11.12 Rank Correlation

Sometimes we may want to look at the relationship between two variables, but one or both of the variables are either ordinal or have a distribution that is far from normal. Significance tests based on the Pearson correlation coefficient will then no longer be valid, and nonparametric analogs to these tests are needed.

Example 11.53 **Obstetrics** The Apgar score was developed in 1952 as a measure of the physical condition of an infant at 1 and 5 minutes after birth [7]. The score is obtained by summing five components, each of which is rated as 0, 1, or 2 and represents different aspects of the condition of an infant at birth [8]. The method of scoring is displayed in Table 11.15. The score is routinely calculated for most newborn infants in U.S. hospitals. Suppose we are given the data in Table 11.16. We wish to relate the Apgar scores at 1 and 5 minutes and to assess the significance of this relationship. How should this be done?

Table 11.15 Method of Apgar scoring

	Score		
Sign	0	1	2
Heart rate	Absent	Slow (< 100)	≥ 100
Respiratory effort	Absent	Weak cry; hypoventilation	Good; strong cry
Muscle tone	Limp	Some flexion of extremities	Well flexed
Reflex irritability	No response	Some motion	Cry
Color	Blue; pale	Body pink; extremities blue	Completely pink

Source: Reprinted with permission of *JAMA*, *168*(15), 1985–1988, 1958.

Table 11.16 Apgar scores at 1 and 5 minutes for 24 newborns

Infant	Apgar score, 1 min	Apgar score, 5 min	Infant	Apgar score, 1 min	Apgar score, 5 min
1	10	10	13	6	9
2	3	6	14	8	10
3	8	9	15	9	10
4	9	10	16	9	10
5	8	9	17	9	10
6	9	10	18	9	9
7	8	9	19	8	10
8	8	9	20	9	9
9	8	9	21	3	3
10	8	9	22	9	9
11	7	9	23	7	10
			24	10	10

The ordinary correlation coefficient developed in Section 11.7 should not be used, because the significance of this measure can be assessed only if the distribution of each Apgar score is assumed to be normally distributed. Instead, a nonparametric analog to the correlation coefficient based on ranks is used.

Definition 11.22 The **Spearman rank-correlation coefficient** (r_s) is an ordinary correlation coefficient based on ranks.

Thus $$r_s = \frac{L_{xy}}{\sqrt{L_{xx} \times L_{yy}}}$$

where the L's are computed from the ranks rather than from the actual scores.

The rationale for this estimator is that if there were a perfect positive correlation between the two variables, then the ranks for each person on each variable would be the same and $r_s = 1$. The less perfect the correlation, the closer to zero r_s would be.

Example 11.54 **Obstetrics** Compute the Spearman rank-correlation coefficient for the Apgar-score data in Table 11.16.

Solution We use MINITAB to rank the Apgar 1-minute and 5-minute scores as shown in Table 11.17 under APGAR_1R and APGAR_5R, respectively. We then compute the correlation coefficient between APGAR_1R and APGAR_5R and obtain $r_s = .593$.

We would now like to test the rank correlation for statistical significance. A similar test to that given in Equation 11.20 for the Pearson correlation coefficient can be performed, as follows:

Equation 11.39 ***t* Test for Spearman Rank Correlation**

(1) Compute the test statistic

$$t_s = \frac{r_s\sqrt{n-2}}{\sqrt{1-r_s^2}}$$

which under the null hypothesis of no correlation follows a t distribution with $n-2$ degrees of freedom.

(2) For a two-sided level α test,

if $t_s > t_{n-2,1-\alpha/2}$ or $t_s < t_{n-2,\alpha/2} = -t_{n-2,1-\alpha/2}$

then reject H_0; otherwise, accept H_0.

(3) The exact p-value is given by

$p = 2 \times$ (area to the left of t_s under a t_{n-2} distribution) if $t_s < 0$

$p = 2 \times$ (area to the right of t_s under a t_{n-2} distribution) if $t_s \geq 0$

(4) This test is valid only if $n \geq 10$.