**Model components**

**Operating model**

The population dynamics model is a single-sex, age-structured model that tracks the number of individuals in an age class *a* by time *t*, *N*a,t, and allows natural mortality, *M*t to vary over time.

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Mean recruitment follows a Beverton-Holt stock-recruit relationship parameterized in terms of steepness (*h*; the expected proportion of virgin recruitment entering the population when spawning biomass is 20% of virgin levels), *R*0 (virgin recruitment) and *B*0 (virgin spawning biomass). Recruitment variation is log-normally distributed with mean 0 and standard deviation σr. Spawning biomass at time *t* (*Bt*) is a function of numbers-at-age (*Na,t*), proportion of mature individuals at a given age (), and weight-at-age (*Wa,t*).

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Annual fishing mortality, *F*t, is comprised of a separable annual selectivity-at-age curve (logistic):

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where *la* is the length at age *a*, and and determine the length at which the probabilities of capture are respectively 50% and 95% at time *t*. Selectivity parameters are specified in the operating model in terms of length (equation 6) and converted to age, conditional upon the length-at-age curve by rearranging equation 8 (which is conditional upon growth). Time variation in selectivity parameters in the operating model is specified by a vector input by the user. Maturity, , is also a logistic function of age and two estimated parameters ( and , which are the time-specific ages at which the probability of maturing is 50% and 95%, respectively):

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Length-at-age follows a growth increment von Bertalanffy form:

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where is the maximum length during time *t*, is the growth rate at time *t* and *a0,t* is the age corresponding to a predicted length of 0 at time *t* (i.e., a scaling parameter). Changes in and are specified as a vector in the operating model. Weight is a function of length:

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Conditional length-at-age for both the catch and survey are calculated from the numbers-at-age matrix by specifying a single standard deviation σlen and the expected length-at-age determined using equation 8. The array, *LAa,z,t* with *a* rows representing the number of age classes and *z* columns representing the number of length bins at time *t,* contains the probability of an individual of age *a* being length *z* (i.e. the proportion of each age class in each length bin). The number of length bins are pre-specified by the user.

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where *Binz* is a vector of the midpoints of the specified length bins. Numbers-at-length *z* are calculated from this matrix of (normalized) probabilities of length-at-age by multiplying each row by the number of individuals at age and then summing over rows (i.e. age).

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The weight of the catch *C* during time *t* is calculated as:

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The survey-selected biomass () at the time of the survey is calculated as:

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where *is* the year-specific survey selectivity-at-age, defined as:

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As with fishery selectivity, the parameters associated with survey selectivity (and , the length at which the probability of being selected in the fishery is 50% and 95%, respectively) are specified according to length and then transformed to age within the operating model conditional upon growth.

**Data simulation**

Catch biomass, catch length frequencies, fishery-independent survey indices of abundance and survey length frequencies are generated using the operating model with error to be used in the estimation models for each year in the simulation. Observed catch biomass and survey biomass are modeled as:

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where εtis a normally distributed random variable with a mean of 0 and a user-specified standard deviation σε. Observed length frequencies for both the catch and the survey are generated by randomly sampling from a uniform distribution of the true numbers at length calculated above for a specified number of times.

**Estimation models**

Production model

The production model specified follows the generalized three-parameter surplus production model specified by Pella and Tomlinson (1969), where biomass at time *t*, *Bt*, modelled as:

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where r refers to the intrinsic population growth rate, K is the population carrying capacity, and m is the shape parameter that informs the productivity of the stock.

Age-structured model

The equations governing the population dynamics within the age-structured estimation model match those of the operating model, with several of exceptions. The estimation model only has the capacity to allow the parameters related to the population processes of growth, fisheries selectivity, or natural mortality to vary over time. When one (or more) of these processes is allowed to vary over time in the estimation model, annual deviations from the mean are modeled as fixed-effects parameters with specified penalties similar to the way in which recruitment and fishing mortality are modeled (see ‘likelihood components’ below). For example, average recruitment (μR) within the assessment method is estimated with annual deviations (*Rdev,t*):

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In a similar manner, fishery selectivity at age () can be estimated within the assessment as deviations around a mean.

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**Likelihood components**

The assessment method is fit to the data generated from the operating model based on four likelihood components. The log-likelihoods (ignoring constants) for catch and the survey index of abundance are log-normal:

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Catch and survey length frequencies are fit under the assumption of multinomial sampling:

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where is the observed proportion at length of the catch at time *t*, is the predicted proportion at length in year *y* of the catch, is the observed proportion at length of the survey biomass in year *y*, and is the predicted proportion at length of the survey biomass in year *y*. The data are weighted with the same CVs and sample sizes with which they were generated (Table 1). Small penalties are added to the objective function to ensure the smoothness of estimated recruitment, fishing mortality, time-varying natural mortality, time-varying selectivity, and time-varying growth in the form of:

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where is the vector of the deviations associated with recruitment, fishing mortality, natural mortality, selectivity and/or growth, and γx is the weight applied to each likelihood component (specified as a CV of ~0.41 for all quantities because it produced estimates that were not overly variable, but still contributed little to the objective function).

**Harvest control rules**

Management targets are needed to parameterized harvest control rules to determine catches in the projections. Fitting stock recruit curves can be difficult to automate within simulations because some tuning is often required. Consequently, proxies for reference points are used instead where the biomass at which maximum sustainable yield occurs (*B*MSY) and the fishing mortality that produces that biomass at equilibrium (*F*MSY) were based on spawning-biomass-per-recruit methods (Clark, 1991; NPFMC (North Pacific Fishery Management Council) & NMFS (National Marine Fisheries Service), 2007). *F*35%, or the fishing mortality that reduces spawning biomass per recruit (SBPR) to 35% of virgin levels is used as a target fishing mortality for Alaskan fisheries. *B*35% is calculated as the SBPR corresponding to *F*35% multiplied by an average recruitment calculated from the entire time series of estimated recruitments. Calculated values of *F*35% and *B*35% are used in conjunction with a control rule to adjust the proportion of *F*35% that is applied to the population based on the status of the population relative to *B*35%. The fishing mortality derived from Eq. 25 is deemed the fishing mortality corresponding to the TAC (which coincides with the OFL), the *F*OFL, and is applied to the population to find the TAC using Eq. 13.

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| Where, | |  | |
|  | | The currently estimated mature biomass in the projected year for TAC determination | |
|  | | Mature biomass resulting from fishing at | |
|  | | Fishing mortality that reduces the mature biomass per recruit to 35% of the unfished level | |
|  | | Determines the slope of the descending limb of the control rule | |
|  | | Fraction of *B*35% below which fishing mortality is set to zero | |

Other simple harvest control rules are available, including a constant catch, a constant fishing mortality, and applying the ‘true’ *FMSY*, which is calculated by a grid search over fishing mortalities within the operating model (i.e., not estimated from the EM).

**References**

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