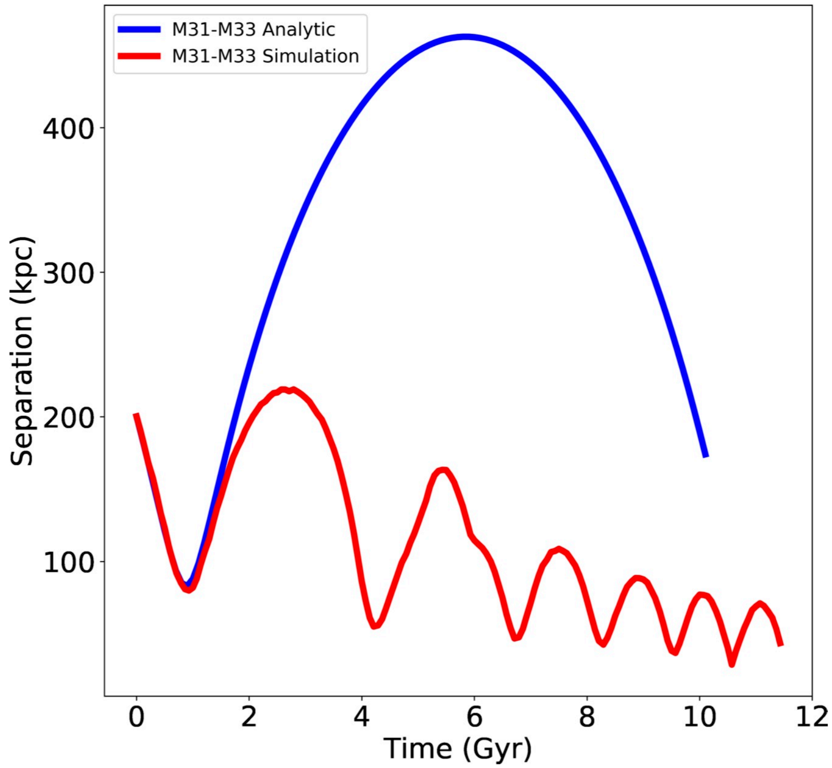
Dynamical Friction

The transfer of energy from the orbital motion of a satellite into random motions of the particles in the medium through which the satellite is moving (e.g. a dark matter halo or a stellar halo).

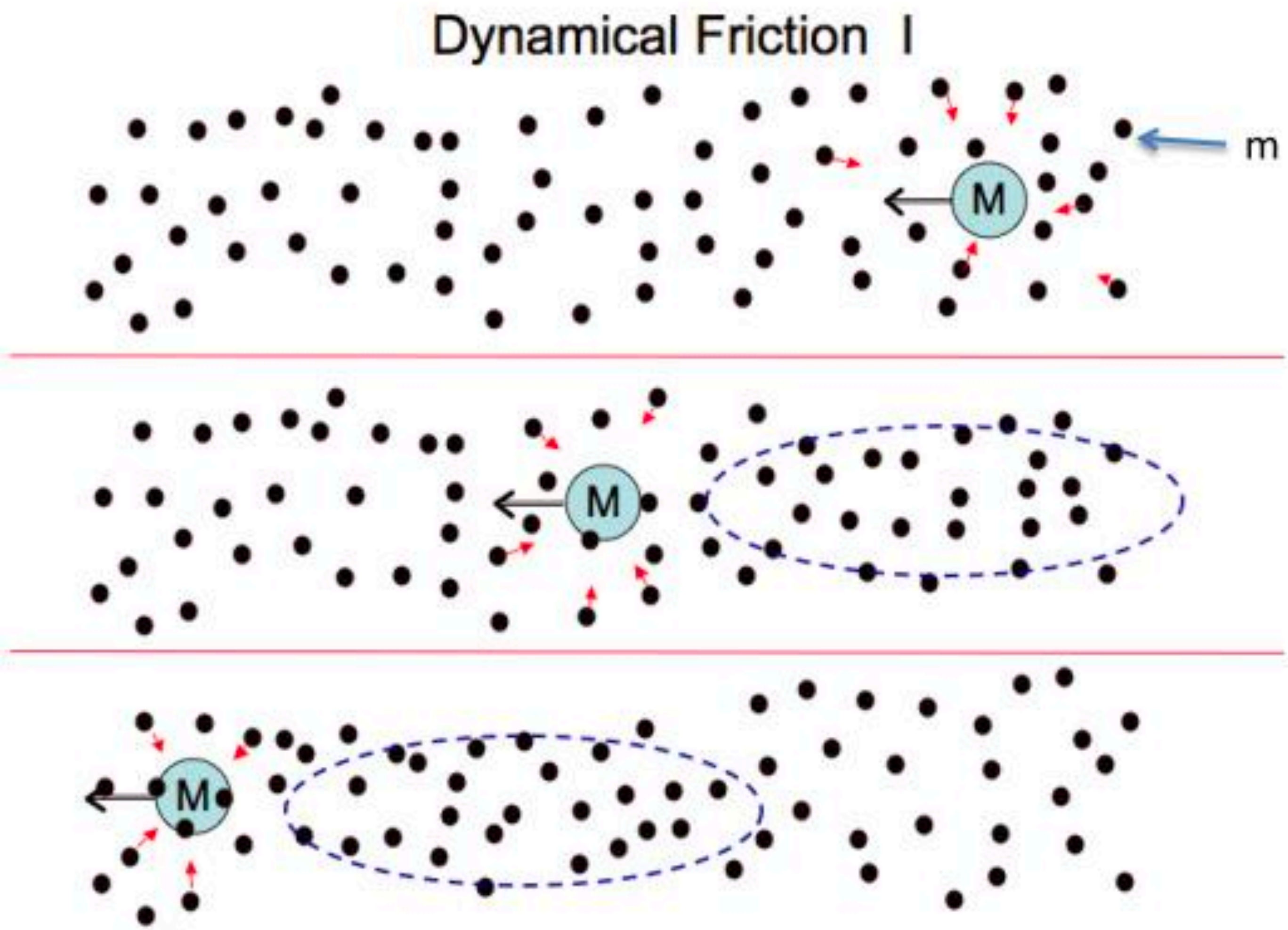
This causes the satellite's orbit to decay.



This Lab: We need to fix our orbit code!

Section 7 in Sparke & Gallagher

Dynamical Friction --- Wake

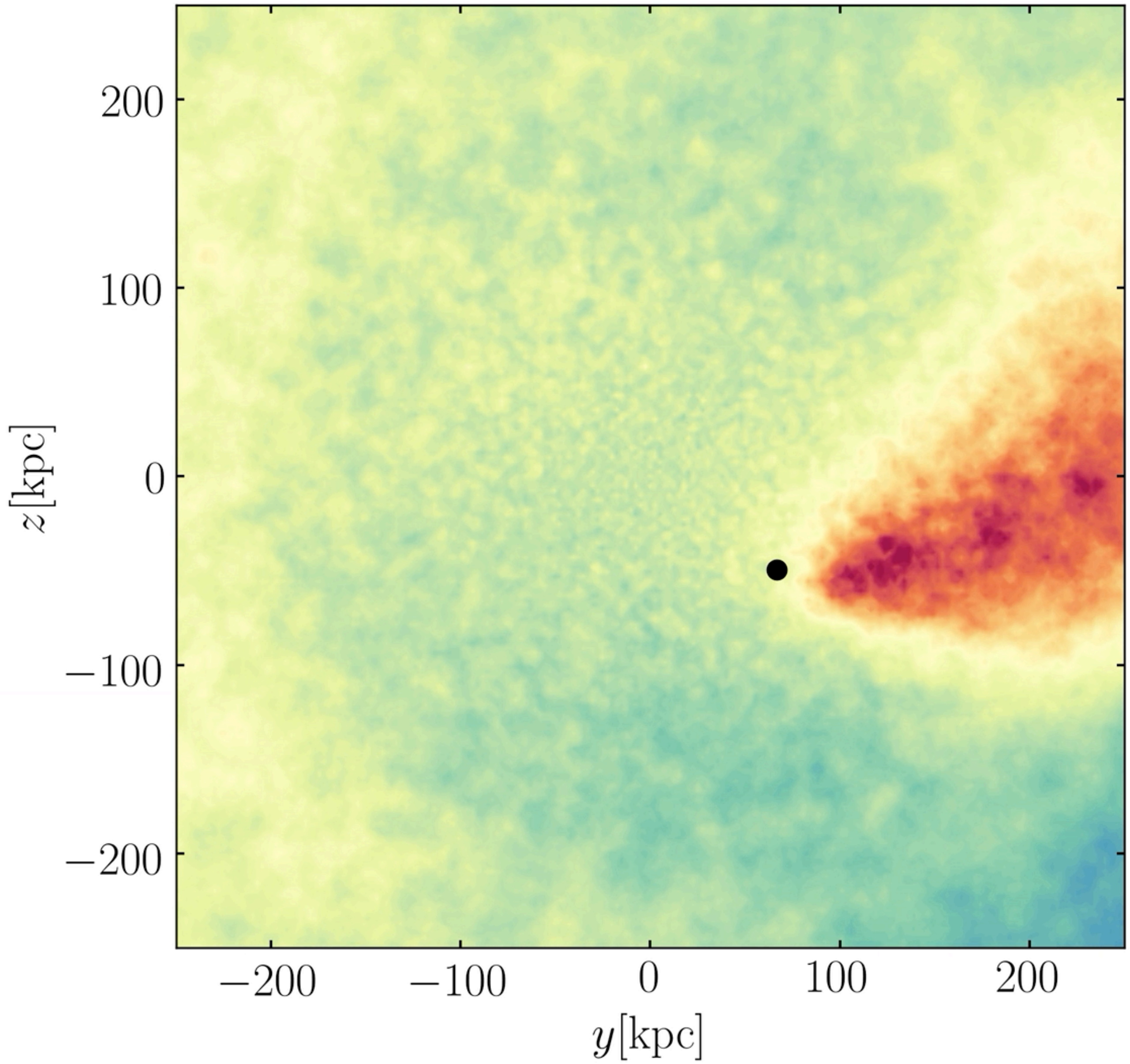


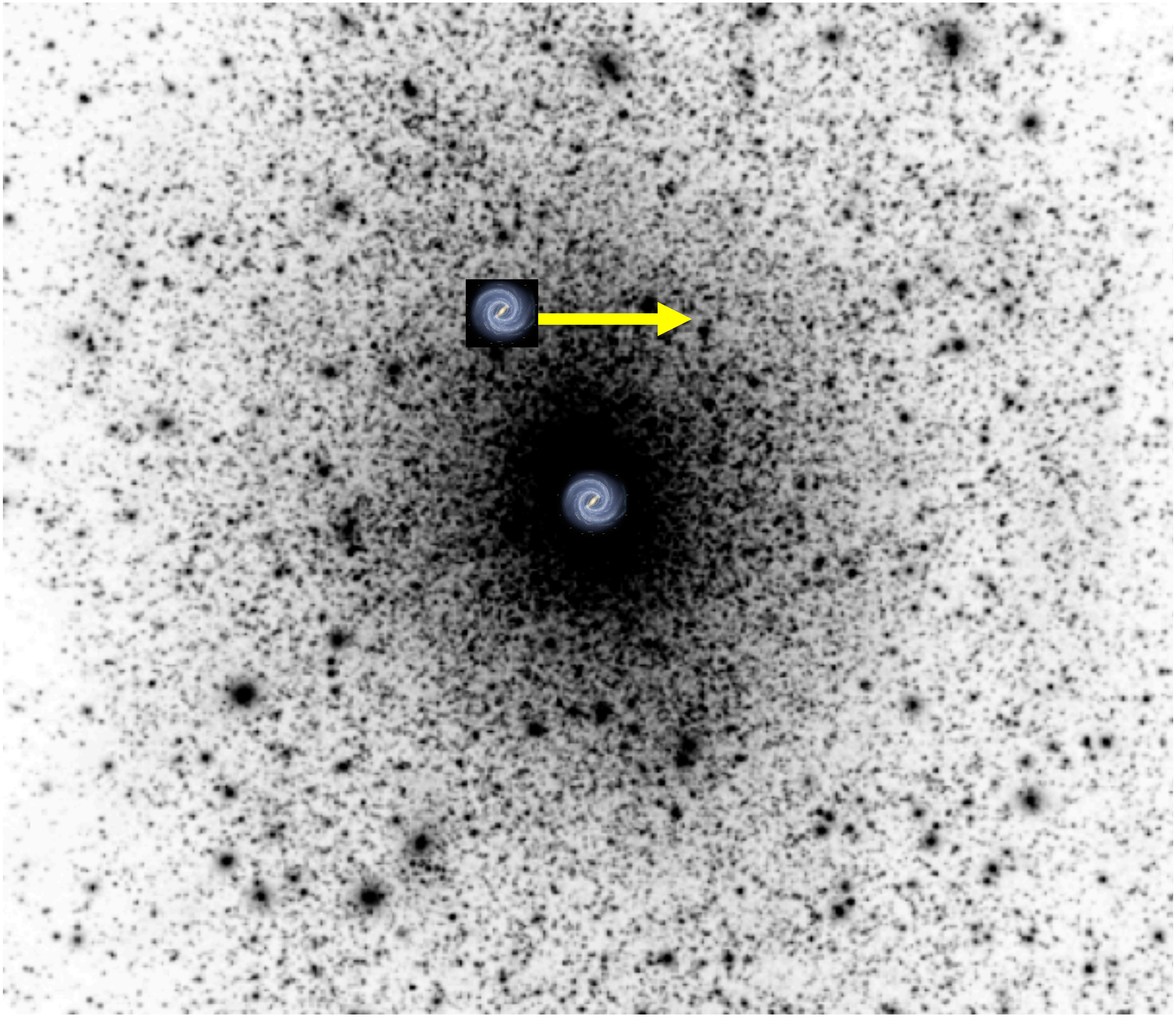
This Over Density then Tugs BACK on M

Dynamical Friction

(details depend on nature of dark matter particle)

Garavito-Camargo, GB+ in prep





Encounter with a Single Star / DM particle

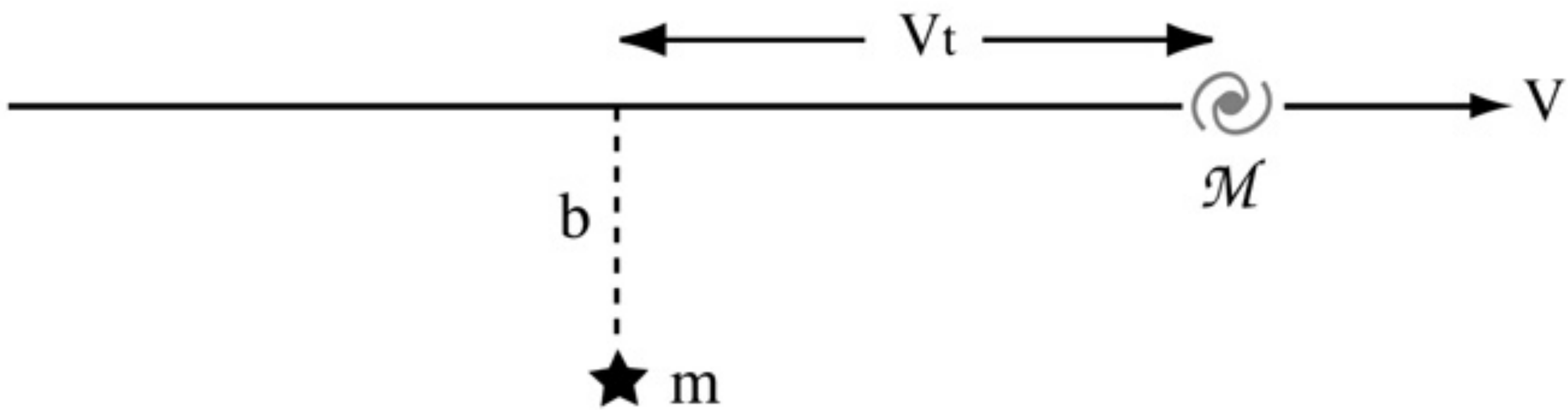
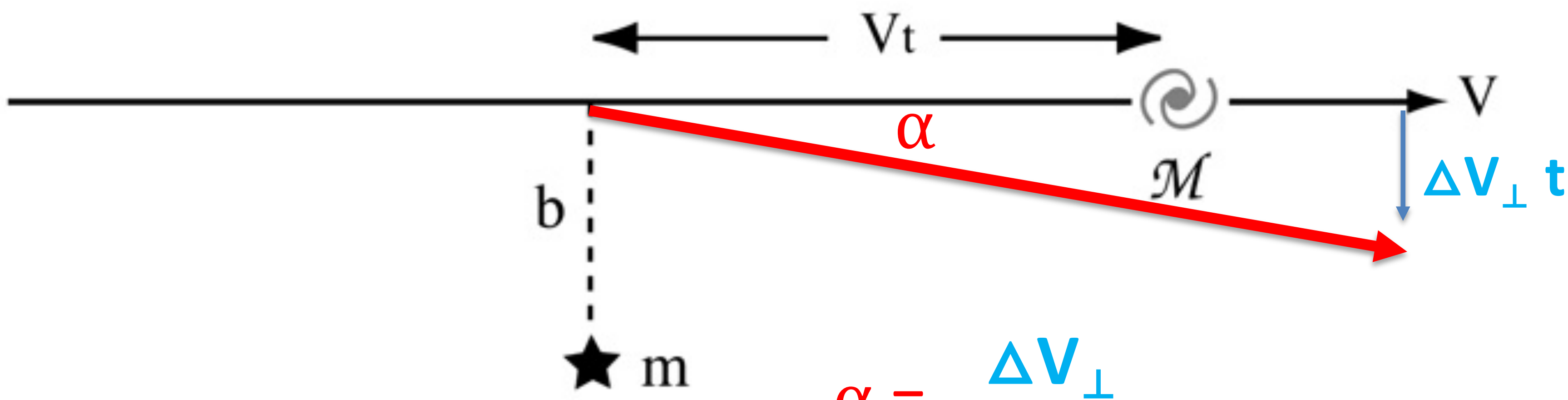


Fig 7.4 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

A galaxy of mass $\mathcal{M}$ moves with speed **V** past a stationary star (or clump of dark matter) of mass **m** in the host halo, a distance **b** from its path.

Encounter with a Single Star / DM particle



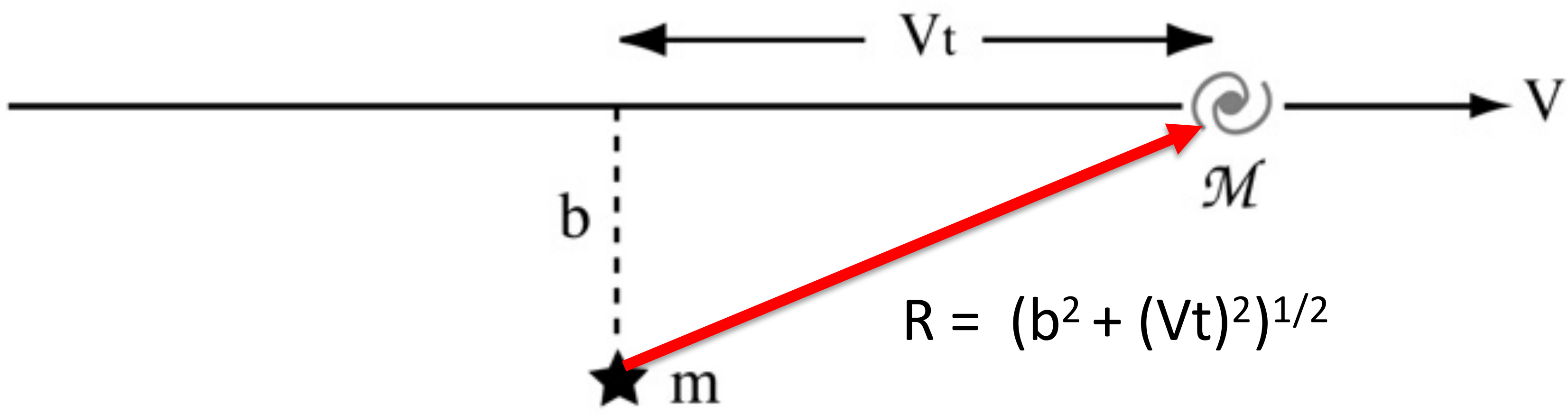
$$\alpha = \frac{\Delta V_\perp}{V}$$

$$F_\perp = \mathcal{M}\,(\Delta V_\perp / \Delta t)$$

Impulse Approximation: $\Delta V_\perp = \mathcal{M}^{-1} \int F_\perp dt$

Where : $F_\perp = \dfrac{G\,\mathcal{M}\,m}{R^2}\,\dfrac{\mathbf{b}}{R}$

Encounter with a Single Star / DM particle



Galaxy is deflected by:

$$\Delta V_{\perp} = \mathcal{M}^{-1} \int \frac{G\, \mathcal{M}\, m \quad b\, dt}{(b^2 + (Vt)^2)^{3/2}}$$

$$= G\, m\, b \left. \frac{t}{b^2 (b^2 + (Vt)^2)^{1/2}} \right|_{-\infty}^{\infty} \quad X\ 2$$

b << Vt so drop b in the brackets

$$= \frac{2\, G\, m}{b\, V}$$

Encounter with a Single Star / DM particle

The star/DM Particle ALSO receives a kick, so the change in energy of the system:

$$\Delta KE_{\perp} = \tfrac{1}{2} \mathcal{M} \, \Delta V_{\perp}{}^{2}{}_{\text{from m}} + \tfrac{1}{2} m \, \Delta V_{\perp}{}^{2}{}_{\text{from } \mathcal{M}}$$

$$= \tfrac{1}{2} \mathcal{M} \left(\frac{2Gm}{b\,V}\right)^{2} + \tfrac{1}{2} m \left(\frac{2G\mathcal{M}}{b\,V}\right)^{2}$$

Note that since $m \ll \mathcal{M}$, the term on the RIGHT dominates

$$= \frac{2G^{2} m \, \mathcal{M}(\mathcal{M} + m)}{b^{2} V^{2}}$$

The DM particle / Star is acquiring MOST of the energy. Since energy is conserved, that energy must come from the motion of the satellite ALONG its orbit (Parallel **II**)

Encounter with a Single Star / DM particle

Determining the change in speed of the satellite **along** its orbit: $\Delta V_{||}$

Initial Kinetic Energy = Total Energy After the Encounter

$$\tfrac{1}{2}\mathcal{M}V^2 = \Delta KE_{\perp} + \Delta KE_{||\,\text{satellite}} + \Delta KE_{||\,\text{particle}}$$

$$= \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\left(V + \Delta V_{||\,\text{from } m}\right)^2 + \tfrac{1}{2}m\left(\Delta V_{||\,\text{from } \mathcal{M}}\right)^2$$

Recall, $\Delta V_{\perp\,\text{from } \mathcal{M}} = \mathcal{M}/m\, \Delta V_{\perp\,\text{from } m}$

So : $\Delta V_{||\,\text{from } \mathcal{M}} = \mathcal{M}/m\, \Delta V_{||\,\text{from } m}$

$$\cancel{\tfrac{1}{2}\mathcal{M}V^2} = \Delta KE_{\perp} + \tfrac{1}{2}\mathcal{M}\left(\cancel{V^2} + 2V\Delta V_{||} + \Delta V_{||}^2\right) + \tfrac{1}{2}m(\mathcal{M}/m)\Delta V_{||}^{\,2}$$

Assuming $\Delta V_{||} << V$, then we can drop $\Delta V_{||}^2$ terms

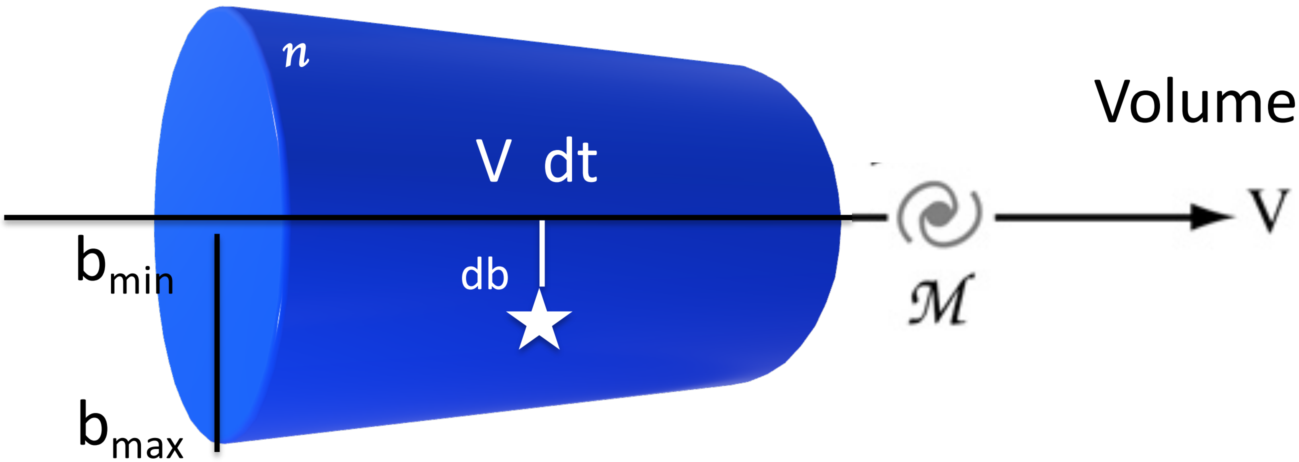
$$\Delta KE_{\perp} = -\tfrac{1}{2}\mathcal{M}\left(2V\Delta V_{||}\right)$$

$$\Delta V_{||} = -\frac{\Delta KE_{\perp}}{\mathcal{M}V} = \frac{-2G^2 m(\mathcal{M}+m)}{b^2V^3}$$

NEXT: (weak) Encounters with **MANY** Stars / DM particles

Consider a satellite galaxy moving through a cylindrical volume of a medium (stellar halo or DM halo) with number density n **= ρ / m**

Volume = 2**π** b V dt



Integrate $\Delta V_{||}$ over **b** to determine the total perturbation to the velocity along the orbit from n weak encounters

$$\Delta V_{||\,\mathrm{TOTAL}} = \int_{b_{min}}^{b_{max}} \Delta V_{||\,\text{per encounter.}}\; n\, 2\,\boldsymbol{\pi}\, V\, dt\, b\, db$$

$$= \frac{-\,2\,G^2\,(\mathcal{M} + m)\,m\,n\,2\,\boldsymbol{\pi}\,V\,dt}{V^3} \int_{b_{min}}^{b_{max}} \frac{b\,db}{b^2} \qquad \ln(\Lambda)$$

Weak Encounters with **MANY** Stars / DM particles

Coulomb Logarithm:

$$\ln(\Lambda) = \ln\,(b_{max}/\,b_{min})$$

b_{max} = Current Separation between satellite and COM of the host galaxy

b_{min} = r_s Radius for a "Strong Encounter"

Strong Encounters: If the change in potential energy is comparable to the initial kinetic energy of the satellite

$$\frac{G(M+m)}{r} \gtrsim \frac{V^2}{2}$$

$$r \lesssim r_s \equiv \frac{2G(M+m)}{V^2}$$

We need our separation b to be larger than this value or we invalidate our energy assumptions (that change in potential energy is small)

V = V_{circ} of the DM halo

Relaxation Time

- The timescale over which the speed of a particle has been changed by as much as its original speed after a series of weak encounters.
- I.e. speed has been sufficiently randomized ($\Delta V = V$), such that the particle loses memory of its initial conditions.
- This enables a system to be **virialized: pressure balances potential**

If we consider instead another particle moving through the cylinder

$$\Delta V_{\text{II TOTAL}} = \frac{-\,2\,G^2\,(m + m)\,m\,n\,2\,\pi\,V\,dt\,\ln(\Lambda)}{V^3} = V$$

$$dt = T_{relax} = \frac{V^3}{8\,\pi\,G^2\,m^2\,n\,\ln(\Lambda)}$$

$$t_{\text{relax}} \approx \frac{2\times10^9\ \text{yr}}{\ln\Lambda}\left(\frac{V}{10\ \text{km}\ s^{-1}}\right)^3\left(\frac{m}{M_\odot}\right)^{-2}\left(\frac{n}{10\ \text{pc}^{-3}}\right)^{-1}.$$

- T_{relax} ranges from about 10^9 years for globular clusters to 10^{12} years for clusters of galaxies.
- So this process is important for globular clusters, young clusters, nuclear star clusters → these are collisional systems.
- BUT within galaxies, encounters between stars are unimportant and they can be treated as **collisionless** systems.
- So how do galaxies virialized after a merger?

Violent Relaxation

- D. Lynden-Bell, MNRAS 136,101 (1967)
- During the process of merging the system is not in equilibrium
- Mass is redistributed by gravitational forces, changing the potential.
- Changes in potential redistribute stellar kinetic energy in a chaotic way (losing memory of initial conditions)
- This process of changes in the dynamics of stars caused by changes in the gravitational potential is called **violent relaxation.**

- **Dynamical Time/Crossing Time:** time needed to cross the system

$$\mathbf{T_{dyn} = R/V} = \sqrt{\frac{R^3}{G\,M}} = \frac{1}{\sqrt{G\,\rho}}$$

The MW :
15 kpc / 220 (kpc/Gyr)
~ 0.1 Gyr

Weak Encounters with **MANY** Stars / DM particles

Dynamical Friction: the corresponding deceleration needed to achieve a change in velocity ΔV_{II} along the direction of motion over some dt

$\rho = m\,n$ density of the halo

$\mathcal{M} + m \sim \mathcal{M}$

$$a_{DF} = a_{II} = \frac{\Delta V_{II}}{dt} = \frac{-2\,G^2\,(\mathcal{M} + m)\,m\,n\,2\,\boldsymbol{\pi}\,V\,\ln(\boldsymbol{\Lambda})}{V^3}$$

$$= \frac{-\,4\,\boldsymbol{\pi}\;G^2\,\mathcal{M}\,\rho\;\ln(\boldsymbol{\Lambda})}{V^2}$$

1. If V of satellite is low, a_{DF} increases
2. Total mass of the halo doesn't matter, it's the density that does.
   - If the density increases (more encounters), the deceleration is stronger
3. A more massive Satellite ($\mathcal{M}$) slows down faster than a lighter one (major encounters will decay faster than minor encounters (MW and M31 vs. M33 and M31), $F = \mathcal{M}\,a_{DF}$

Weak Encounters within a HALO

We ignored the fact that stars/DM particles in the host also have some internal velocity distribution (we assumed the initial speed of the halo particles was 0).

$$a_{DF} = -2\, G^2\, \mathcal{M}\, m\, n\, 2\, \boldsymbol{\pi}\, \ln(\boldsymbol{\Lambda}) \int f(v)\, \mathbf{v} \,/\, |\mathbf{v}|^3$$

$$f(v) = \frac{n_o}{(2\pi\sigma^2)^{3/2}} exp(-\frac{v^2}{2\sigma^2})$$

Isotropic, Maxwellian velocity distribution function

$$\frac{dv}{dt} = -\frac{4\pi G^2 M \rho}{v^2} \ln\Lambda \left[erf(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \frac{\mathbf{v}_x}{v}$$

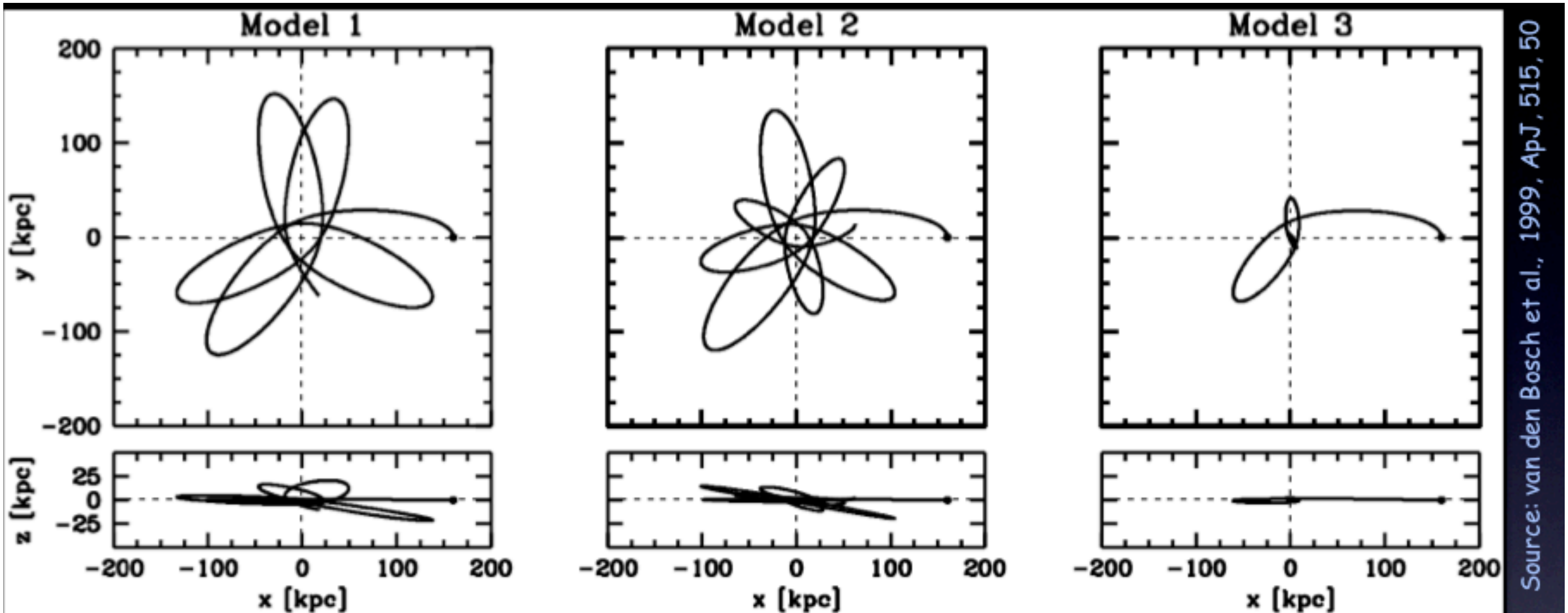
$X = v/(\sqrt{2}\sigma)$ and erf is the error function. Consider an isothermal sphere with a flat rotation curve Vc and velocity dispersion $\sigma = v_c/\sqrt{2}$, so $X = 1$

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}$$

$$\boxed{\mathbf{a} = -0.428 \frac{G M_{\rm sat} \ln(\Lambda)}{r^2} \frac{\mathbf{v}}{v}}$$

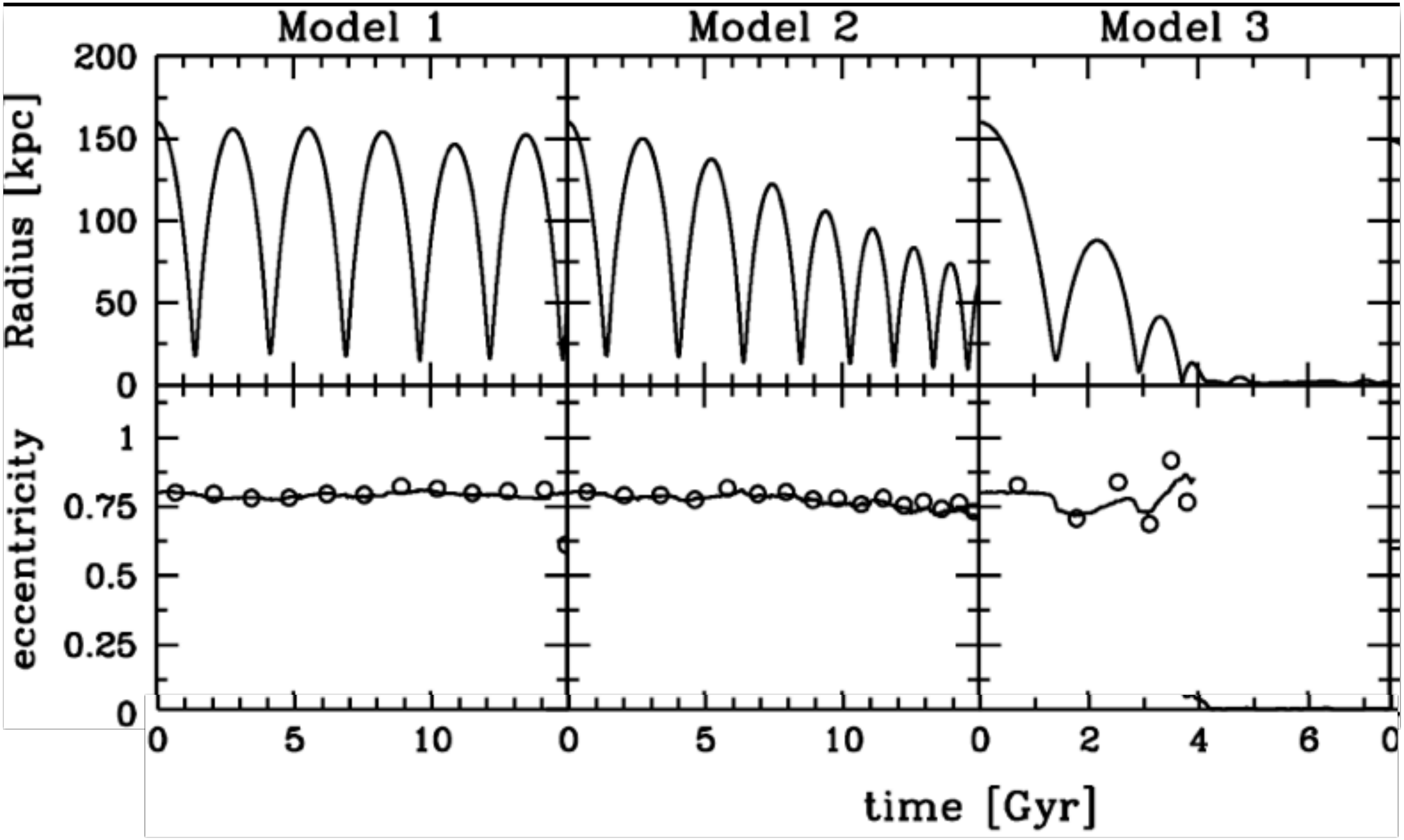
Dynamical Friction for a satellite moving through a density distribution that follows the profile for an Isothermal Sphere.

Dynamical Friction & Orbital Decay

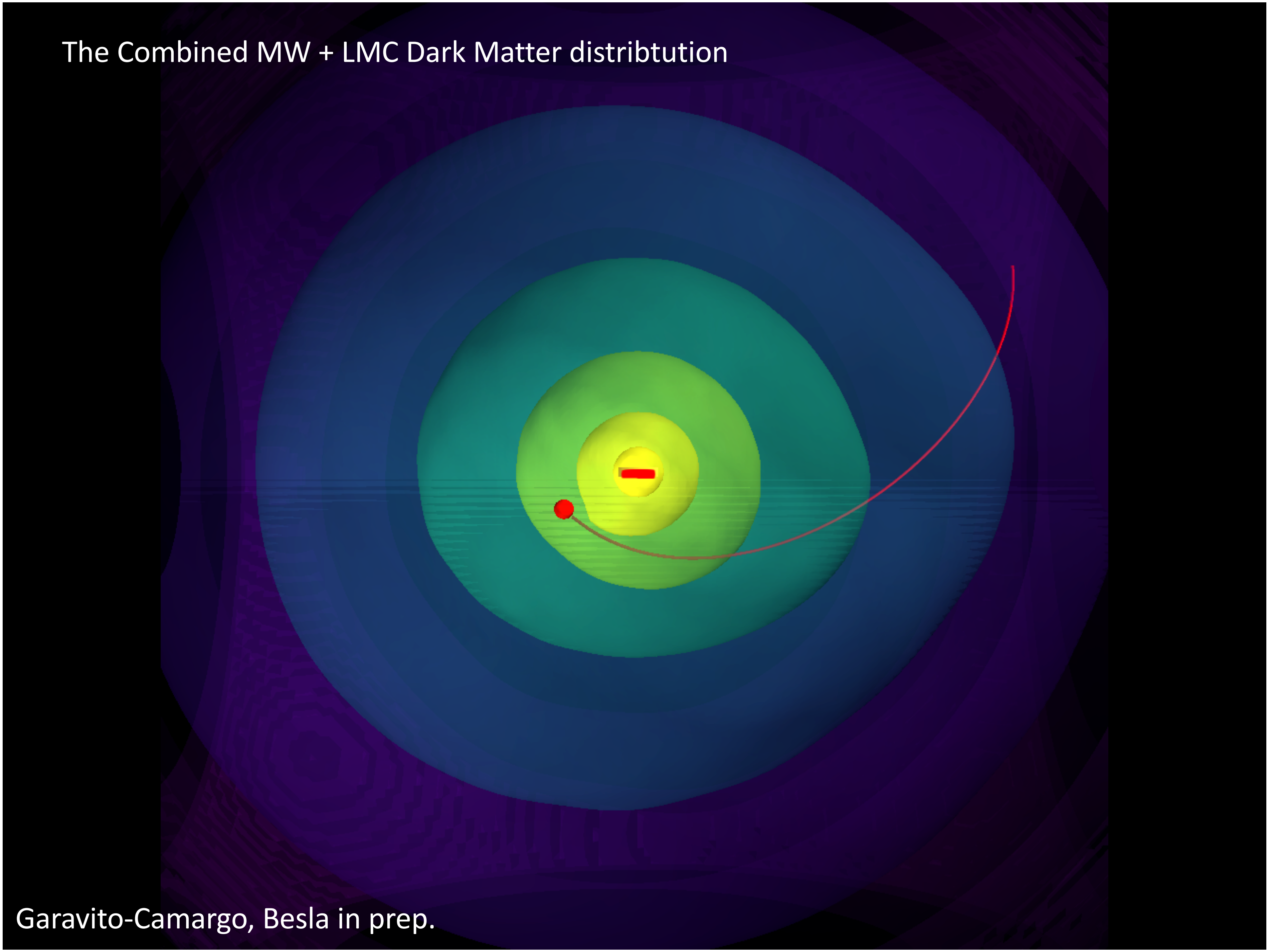


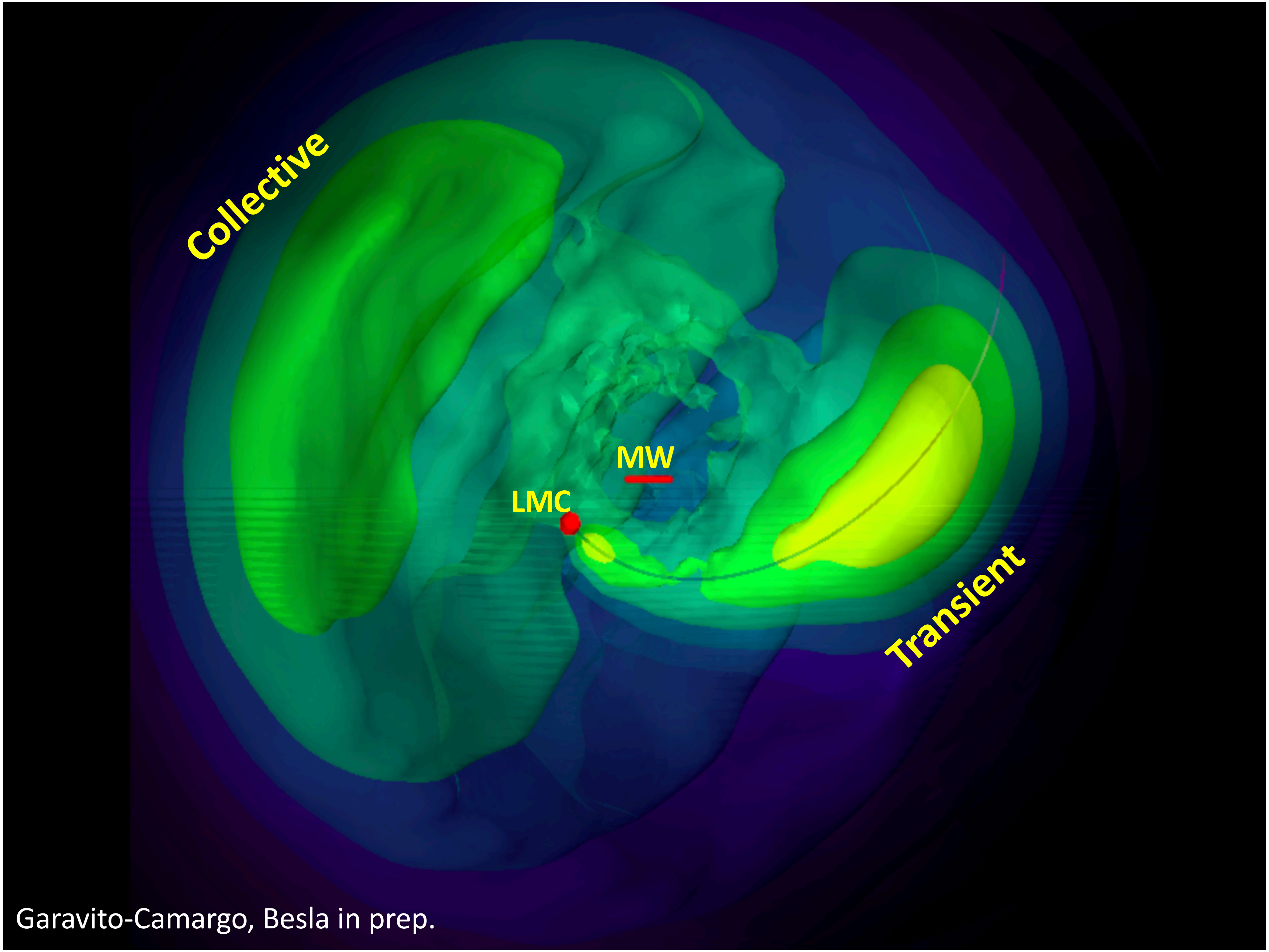
Model 1: $M_S/M_h = 2 \times 10^{-4}$
Model 2: $M_S/M_h = 2 \times 10^{-3}$
Model 3: $M_S/M_h = 2 \times 10^{-2}$

- All three orbits have initial eccentricity e=0.8
- Orbits of more massive subjects decay faster
- No (obvious) orbit circularization

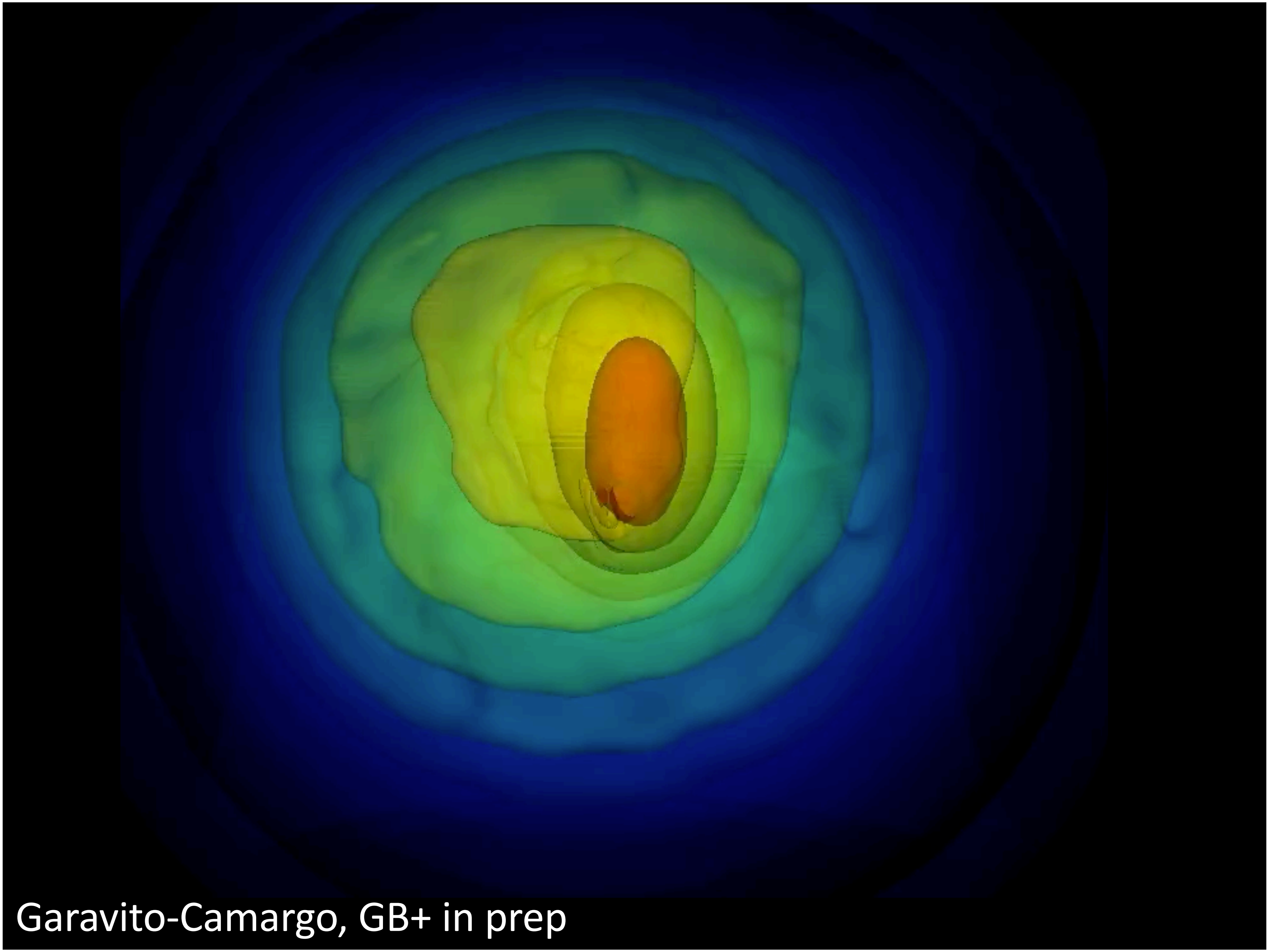


Van den Bosch + 1999


The Combined MW + LMC Dark Matter distribtution
Garavito-Camargo, Besla in prep.



Garavito-Camargo, Besla in prep.


Garavito-Camargo, GB+ in prep