1.1

X.T*X = [[8 14 16 18 22 30]
[14 25 29 32 39 53]
[16 29 34 37 45 61]
[18 32 37 41 50 68]
[22 39 45 50 61 83]
[30 53 61 68 83 113]]

|X.T * X – theta * E| = 0

Eigenvalues:

Theta = [2.81035710e+02 2.61743490e-15 9.64290269e-01 -7.95206469e-15
-1.28284536e-15 2.56556957e-15]

1.2

X * X.T = [[112 137], [137 170]]

|X.T * X – theta * E| = 0

Eigenvalues:

Theta = [0.96429027 281.03570973]

eigenvectors:

[[-0.7768816 -0.62964671]
[0.62964671 -0.7768816]]

1.3

Almost the same. The first matrix's values have two identical elements with the second one.

When these two matrices are real symmetric matrices, they can be diagonalized, so the same eigenvalues must be similar.

1.4

Mean = [[4.], [5.]] D = X – mean

[[-2. -1. -1. 0. 1. 3.]
[-3. -1. 0. 0. 1. 3.]]

|X.T * X – theta * E| = 0

Eigenvalues:

Theta = [0.88275723, 35.11724277]

eigenvectors:

[[-0.74727547 -0.66451439]
[0.66451439 -0.74727547]]

Cd = [-0.66451439, -0.74727547]

Because 35.11724277 is much bigger than 0.88275723, I would choose correlative eigenvector [-0.6970804 -0.71699297].

Finally:

D = dot(cd, x)

[-2.82357972 -4.98264505 -5.72992052 -6.39443491 -7.80622477
-10.62980449]

2. 1:
We can’t solve this problem by log, because this is a sum which can’t be solved by log.
2.2:
[0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]
2.3:
theta = [0.5 0.6 0.6]
2.4:
[0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]
2.5:
theta = [0.5 0.6 0.6]
2.6:
They are different.
Theta = [0.40641711229946537 0.5368421052631579 0.6432432432432431]
2.7:
They are different.
[0.5 0.6 0.6]
[0.40641711229946537 0.536842105263158 0.6432432432432431]