Producing and Reflecting X-rays

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The project I designed involved creating an X-ray source and using assorted mirrors at grazing-incidence angles to try and reflect X-rays. Measurements showing that the X-rays had reflected and the strength of those X-rays were planned. However, I failed to produce any X-rays because of a safety trip mechanism in the high voltage source that prevented me from feeding a large enough current to the X-ray source.

Introduction

My project focused on building an apparatus that could emit and reflect X-rays. The project as designed had two major measurements. The first measurement was a binary yes or no whether or not a shallow enough angle of reflection would allow me to reflect X-rays in a manner similar to visible light. The second was a quantitative measurement of the critical angle of the mirrors I used to achieve this reflection. However, the focus of the project fell on the experimental verification of reflection and the production of X-rays.

The reflection of X-rays has many industrial and research uses. The CHANDRA X-ray telescope requires X-ray reflection to focus X-ray light into high resolution images and spectra of dim and distant astronomical sources, including most recently an optical counterpart to neutron star mergers[1]. X-ray reflection also has industrial uses, including Grazing-Incidence Small-Angle X-ray Scattering (GISAXS) which can be used to study or map the distribution of nano-scale features, films, or patterns on the surface of objects[2].

Unfortunately, because of issues with the X-ray source, neither measurement was made.

Procedure

The first part of the experiment is making the X-ray source. The physical science of how this works is explored in the theory section, but the circuit simply requires one end of the tube (the anode) to supplied with high voltage, while the filament end is supplied with high current. Figure 1 below is simple circuit diagram demonstrating the setup required to generate X-rays. The whole setup will be surrounded with lead blocks to prevent irradiating the lab and students within it, with a gap in the shielding to allow X-rays to exit in a single direction. A Geiger counter will then be placed in front of the aperture to test if X-rays are being emitted.

The vacuum tube used was the 2X2A model made by Mullard Limited. The high voltage source was the Bertan 380X.

It should be noted that most of these connections used alligator clips which were exposed, and that touching any metal directly connected to the circuit is incredibly dangerous.

At this point, the various films would be placed in front of the aperture at variable distances to find their response to the X-rays. Primarily, I would look for how long I need to expose the film to get a visible stain or image on the film. I would vary distance by 25 cm and start with exposures of 15 seconds, and increase exposure time by 15 seconds in future trials. This data would be organized into a table with distance on one axis and exposure time on another, with the cells indicating whether any response was visible in the film.

I would do the same experiment by placing the Geiger counter probe at the same distances as the films, and measuring the milliroentgens per hour.

Once the apparatus is producing X-rays, the first part of the experiment calls for verification that the equipment available can reflect X-rays and measure X-rays. By placing one mirror such that X-rays from the aperture have an incidence of 1 degree and placing the film or Geiger counter along the path of the reflected light with a baffle blocking the original source, to show that the response is due to reflected X-rays. Placing the baffle such that it blocks the source but not the reflected ray might be difficult at short distances, because the path of the reflected light rays and the incident light rays directly from the source will not differ by much. Moving the source further away may alleviate this issue, but the X-rays may not be strong enough to register a response on film or with the Geiger counter at greater distances.

Another proposed setup would use two mirrors to reflect the light twice. The second mirror would likewise reflect the incident light at and angle of 1 degree, except the incident light is the reflected light from the first mirror. Figures 2 and 3 show both of these setups in detail. The total change in the direction of the light ray will be much larger with a shorter path, but this setup causes more X-rays to be lost at each reflection, as some amount of the X-ray photons striking each

Figure 1: The diagram shows the circuit used to generate X-rays. The high voltage in the tube accelerates the electrons that are boiled off the filament by high current. When the electrons hit the glass or anode, they emit radiation.

Figure 2: The top-down view of the experimental verification using a single mirror. A single mirror is easier to set up, but might not allow for clear distinction between the reflected and incident light. Angles are not drawn to scale.

Figure 3: The top-down view of the experimental verification using two mirrors. The usage of two mirrors reduces the chance of whichever detection method we choose picking up X-rays from the source directly. Angles are not drawn to scale.

mirror will transmit instead of reflect.

In either case, if a response is measured on film or with a Geiger counter, multiple trials should be run with and without the mirrors to ensure that the response is because of reflected X-rays.

The final stage of the experiment would be attempting to measure the critical angle of a reflector. Based on the materials I had available, I am skeptical if this part of the experiment was ever possible. However, I would first see if reducing the incidence angle on the mirror caused a visible difference in the intensity of the X-rays as measured by the Geiger counter or seen in the response of the film, almost certainly the former. If the difference is measurable, I would continue reducing the incidence angle of the light striking the mirror, until I see no more increase in the intensity of the reflected X-rays. At that point, reflection is maximized so decreasing the incidence angel no longer increases the amount of reflected light I measure.

Another issue with this procedure is that all these angular placements have to be done by hand. For the previous part of the project, I did not choose and incidence angle of 1 degree arbitrarily, as that is the smallest angle that I am absolutely sure I could accurately measure and set up. Even in that case the set up would consume large amounts of lab time

Theory

I will start by proving that I do not need to find the critical angle of reflection for the X-rays to reflect X-rays, and that a noticeable amount of reflection can occur at non-critical angles. This is because from the outset of the experiment there was the possibility that secondary goal of measuring the critical angle of reflection was completely out of reach, simply because of how I designed the experiment.

I will begin with the most general description of a light wave as an electric and magnetic field, which will allow me to employ field boundary conditions at the interface of our reflector and the open air.

I must also assume for the purpose of this derivation that all the materials involved are linear, isotropic, and homogeneous. These assumptions respectively posit that the field strength within the medium increases linearly with the external or applied field, that the field within the medium propagates the same way in all directions, and that the field in the medium propagates the same way everywhere within the medium. These assumptions are problematic in most real-world applications, but here allow us to write the following relationships between the electric E and displacement field D and the magnetic field strength H and magnetic flux density B in the problem.

$$D = \epsilon E \ (1)$$

$$B = \mu H \ (2)$$

Where ϵ and μ are the electric permittivity and the magnetic permissivity, which reduce to scalars in the approximation specified above. I can now employ simplified versions of Maxwell's laws that apply to light waves.

$$\omega B = k \times E$$

$$\omega D = -k \times H$$

The parameter ω is simply the frequency of the oscillation in the electric field (and also the frequency of the light), and k is the wave-vector. The wave-vector points in the direction of propagation of the light wave, and is proportional to the wavenumber. Figure 4 below demonstrates the orientation of the vectors in question when describing the light-wave.

The figure shows that E, H, and k vectors are all orthogonal to each other, meaning these cross products easily reduce to scalar equations describing the magnitudes of each of our fields.

$$\omega B = kE \ (3)$$

$$\omega D = -kH \ (4)$$

The wavenumber k can be re-expressed in terms of the index of refraction of the material, using the definition of the wavenumber:

$$k = \frac{2\pi}{\lambda} \ (5)$$

The variable λ is the wavelength of the light wave, which in turn allows us to relate the wavenumber to the wave-propagation speed v, using the classical definition. I also include the relationship between index of refraction and the speed of light in vacuum to the wave-propagation speed.

$$v = \frac{c}{n} = \nu\lambda \ (6)$$

By rearranging Equation 5 to acquire λ in terms of k, I can then plug that expression into Equation 6, and solve for k. I also employ the relation between linear and angular frequency, as this will cause the angular frequency term to fall out of the final expression.

$$\frac{c}{n} = \nu\frac{2\pi}{k}$$

$$\frac{kc}{n} = \omega$$

$$k = \frac{n\omega}{c} \ (7)$$

With this relationship, I can substitute k in Equations 3 and 4.

$$\omega B = \frac{n\omega}{c}E$$

$$\omega D = -\frac{n\omega}{c}H$$

Now I use Equation 1 to eliminate any D terms and Equation 2 to eliminate any B terms. At this point, the ω on either side of each equation cancel out as well.

$$\mu H = \frac{n}{c}E$$

$$\mu c H = nE \ (8)$$

$$\epsilon E = \frac{n}{c}H$$

$$\epsilon c E = nH \ (9)$$

By solving for c we can find a linear relationship between H and E. This relation will be used when the field boundary conditions are applied.

$$c = \frac{nH}{E\epsilon}$$

$$\mu\frac{nH^2}{E\epsilon} = nE$$

$$H^2 = \frac{E^2\epsilon}{\mu}$$

$$H = \sqrt{\frac{\epsilon}{\mu}}E \ (10)$$

Equation 10 is usable for my purposes, but it can be reduced even further. If I instead multiply Equations 8 and 9 together and solve for the index of refraction, I can find its value in terms of the relative permittivity, permissivity.

$$\epsilon\mu c^2 HE = (n)^2 EH$$

$$n = c\sqrt{\epsilon\mu} \ (11)$$

By re-expressing the coefficient in Equation 10 using Equation 11, we can re-write the linear relationship in terms of the index of refraction.

$$H = \frac{n}{c\mu}E$$

But even this can be simplified further. In my experiment, I only intended to use non-magnetic materials, so for my purposes one could easily say that μ is equal to μ_0, the magnetic permissivity of free-space.

$$H = \frac{n}{c\mu_0}E \ (12)$$

With this relationship in hand, we can at last turn to the boundary conditions for electric field. The condition I want to use is that the tangential (i.e. parallel to the interface) component of the field will be continuous. So for an arbitrary field A, we can write the simplest form of the boundary condition.

$$A + A_r = A_t \ (13)$$

Above, A is the incident field component and is simply equal to the field strength, A_r is the reflected wave component, and A_t is the transmitted wave or field, all evaluated at the boundary. Figure 5 shows that the transmitted wave is the only part of the wave that makes it across the boundary, so the field created by the reflected and incident waves must be equal in magnitude of the field created by the transmitted wave.

Figure 4: The electromagnetic components that form a light wave, and that will allow me to use the boundary conditions of that all electromagnetic fields must obey when propagating past an interface.

Figure 5: The two-dimensional representation of the wave striking the interface. Either the magnetic or electric field component of the will point out of the page. The plane of incidence is the page itself.

Furthermore, we can approximate A_r and A_t as linearly proportional to the external or incident field.

$$A_r = rA$$

$$A_t = tA$$

The coefficients r and t effectively give what fraction of the wave's amplitude is transmitted or reflected at the boundary. I can now re-write Equation 13 in a much simpler form.

$$1 + r = t \ (14)$$

This only describes the case where the wave is entirely tangential to the interface, however. In Figure 5, this corresponds to a wave coming into or out of the page. However, because our E and H fields are orthogonal to each other, so while one will be coming out of the page, the other will have to be on the plane of incidence and thus have only some component in the tangential direction. Figure 5 allows me to find these components for the incident, reflected, and transmitted field. Note that angles are defined relative to the interface, not the normal of the interface. In the case of grazing incidence optics, the convention I use here is more convenient then the typical definition.

$$A_{tan} = Asin(\theta_i)$$

$$A_{r_{tan}} = A_r sin(\theta_r) = A_r sin(\theta_i)$$
$$A_{t_{tan}} = A_t sin(\theta_t)$$

Following essentially the same derivation above, we can relate r and t in much the same way as before.

$$sin(\theta_i) - rsin(\theta_i) = tsin(\theta_t) \ (15)$$

Of course, this raises the question of which field is in which direction. The convention calls the case in which E is in the plane of incidence p-polarization, while the case where E is coming out of or going into the page is called s-polarization. It is thus necessary to solve for r both cases. Starting with p-polarization we find our boundary conditions.

$$Esin(\theta_i) - E_r sin(\theta_i) = E_t sin(\theta_t)$$

$$sin(\theta_i) - r_p sin(\theta_i) = t_p sin(\theta_t) \ (16)$$

It may be tempting to jump straight to the Equation 14 when writing the H equation, but I know from Equation 12 that the coefficient of H_t depends on the index of refraction of the material, which in general will differ for the transmitted wave.

$$H + H_r = H_t$$

$$\frac{n_1}{c\mu_0}E + r_p\frac{n_1}{c\mu_0}E = t_p\frac{n_2}{c\mu_0}E$$

$$n_1 + r_p n_1 = t_p n_2 \ (17)$$

n_1 is the index of refraction of the material the wave propagates through to reach the interface (usually air), and n_2 is the material of the interface. By solving for t_p in Equation 17 and plugging it into Equation 16 I can find a value of the reflection coefficient.

$$t_p = \frac{n_1}{n_2} + \frac{r_p n_1}{n_2}$$

$$sin(\theta_i) - r_p sin(\theta_i) = \frac{n_1}{n_2}sin(\theta_t) + \frac{r_p n_1}{n_2}sin(\theta_t)$$

$$-r_p sin(\theta_i) - \frac{r_p n_1}{n_2}sin(\theta_t) = \frac{n_1}{n_2}sin(\theta_t) - sin(\theta_i)$$

$$r_p = \frac{\frac{n_1}{n_2}sin(\theta_t) - sin(\theta_i)}{-sin(\theta_i) - \frac{n_1}{n_2}sin(\theta_t)}$$

$$r_p = \frac{n_1 sin(\theta_t) - n_2 sin(\theta_i)}{-n_2 sin(\theta_i) - n_1 sin(\theta_t)}$$

$$r_p = \frac{n_2 sin(\theta_i) - n_1 sin(\theta_t)}{n_2 sin(\theta_i) + n_1 sin(\theta_t)} \ (18)$$

I can do the same derivation for s-polarization, where the only difference is that the boundary conditions for the E and H fields switch such that H has the component in the tangential direction.

$$Hsin(\theta_i) - H_r sin(\theta_i) = H_t sin(\theta_t)$$

$$n_1 sin(\theta_i) - n_1 r_s sin(\theta_i) = n_2 t_s sin(\theta_t) \ (19)$$

$$1 + r_s = t_s \ (20)$$

Then the algebra is effectively the same as the derivation for p-polarization.

$$n_1 sin(\theta_i) - n_1 r_s sin(\theta_i) = n_2(1 + r_s)sin(\theta_t)$$

$$-(n_2 r_s sin(\theta_t) + n_1 r_s sin(\theta_i) = n_2 sin(\theta_t) - n_1 sin(\theta_i)$$

$$r_s = \frac{n_1 sin(\theta_i) - n_2 sin(\theta_t)}{n_1 sin(\theta_i) + n_2 sin(\theta_t)} \ (21)$$

The reflection coefficient itself is a dimensionless wave amplitude, and does not give the proportion of the light that is reflected. I must instead square each of these terms to get the proportion of the incoming power of the light wave that is reflected, R.

$$R_p = \left(\frac{n_2 sin(\theta_i) - n_1 sin(\theta_t)}{n_2 sin(\theta_i) + n_1 sin(\theta_t)}\right)^2 \ (22)$$

$$R_s = \left(\frac{n_1 sin(\theta_i) - n_2 sin(\theta_t)}{n_1 sin(\theta_i) + n_2 sin(\theta_t)}\right)^2 \ (23)$$

By using conservation of energy I can also relate the transmitted power coefficient to the reflected power coefficient.

$$T + R = 1 \ (24)$$

By plugging Equations 22 and 23 into Equation 24, I can quickly find T_s and T_p.

$$T = 1 - R$$

$$T_p = \left(\frac{n_2 sin(\theta_i) + n_1 sin(\theta_t)}{n_2 sin(\theta_i) + n_1 sin(\theta_t)}\right)^2 - \left(\frac{n_2 sin(\theta_i) - n_1 sin(\theta_t)}{n_2 sin(\theta_i) + n_1 sin(\theta_t)}\right)^2$$

$$T_p = \frac{2n_1 n_2 sin(\theta_i) sin(\theta_t)}{(n_2 sin(\theta_i) + n_1 sin(\theta_t))^2} \ (25)$$

$$T_s = \left(\frac{n_1 sin(\theta_i) + n_2 sin(\theta_t)}{n_1 sin(\theta_i) + n_2 sin(\theta_t)}\right)^2 - \left(\frac{n_1 sin(\theta_i) - n_2 sin(\theta_t)}{n_1 sin(\theta_i) + n_2 sin(\theta_t)}\right)^2$$

$$T_s = \frac{2n_1 n_2 sin(\theta_i) sin(\theta_t)}{(n_1 sin(\theta_i) + n_2 sin(\theta_t))^2} \ (26)$$

As θ_i approaches 0 and the reflecting angle becomes shallower, the coefficient T in the case of either p-polarization or s-polarization will approach 0, meaning the R coefficient approaches 1 and all the light reflects. Note that this does not directly depend on the wavelength or energy of the light, although the index of refraction does. However, the derivation shows what I need it to, and that is to reflect X-rays I do not need to find the critical angle to reflect the X-rays. Therefore, the first part of the experiment is achievable, even if the measurement of critical angle is not.

I should briefly note that the X-Rays produced in this experiment will not necessarily be uniformly polarized in any particular direction. Describing the reflection of waves without one of these two polarisations is a geometrically and algebraically intensive problem., so the effective reflected power coefficient is given by averaging the two reflection coefficients together.

$$R = \frac{1}{2}(R_s + R_p) \ (27)$$

The derivation of the working equation is in comparison trivial. Nothing more is required then Snell's Law.

$$n_1 cos(\theta_i) = n_2 cos(\theta_t) \ (28)$$

Note that I am still using the convention of the previous derivation of measuring angles from the interface instead of the normal of the interface. In any case, we place the constraint that θ_t can not be less than 0, at which point the wave is completely reflected. In such a case, the incident angle is denoted as the critical angle.

$$n_1 cos(\theta_c) = n_2$$

$$\theta_c = cos^{-1}\left(\frac{n_2}{n_1}\right) \ (30)$$

Normally this phenomenon is called total internal reflection, since the light reflects back into the denser medium at the interface. However, because the index of refraction is less than 1, the reflection in this case occurs when the light tries to move from a less dense to more dense medium. This is called total external reflection to differentiate from the related effect.

In the case of X-rays, the index of refraction tends to be very close to unity[3] regardless of material and thus the ratio of the indices of refraction are also around unity. This means that the critical angle approaches 0 because of the inverse cosine dependence. The following equation approximately describes the real component of the index of refraction.

$$n = 1 - \frac{n_e r_0 \lambda^2}{2\pi} \ (31)[3]$$

λ is the wavelength of light moving through the material, r_0 is the classical electron radius, and n_e is the electron density of the material. I will re-write electron density as the density of atoms times the number of electrons per atom (equal to the atomic number, denoted Z) since those quantities can be found online or in indices with some ease.

$$n = 1 - \frac{Z n_a r_0 \lambda^2}{2\pi} \ (32)$$

Equation 32 suggests that the higher the atomic number and the atomic density of the material, the greater the difference between n and unity. For the sake of demonstration, I will use parameters from aluminum, which is commonly used as film for reflective surfaces, including the mirrors I intended to use in the experiment. Osmium has a density of 2.702 grams per cubic centimeter, an atomic weight of 26.98 amu and an atomic number of 13 [4]. This gives me an atomic density of 6.033×10^{22} atoms per cubic centimeter. The X-ray band has wavelengths ranging from 10 to 0.01 nm. After plugging all these parameters into Equation 32, I acquire a refractive index of 0.9648 or 1-3.516 $\times 10^{-8}$ If I put these in Equation 30 as n_2 and say that n_1 is unity, on the upper bound I get a critical angle of 15 degrees, which seems reasonable. However, the lower bound gives a critical angle of about one one-thousandth of the upper bound. Most of the X-rays produced would be somewhere between those values.

Finally, I must discuss what I used as a source for my X-rays. The atmosphere is opaque to X-rays[5], and X-ray producing tubes are cost-prohibitive, necessitating the creation of my own source. I chose to use an archaic vacuum tube rectifier. The device consists of a filament that can be fed with high current and an anode. By having high enough voltage on the anode, the hope is that electrons boiled off of the filament will gain large amounts of kinetic energy as they move towards the anode. Then when the electrons approach the positively charge nuclei of the anode or the glass of the tube, they will be subjected to electromagnetic forces that slow them down. The kinetic energy of the electron is then converted into photons in a form of radiation called brehmsstrahlung[6], and if the kinetic energy was high enough, those photons will by X-ray photons. This is the same basic concept behind an actual X-ray tube[7] but far cheaper, less powerful, and more likely to melt or break unexpectedly.

I need to demonstrate that this setup can work with sufficiently high voltage. The minimum wavelength of light produced by brehmsstrahlung is given by the maximum energy that any electron can give to a photon. This will be the maximum kinetic energy of an electron the tube, which is given below.

$$E_k = eV \quad (33)$$

V is the voltage supplied to the tube, and e is the charge of the electron. We want to equate this with the energy of a photon generated by brehmsstrahlung, so we use the Einstein-Planck relationship.

$$E_{photon} = h\nu$$

ν is the frequency of the photon, and h is Planck's constant. I can relate the frequency to the wavelength easily.

$$E_{photon} = \frac{hc}{\lambda} \quad (34)$$

By setting Equation 33 and 34 equal to each other, the λ in Equation 34 becomes $\lambda_m in$ as the energy of the photon is maximized, and it can be solved for with ease.

$$\frac{hc}{\lambda_{min}} = eV$$

$$\lambda_{min} = \frac{hc}{eV} \quad (35)$$

Equation 35 is called the Duane-Hunt relationship[8], and it gives the absolute maximum energy of any photon emitted by the apparatus. Brehmsstrahlung does not always result in the full stop of the electron, and thus many of the emitted photons will have a higher wavelength the the minimum given by the Duane-Hunt Relationship. In the original design of the experiment, I was planning to supply 40 kV to the vacuum tube. All the other constants are well known, so I can predict that the minimum wavelength of light produced by brehmsstrahlung in the tube would be hundredths of nanometers, which is well within the range of X-rays.

Results & Discussion

Not only did I not reflect X-rays, the source I was using did not manage to produce X-rays. The high voltage source (Bertan 380X) I was using had a maximum current allowance of 0.8 milliamps. If the circuit hooked up to the high voltage registered a current higher than this, it would turn off the high voltage. I found that the maximum voltage that could be supplied to the circuit without tripping it was 5.9 kV. That should be acceptable, as the $\lambda_m in$ calculated from Equation 35 is about a tenth of a nanometer, which is still within the X-ray band.

I also need to provide large currents to the filament, which will increase the current in the high voltage supply. The end result was that I could not supply a filament current greater than 2 amps without tripping the circuit. Reducing the voltage in the high-voltage part of the circuit did not appear to effect the current at which the circuit would trip. I suspected that the current might be conducting to the high-voltage end of the vacuum tube because of hand oils on the bulb, but cleaning the bulb did not improve the results. The current may be jumping through the air, either across the alligator clips used to attach the high current source to the bulb, from those clips to the high voltage end of the tube, or from the positive terminal of the high current supply to the ground of the high voltage supply, which was placed at the negative terminal of the high current supply. Of course, it is possible that the current is moving from filament to anode as its supposed to, and the high voltage supply still can not handle the current in any case, in which case the only solution is another supply.

Since my source never produced X-rays, I was never able to reflect them and the other parts of my experiment were never tested.

Conclusions

Since my project did not work, I feel this section is best dedicated to exploring possible improvements to the experiment and possible solutions to problems that may have arisen later on in the experiment.

The first and most obvious improvement is a less safe high voltage source, as this would very quickly solve the issue that ultimately impeded the experiments progress. Also, an even higher voltage would be preferable. While 10kV or even 5kV are more than sufficient to produce some X-rays, they might not produce enough. As I have mentioned, X-rays are lost at each reflection and over distance, so I want my source to have as high an intensity as is feasible. The issue with this, and the reason I did not just get a higher voltage source from the outset, is because the higher voltage sources are more expensive, and most high voltage sources have the built-in circuit safety that impeded me in the course of the experiment.

Another key improvement is some better way to quantitatively detect X-rays. The film on hand may not have been sensitive to X-rays, but there is fairly readily available medical film that is sensitive. However, even use of that film would not solve the issue of making quantitative measurements. The Geiger counter may enable indirect quantitative measurements. However, the counter works by detecting ionizing radiation [9], and X-rays actually tend pass straight through material without ionizing it, as seen with the near unity refractive indices. It is thus possible that the Geiger counter is not very sensitive to X-rays, and thus makes a poor instrument for measuring differences in X-ray intensity due to distance or reflection.

One solution proposed by a teaching assistant is an electronic spectrograph. The idea is that instead of trying to eyeball any changes in intensity, the X-ray would be reflected at a slightly different wavelength because of the Kerr Effect[10]. Therefore in the spectrograph, there should be two visible peaks in the X-ray region, and their relative intensities should also be straight forward to measure with software. However, this method involves its own complications.

First it assumes that the source is emitting at discrete peaks, and not continuously. In the latter case the spectrograph will not work, since the reflected and incident spectra will just overlap. Further, I would need to make a spectrograph that is capable of resolving the difference between x-ray peaks. The index of refraction for x-rays is always close to unity, and it can be reasonably assumed that the different X-ray wavelengths will have fairly small difference in their refractive indices. This means the spectral lines will appear so close together that they may register as one emission line unless I use a long spectrograph.

Employing the Kerr effect involves its own issues. Only certain materials display the effect strongly, and they tend to be toxic liquids[11]. Furthermore the Kerr Effect is proportional to the wavelength[11], making it less than ideal for low wavelength X-rays. The Kerr effect also discards the assumption of a linear, isotropic, and homogeneous reflector, which was critical to the derivation in the Theory section.

One issue that I actually anticipated was the issue with smoothness. The less smooth a surface is, the more likely and incident ray will be reflected off at some strange angle, as demonstrated in Figure 6.

This phenomenon is easily observable in nature; a calm pool of water will show a reflection, but a rippling or disturbed pool will not. I also found that the smaller the wavelength, the more likely the light will reflect at some odd angle because of a surface feature. The reflectivity will be modified by some factor, given below.

$$R_{eff} = R_0 e^{-\frac{4\pi\sigma}{\lambda^2}} \quad (36)$$

R_0 is the reflectively coefficient given by Equation 27 while σ is the standard deviation from a perfectly smooth surface. Notice also that the exponential term decreases with the wavelength of light, requiring correspondingly lower σ (or more smoothness) to achieve the same reflection coefficient. For this reason the Chandra X-ray Telescope's mirrors are perfectly smooth to the width of a few atoms[13]. I do not necessarily need a mirror that smooth to reflect X-rays at all, but I do not know how smooth the mirrors used in the experiment are. To preempt this issue, I turned to the lessons learned in theoretical mechanics. Namely, a liquid experiencing surface tension will try to minimize its surface area in the absence of any external forces, so I imagined that I could use a calm water surface to make a plane flat enough to reflect X-rays. Although some density is lost when compared to aluminum or other metals, the efficiency gain from the increased smoothness may make up the difference and then some.

Of course, the solution is not tested, and is not perfect even from a theoretical standpoint. Getting a sufficiently smooth surface of water might be impossible in the middle of an active physics lab. Also, the presence of gravity means this method probably only realistically allows for a single reflection, as placing more "mirrors" at different angles will cause their structure to sag.

While this experiment obviously failed in the end, listing possible future developments of the project is still a worthwhile discussion. The obvious next step is moving from reflecting X-rays to focusing X-rays, since that is the paramount goal in astronomical contexts. The Wolter telescope designs were created for this purpose[14] and Wolter Type I is the optical design employed in Chandra. The designs technically require the mirrors (as every design calls for at least four[14]) to not only lie on a paraboloid and hyperboloids, but also have the curvature of a paraboloid or hyperboloid. However, Wolter Type I only has a little curvature and probably could employ flat mirrors, at the cost of a slightly blurrier focus.

I mentioned earlier that the largest angular set-up I trusted in my manual ability to measure with the tools on hand was a single degree. If one were to pursue the loftier goal of focusing the X-rays, then finer control of the angle of the mirrors would be necessary. Building a rotating apparatus with angular gradations to control the mirrors instead manually placing them would reduce the time required to set up the experiment as written, and make the focusing of X-rays possible.

Figure 6: If a surface is not perfectly flat, the light will strike at a different incidence angle, and be reflected at a strange angle instead of the apparent incidence angle as seen from a macroscopic scale.

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I referred to this textbook heavily when deriving the Fresnel Equations in the Theory section. While it does not include the derivation, it does include the boundary conditions of a field at an interface, and numerous other tidbits of information used in the derivation.

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