopsis os your planned thesis.)

For the thesis, a functional programming language based on dependent type theory will be implemented in Haskell. The software will contain the definition of the language with a type checker and an interpreter.

The first part of the program is lexical and syntactic analysis. This will be implemented with the help of the monadic parser combinator library called Megaparsec.

The next part is semantic analysis, which uses type checking and type inference to check if a term is well-typed. Due to the dependent type system, terms can appear in types as well, thus type checking involves evaluation of certain parts of the program. The language can contain holes and implicit arguments, which can be inferred by the software.

If the type checking succeeds, then the program can be evaluated to its normal form.

The software will have a terminal user interface. The type checker reports any type errors and unfillable holes to the user. Simple functional programs will be written in the language to test the type checker and the interpreter.

Budapest, 2020.11.23.

Contents

Chapter 1

Introduction

This thesis specifies and implements a type checker and interpreter of a simple functional programming language with dependent types. It features row polymorphism and extensible records.

Chapter 2 intodoces the background concepts for the thesis.

Chapter 3 contains the usage and the specification of the language.

Chapter 4 covers the implementation of the type checker and the interpreter.

Chapter 2

Background

2.1 Functional programming

Functional programming is the idea of structuring software by composing and applying functions, where mutable state and side effects are isolated and kept track of. The functions are similar to mathematical functions, they take in values as parameters and return new values based on the given arguments. In contrast to imperative procedues, which are defined by sequences of statements with side effects, function definitions are expression trees of functions, operators, and values [3, 4].

Functions are treated just like any other values in functional languages, they can be given in function arguments, returned from functions, stored in data structures, and defined in any context [1, 3].

If one defines a function that takes other functions as arguments, it is called a *higher-order functions*. For example,

$$\mathit{twice}(f, x) := f(f(x))$$

is a higher order function, it takes a function and a value, returns a function applied to that value twice [3, 8]. Higher-order functions allow one to refactor functions with similar structures by having parts of the definition be parameters of the new function.

The ability for a function to return another function gives rise to the technique called *currying*, where instead of having function arguments be given in tuples of values, the function is instead defined to have a single argument then directly return another function that takes another argument and so on [2, 3, 8]. For example,

$$f(x)(y)$$

$$
\begin{aligned}
t, u ::= {} & x && \text{variable} \\
\mid {} & \lambda x.\, t && \text{lambda abstraction} \\
\mid {} & t\, u && \text{function application}
\end{aligned}
$$

Figure 2.1: The syntax of untyped lambda calculus

is a curried function applied to two arguments.

A desirable trait in functional programming is *referencial transparency*. It allows one to replace any variable with its definition or factor out parts of the expressions into a new variable without changing the semantics of the program [3, 4]. This property is lost if side effects are unrestricted in functions.

Imperative loops need mutable variables to function, so to avoid mutable state, recursion is used instead in functional programming [3]. Often higher-order combinators which use recursion under the hood are used instead of explicit recursion, such as maps, folds, and recursion schemes, using them one can be sure that a particular function terminates [5, 7]. With an optimizing compiler, recursive functions can be just as performant as imperative loops [1].

2.2 Lambda calculus

Lambda calculus is a model of computation based on mathematical functions, it is the basis of functional programming languages. The simplest untyped version only has three syntactic constructs (see fig. 2.1), variables, lambda abstraction, which binds variables and creates an anonymous function, and function application. It has a single rule for computation called *β-reduction*, the rule is as follows:

$$(\lambda x.\, t)\, u \mapsto t[x \coloneqq u]$$

Types

Dependent types

Proofs

Chapter 3

User documentation

3.1 Installation

1. Install Stack [9].
2. Clone the repository:

   ```
   git clone https://github.com/szumixie/unnamed.git
   ```

3. Enter the directory: `cd unnamed`.
4. Run `stack install --flag unnamed:release`.

3.2 Usage

An Unnamed program can be written in a file with any source code editor.

Comments in the language are similar to Haskell and its derived languages.

```
-- This is a line comment
{- This is a
   block comment -}
```

Types can be defined with their church encoding

```
Bool = ∀ {A} → A → A → A

true  : Bool = λ x y → x
false : Bool = λ x y → y
```

$$\frac{}{\Gamma,\ x : A \vdash x : A} \qquad \frac{\Gamma \vdash t : A \qquad \Gamma \vdash u : B[x \coloneqq t]}{\Gamma \vdash \mathsf{let}\{x : A = t;\ u\} : B}$$

$$\frac{}{\Gamma \vdash \mathsf{U} : \mathsf{U}} \qquad \frac{\Gamma \vdash A : \mathsf{U} \qquad \Gamma,\ x : A \vdash B : \mathsf{U}}{\Gamma \vdash \forall (x : A) \to B : \mathsf{U}}$$

$$\frac{\Gamma,\ x : A \vdash t : B}{\Gamma \vdash \lambda x \to t : \forall (x : A) \to B} \qquad \frac{\Gamma \vdash t : \forall (x : A) \to B \qquad \Gamma \vdash u : A}{\Gamma \vdash t\, u : B[x \coloneqq u]}$$

$$\frac{\Gamma \vdash A : \mathsf{U}}{\Gamma \vdash \mathsf{Row}\, A : \mathsf{U}} \qquad \frac{}{\Gamma \vdash \#\{\} : \mathsf{Row}\, A} \qquad \frac{\Gamma \vdash t : A \qquad \Gamma \vdash r : \mathsf{Row}\, A}{\Gamma \vdash \#\{l : t \mid r\} : \mathsf{Row}\, A}$$

$$\frac{\Gamma \vdash R : \mathsf{Row}\, \mathsf{U}}{\Gamma \vdash \mathsf{Rec}\, R : \mathsf{U}} \qquad \frac{}{\Gamma \vdash \mathsf{rec}\{\} : \mathsf{Rec}\, \#\{\}} \qquad \frac{\Gamma \vdash t : A \qquad \Gamma \vdash u : \mathsf{Rec}\, R}{\Gamma \vdash \mathsf{rec}\{l = t \mid u\} : \mathsf{Rec}\, \#\{l : A \mid R\}}$$

$$\frac{\Gamma \vdash t : \mathsf{Rec}\, \#\{l : A \mid R\}}{\Gamma \vdash t.l : A} \qquad \frac{\Gamma \vdash t : \mathsf{Rec}\, \#\{l : A \mid R\}}{\Gamma \vdash t.\!-\!l : \mathsf{Rec}\, R}$$

Figure 3.1: Typing rules

3.3 Lexical structure

The language is indentation sensitive, similarly to Haskell [6].

3.4 Syntax

$$::=$$

3.5 Type system

The typing rules of the language are presented in fig. 3.1.

3.6 Semantics

Figure 3.2

3.7 Errors

$$
\frac{}{x \Downarrow x}
\qquad
\frac{u[x \coloneqq t] \Downarrow u'}{\mathsf{let}\{x = t;\ u\} \Downarrow u'}
$$

$$
\frac{}{\mathsf{U} \Downarrow \mathsf{U}}
\qquad
\frac{A \Downarrow A' \qquad B \Downarrow B'}{\forall(x : A) \to B \Downarrow \forall(x : A') \to B'}
$$

$$
\frac{t \Downarrow t'}{\lambda x \to t \Downarrow \lambda x \to t'}
\qquad
\frac{t \Downarrow \lambda x \to v \qquad v[x \coloneqq u] \Downarrow v'}{t\,u \Downarrow v'}
\qquad
\frac{t \Downarrow n \qquad u \Downarrow u'}{t\,u \Downarrow n\,u'}
$$

$$
\frac{A \Downarrow A'}{\mathsf{Row}\,A \Downarrow \mathsf{Row}\,A'}
\qquad
\frac{}{\#\{\} \Downarrow \#\{\}}
\qquad
\frac{t \Downarrow t' \qquad r \Downarrow r'}{\#\{l : t \mid r\} \Downarrow \#\{l : t' \mid r'\}}
$$

$$
\frac{R \Downarrow R'}{\mathsf{Rec}\,R \Downarrow \mathsf{Rec}\,R'}
\qquad
\frac{}{\mathsf{rec}\{\} \Downarrow \mathsf{rec}\{\}}
\qquad
\frac{t \Downarrow t' \qquad u \Downarrow u'}{\mathsf{rec}\{l = t \mid u\} \Downarrow \mathsf{rec}\{l = t' \mid u'\}}
$$

$$
\frac{t \Downarrow \mathsf{rec}\{l = u \mid v\}}{t.l \Downarrow u}
\qquad
\frac{t \Downarrow \mathsf{rec}\{l' = u \mid v\} \qquad l \neq l' \qquad v.l \Downarrow w}{t.l \Downarrow w}
\qquad
\frac{t \Downarrow n}{t.l \Downarrow n.l}
$$

$$
\frac{t \Downarrow \mathsf{rec}\{l = u \mid v\}}{t.{-l} \Downarrow v}
\qquad
\frac{t \Downarrow \mathsf{rec}\{l' = u \mid v\} \qquad l \neq l' \qquad v.{-l} \Downarrow v'}{t.{-l} \Downarrow \mathsf{rec}\{l' = u \mid v'\}}
\qquad
\frac{t \Downarrow n}{t.{-l} \Downarrow n.{-l}}
$$

Figure 3.2: Big-step operational semantics

Chapter 4

Developer documentation

4.1 Project structure

The project is implemented in Haskell, using the Glasgow Haskell Compiler with many of its extensions enabled.

Figure 4.1

4.2 Raw Syntax

Listing 1

4.3 Core Syntax

Listing 2

4.4 Parsing

The Haskell library Megaparsec is used to do both lexing and parsing at the same time. Megaparsec is a monadic parser combinator library.

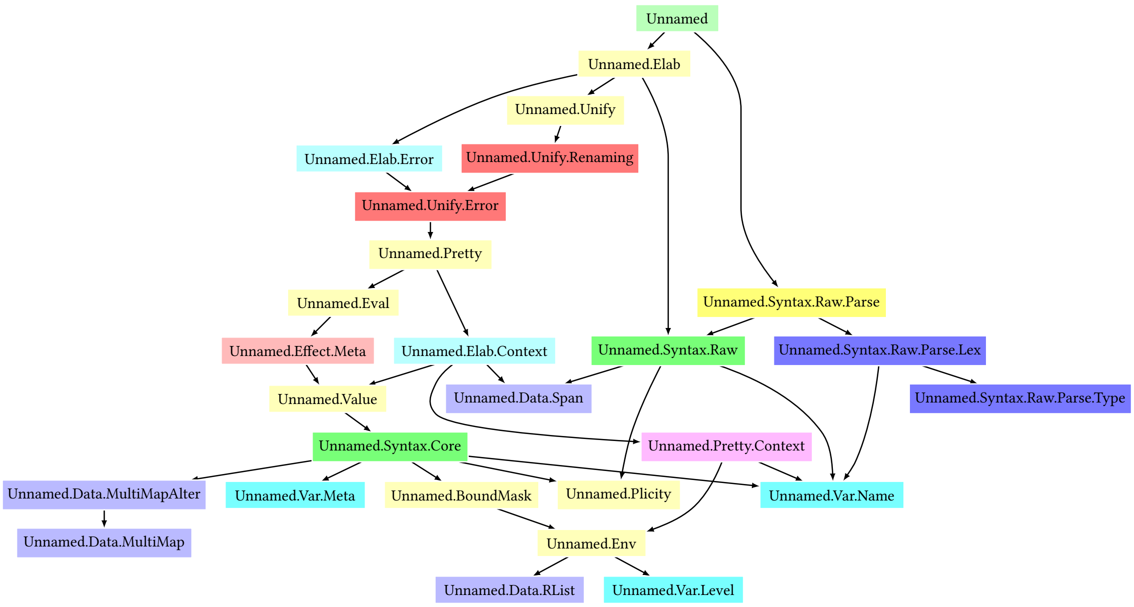

Figure 4.1: Transitively reduced dependency graph of the Haskell modules

```haskell
data Term
  = Span Span Term
  | Var Name
  | Hole
  | Let Name (Maybe Term) Term Term
  | U
  | Pi Name (Maybe Term) Term
  | Lam Name (Maybe Term) Term
  | App Term Term
  | RowType Term
  | RowEmpty
  | RowExt Name Term Term
  | RecordType Term
  | RecordEmpty
  | RecordExt Name (Maybe Term) Term Term
  | RecordProj Name Term
  | RecordRestr Name Term
```

Listing 1: Raw syntax ADT

```
data Term
  = Var Level
  | Meta Meta (Maybe BoundMask)
  | Let Name Term Term
  | U
  | Pi Name Term Term
  | Lam Name Term
  | App Term Term
  | RowType Term
  | RowLit (MultiMap Name Term)
  | RowExt (MultiMap Name Term) Term
  | RecordType Term
  | RecordLit (MultiMap Name Term)
  | RecordProj Name Int Term
  | RecordAlter (MultiMapAlter Name Term) Term
```

Listing 2: Core syntax ADT

4.5 Evaluation

4.6 Unification

4.7 Elaboration

4.8 Main

4.9 Testing

Chapter 5

Conclusion

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