Using the Photoelectric Effect to Measure Planck's Constant

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Our experiment employed the photoelectric effect to find the value of Planck's constant. By exciting electrons with differing frequencies of light and finding the stopping voltage of those electrons, we constructed a linear regression stopping voltage as a function of light frequency, with a slope of $\frac{h}{e}$. Our measured value of Planck's constant was $1.0 \pm 2.4 \times 10^{-34}$ J s. This experiment demonstrates how precisely measured macroscopic effects aid in determining near infinitesimal values with an acceptable degree of accuracy. The experiment also demonstrated the necessity of large samples when using methods of linear regression.

Introduction

The first observation of the photoelectric effect was recorded by Heinrich Hertz in 1887, when he noticed that ultraviolet could cause metallic plates to emit flashes of light, then understood to be electrons leaving the surface of the metal[1]. The electrons on the plate were able to move freely around the solid structure of the metal, and thus could easily gain enough energy form incident light to fly off the metal. However, the amount of electrons ejected by the effect appeared to be independent of the intensity of the light used to excite the electrons. Indeed, some light could never excite electrons, regardless of how intense or focused the light became. Light was then considered a wave, and if this effect depended on the energy of light, surely more light meant intensity, more energy, and more electrons. Empirically, this was not the case, an immensely foreboding development for the consideration of light as purely a wave. In 1905, Albert Einstein came to the conclusion that the light acted not as a wave, but instead as a beam of particles with a discrete energy based on the frequency of light, and that the energy of each individual particle (or photon) excited (or failed to excite) the electrons in the metal. This momentous discovery opened the door to the particle-like behavior of light and won Einstein the Nobel Prize in 1821 [1].

The goal of this experiment was to exploit the effect to determine Planck's constant, which links the amount of energy held by a photon to the frequency of the photon. By finding the stopping voltage necessary to prevent electrons excited by the effect from flying off the metal, we can find the maximum kinetic energy imparted to the electrons by any given wavelength of light. We then equated this stopping voltage to the kinetic energy of the electrons to perform linear regression, with the constant $\frac{h}{e}$ as the slope, where e is the elementary charge of the electrons. The frequency nu is treated as the x-parameter, so the experiment required different colors of light. Finding the stopping voltage of each color of light also required linear regression. Our final result was of $1.0 \pm 2.4 \times 10^{-34}$ J s.

Theory

The theoretical derivation of the photoelectric effect mostly arises from empirical observations made by Heinrich Hertz and later quantified by Einstein's work. The physical explanation is simple: a light packet or photon strikes the freely moving electrons on the surface of a metal, and imparts kinetic energy to electrons, potentially ejecting them from the surface. The photoelectric effect is described by the equation below, first formulated by Einstein.

$$E_k \leq E_p - \Phi \ (1)$$

The kinetic energy of an ejected electron E_k is less than or equal to the energy of the photon E_p minus the work function Φ of the metal in question. The work function is the measure of how tightly the metal holds onto electrons. More precisely, this work function is equal to the difference in potential energy near the surface of the metal minus the Fermi level [2]. The Fermi level is essentially the potential energy of the electron when it is bonded to the metal. The work function varies from metal to metal, but are typically of order unity electron-volts.

Note also that the work function represents the minimum binding energy of the electrons, and other electrons may require more energy to fully eject from the metal. This is the cause of the inequality in Equation 1, as the kinetic energy could be much less than the maximum possible energy, if the photon could still eject a more heavily bound electron. However, for the sake of this experiment, we are interested in the stopping voltage, where all the electrons are turned back, so for our purposes we can just as well use an equality, since any electrons of kinetic energy less than the maximum will naturally also stop.

We must also find energy of the photon, which is given by the Planck-Einstein relation, namely

$$E_p = h\nu \ (2)$$

Where h is Planck's constant, and ν is the frequency of the of the light. This also follows from Einstein's landmark work, but originated as a stop-gap measure introduced by Planck to make sense of black-body radiation, then called Planck's Postulate [3]. However, this problem solving trick actually suggested ontological understanding of light as particles with quantized energy.

We have no way of directly measuring the kinetic energy of the electrons, and must instead equate the kinetic energy to voltage, which we can measure. The potential energy barrier created by a voltage difference follows from the first principles of electromagnetic field theory:

$$E_v = eV \ (3)$$

Where e is the elementary charge of an electron, and V is the voltage we measure or induce between the anode and cathode. Note that we only intend to use the magnitude of the elementary charge, as the stopping voltages in our trials are all positive values. All this represents is that we measured the voltage cathode to anode instead of anode to cathode, effectively "switching" the sign on the voltage.

By saying that the energy in Equation 3 is the kinetic energy of Equation 1, we find the stopping voltage, namely the voltage at which all electrons ejected by the effect can not overcome the potential difference between the anode and cathode, and thus can not enter the circuit to generate current. We also substitute Equation 2 instead of E_p, and replace the inequality with an equality as elaborated above, we find a new expression.

$$eV_s = h\nu - \Phi \ (4)$$

We only want V_s as the dependent part of our equation and linear regression, so we divide both sides by the elementary charge e, to get our final working equation.

$$V_s = \frac{h\nu}{e} - \frac{\Phi}{e} \ (5)$$

By plotting the stopping voltage as a function of the frequency of light and performing the linear regression, we can find the slope of the plot, which should represent the quantity $\frac{h}{e}$. By multiplying our slope by e we then find Planck's constant.

Procedure

The experiment employed a mercury cadmium lamp to produce emission spectra, which could be separated into different colors by reflection grating. By angling the reflection grating, we induce rays of specific colors to strike a cathode and eject electrons. The setup is shown in more detail in Figure 1. We used four emission lines, corresponding to yellow, light green, darker green, and violet. The corresponding linear regressions of the current as a function of voltage were used to find the y-intercept, which

we will use as the stopping voltage V_s. Note also that the ambient light within the room was reduced as much as practical to prevent other light sources causing the photoelectric effect. An anode then picks up these electrons and allows them to generate a current in our circuit, to which we have connected both an ammeter and voltmeter. We also have a voltage control, to change the potential difference between the anode and cathode. By increasing the voltage between the anode and cathode, we can stop the movement of electrons from anode to cathode entirely. This is the stopping voltage we need for the final linear regression to determine h.

However, finding the stopping voltage of the electrons by hand is incredibly difficult, as the current in the circuit is incredibly small, and vulnerable to tiny changes from semi-random and difficult to control conditions like touching wires. In addition, it may not be possible to induce the voltage necessary to fully stop electrons. All of this necessitates the use of linear regression to find the actual stopping voltage. By measuring the current for any given voltage many times, we found that as the current approaches zero, the dependence of current on voltage became approximately linear. By fitting a trend line to this linear behavior, we hope to find a far more accurate representation of the stopping voltage in the y-intercept of the trend line.

It should be noted that locating this linear behavior required many trials to find graphically when the linear part began. In reality, the voltage will cause a plateau of current for higher values, followed by a power law dependence, followed by the linear behavior we want, followed by another plateau past the stopping voltage. Multiple trials were taken to simply find at which voltage current begins to decay linearly, and then trials were taken again to collect as many data points with linear behavior as was possible. Even then, many of our graphs still contained data points that obviously did not align with the linear decay we wanted to measure. These points were excluded from the linear fit by eye and estimation. If it did not look like linear behavior, we excluded it from the regression. Each color required its own regression, so Figures 2, 3, 4, and 5 correspond to yellow, light green, dark green, and violet bands respectively.

With these stopping voltages, we can used the spectral emission lines to match the color we used to its emission wavelength, which in turn can be used to find the frequency of the light and perform the final regression, represented by Figure 6. In general, matching the color of light you see to the wavelength of light on a spectra can be a tricky business, as even spectra with labelled emission lines can fall victim to personal preference of what constitutes "light" or "dark" green, or indeed yellow instead of orange. However, in the case of the mercury cadmium lamp, the emission lines are so far apart and comparatively sparse that no such confusion can truly arise. Only two green lines exist in any case, and there is only one line that could be called yellow or orange.
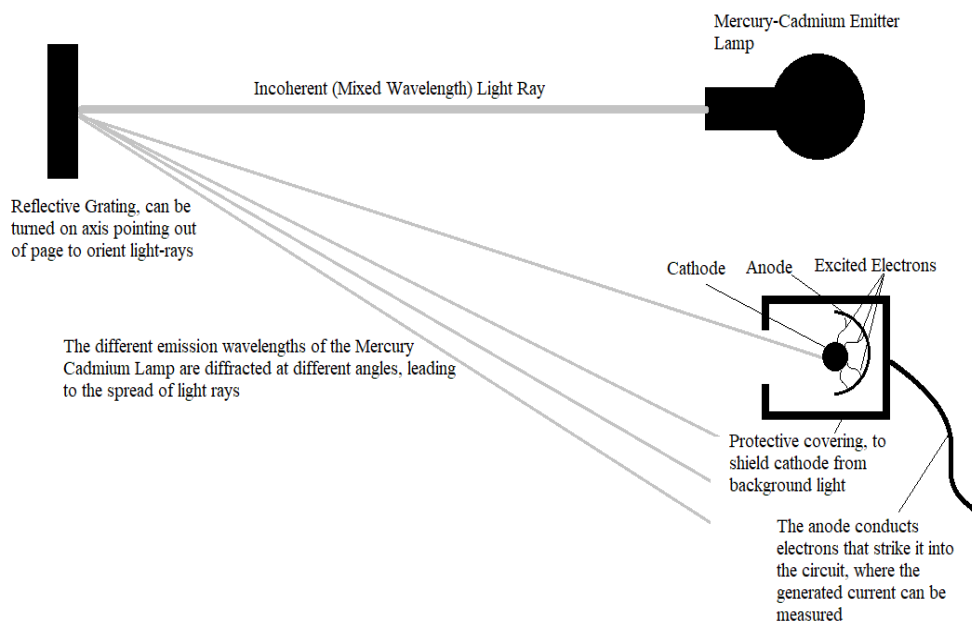
Figure 1: The top-down view of our experimental setup. By inducing the light to strike the reflective grating, we can separate the incoherent source into several singe-colored rays of light that are deflected at different angles. We then aim the deflected light into the anode to induce the photoelectric effect.
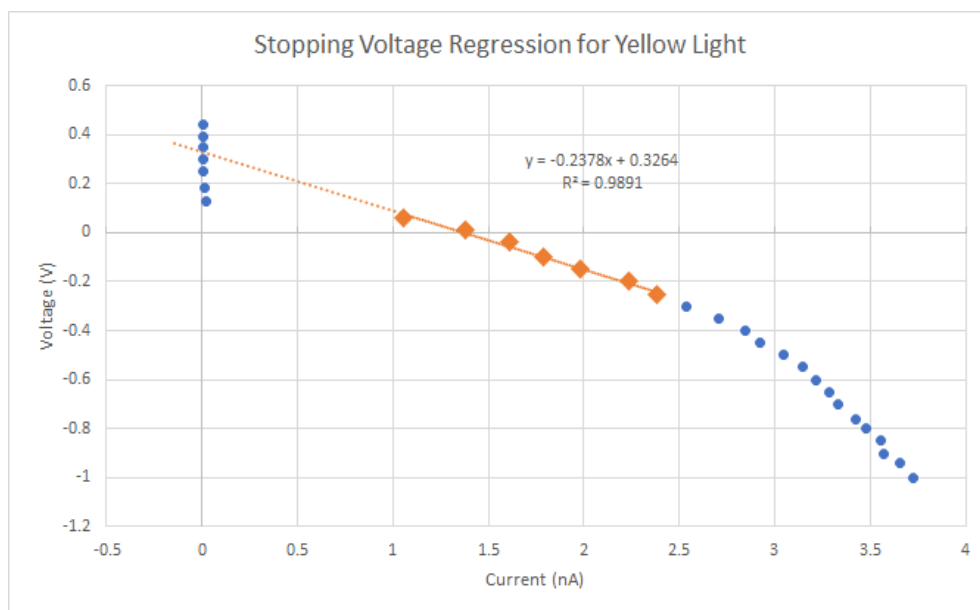
Data Analysis

Figure 2: The current vs voltage graph for yellow light, which we used to find the stopping voltage of electrons excited by the light. This light has wavelength $\lambda = 578$ nm and frequency $\nu = 519$ terahertz. Error bars are excluded from the graph, as they are too small to see, but the error in each voltage measurement in $\pm$ 0.01 V, and $\pm$ 0.01 nA for current. The diamond shaped points were those used in the regression. The other points were excluded, as they did not exhibit the linear behavior we wanted.
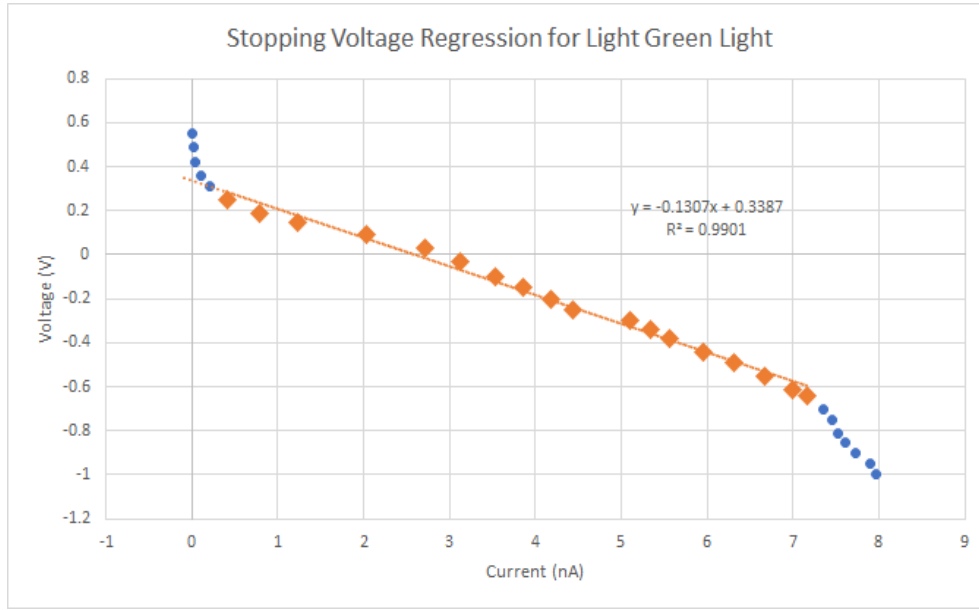
Figure 3: The current vs voltage graph for light green light, which we used to find the stopping voltage of electrons excited by the light. This light has λ = 546 nm and ν = 549 terahertz. Error bars are excluded from the graph, as they are too small to see, but the error in each voltage measurement in ± 0.01 V, and ± 0.01 nA for current. The diamond shaped points were those used in the regression. The other points were excluded, as they did not exhibit the linear behavior we wanted.
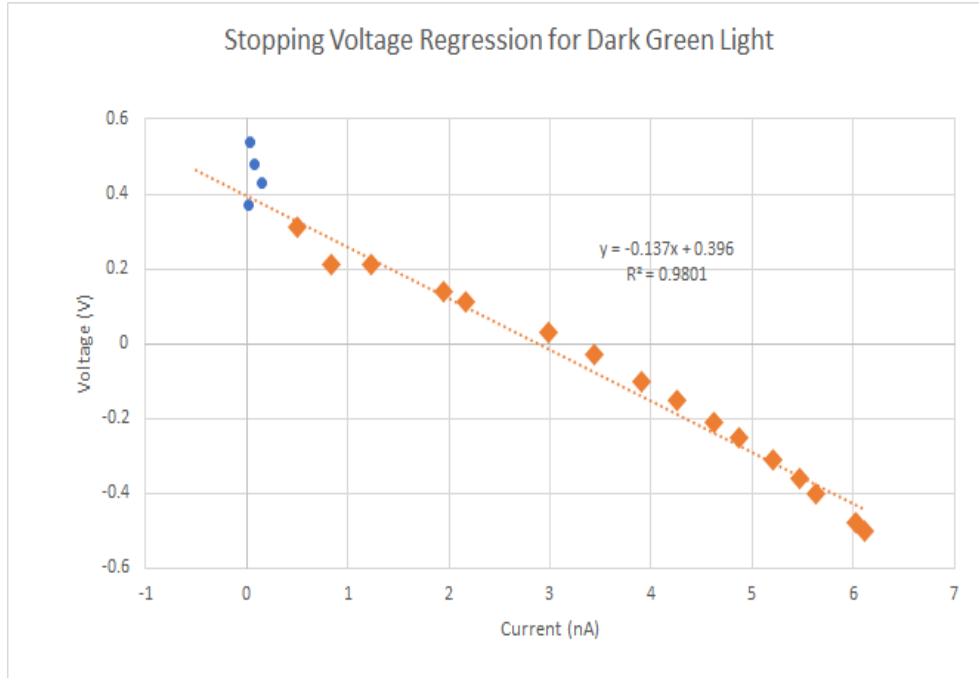

Figure 4: The current vs voltage graph for dark (blueish) green light, which we used to find the stopping voltage of electrons excited by the light. This light has λ = 508.5 nm and ν = 590 terahertz. Error bars are excluded from the graph, as they are too small to see, but the error in each voltage measurement in ± 0.01 V, and ± 0.01 nA for current. The diamond shaped points were those used in the regression. The other points were excluded, as they did not exhibit the linear behavior we wanted.
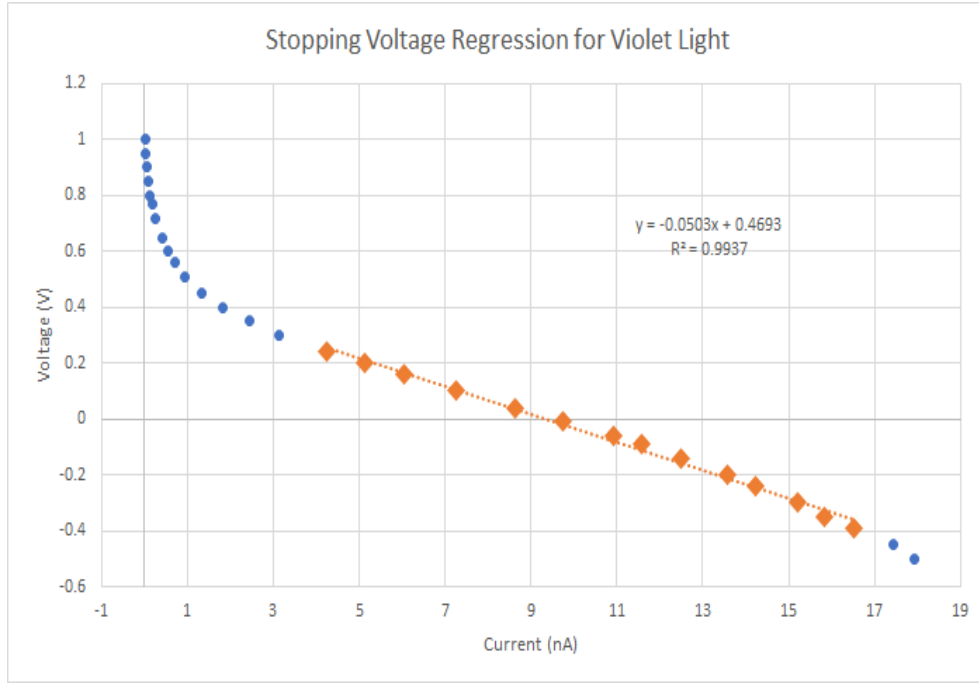
Figure 5: The current vs voltage graph for violet light, which we used to find the stopping voltage of electrons excited by the light. This light has λ = 404.7 nm and ν = 741 terahertz. Error bars are excluded from the graph, as they are too small to see, but the error in each voltage measurement in ± 0.01 V, and ± 0.01 nA for current. The diamond shaped points were those used in the regression. The other points were excluded, as they did not exhibit the linear behavior we wanted.
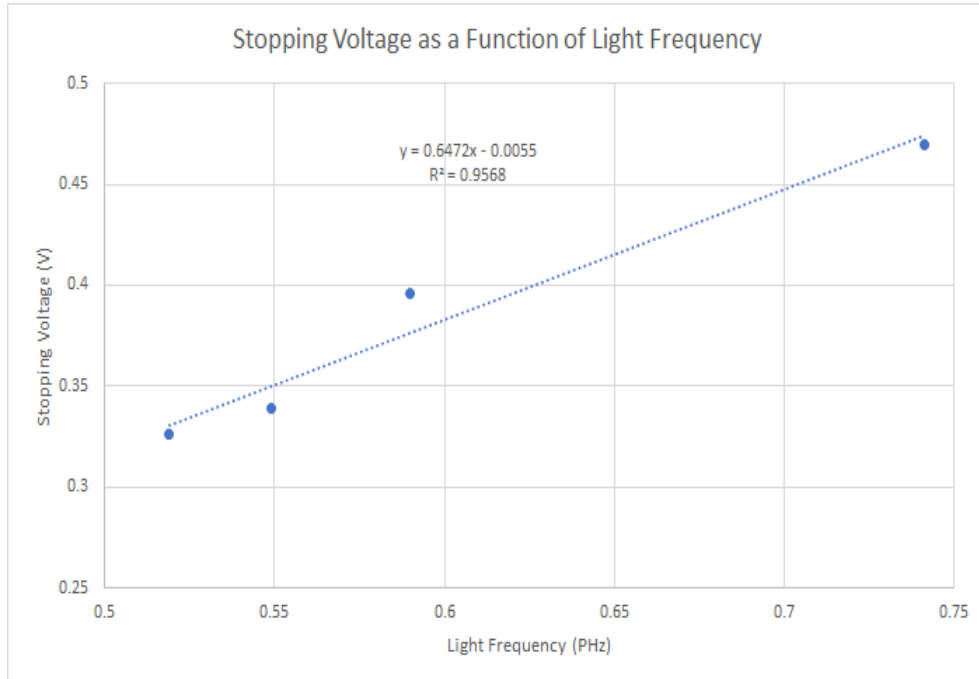

Figure 6: The final regression of stopping voltage as a function of the frequency of light, used to acquire the slope $\frac{h}{e}$ which we then multiply by e to acquire Planck's constant. The error uncertainty in each of these voltages is replicated in Table 2, as they were far too small to appear on this figure.
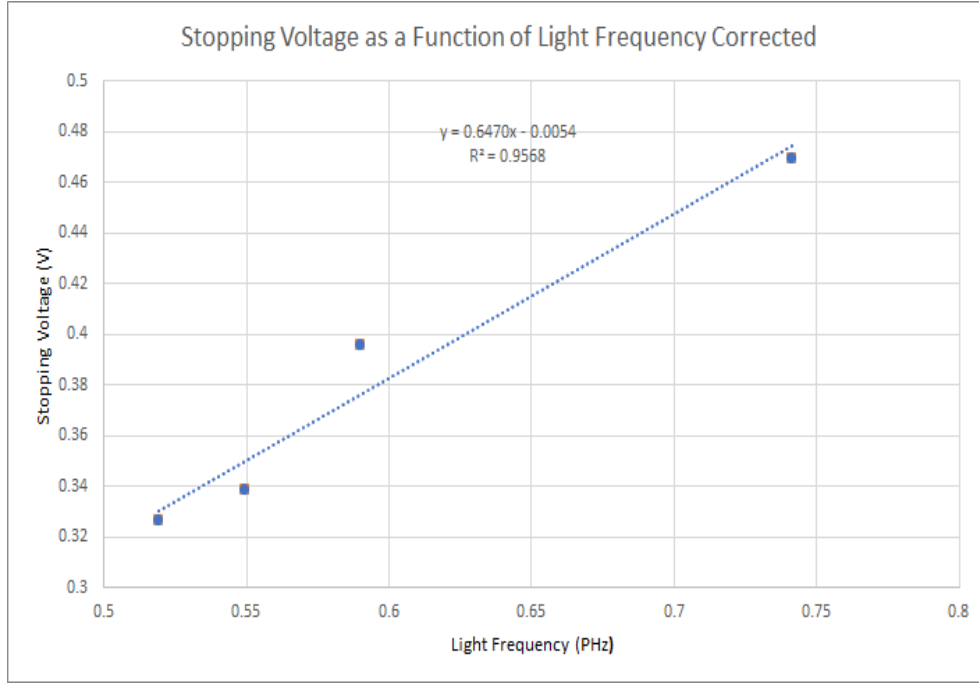
Figure 7: The above regression of stopping voltage as a function of the frequency of light, but with the correction for dark current. The corrected slope differs only by 0.0002 $\frac{V}{PHz}$.

Voltage (V)	Dark Current (nA)
-4.87	0.0008
-3.86	0.0006
-2.98	0.0006
-1.93	0.0005
-0.93	0.0004
-0.04	0.0003
0.94	0.0002
1.90	0.0001
2.88	0.0001
3.87	0.0000
4.94	-0.0001

Table 1: These are our measurements of residual current in the circuit with approximately no photo-electrons moving through the circuit. This table represents a small subset of the data collected, as most of said data was not used. The value we did use is included, as most of our trials had voltages ranging between -1 and 1 volts.

Color	Stopping Voltage (V)	σ_{V_s} (V)
Yellow	0.3264	0.0158
Light Green	0.3387	0.0053
Dark Green	0.3960	0.0056
Violet	0.4693	0.078

Table 2: These are the uncertainties of the stopping voltage, acquired via linear regression error analysis. They were too small to appear on the actual figure of said data, and so are reproduced here with the corresponding stopping voltage for ease of use.

Error Analysis

The only uncertainty is measurements was taken from the voltmeter and picoammeter we used. The wavelengths of light are provided by the emission spectra posted within the work area, and the value of the elementary charge and speed of light are so well known that their contribution to our random error are negligible. We watched for fluctuations in the reading we saw on the various devices, and said that the measurement was uncertain in that digit by ± 1. We found that the hundredths digit for tended to shift randomly for both of our detectors, so the uncertainty for voltage was 0.01 volts, and the uncertainty for current was 0.01 nanoamps, since the read-off was in nanoamps. This also made our error constant across each voltage and current measurements. Note that the methods of linear regression we learned only makes use of the error in the y-values used in linear regression, in our case the latter. This constant error will lead to different values in the errors of each y-intercept or stopping voltage in each color of light. These variable errors must in turn be propagated through the final linear regression of stopping voltage V_s. light frequency to get the final error. The uncertainty in our final answer thus entirely derives from the error of the picoammeter measurement, and propagates through linear regression. This error is too small to be expressed in the relevant figures, and is instead seen in Table 2.

However, there is a source of error that is not represented in the quoted uncertainty. This is the error that arises from our random selection of points that are or are not part of the linear regression. This selection involves a lot of guesswork on our part, and by nature can not be represented in the final uncertainty quoted by the measurement. This falls

under the purview of random error, since whether we include or exclude too many points on each regression would randomly tend one way or the other, but nevertheless I find it troubling enough to mention here.

Discussion

The final measured value we have for Planck constant h is $1.0 \pm 2.4 \times 10^{-34}$ Js while the accepted value is 6.6×10^{-34} Js. This means or apparently quite precise value is also rather inaccurate, as there is a 3σ difference between the accepted and measured value. However, for a constant this small, using methods this imprecise, getting within the correct order of magnitude is not terrible.

The main source of systematic error is the dark current present through our circuit. The source of dark current is electrons randomly generated in the circuit elements such as the photo-emitter as well as the wiring itself, as a result of impurities in the components of the circuit and ambient heat. The presence of dark current is a persistent issue in higher range optics where it causes electrons to fill CCD potential wells [4], and in our case induces electrons to move through our circuit. The strength of dark current appears to depend on temperature of the circuit and the various electron producing circuit elements [4]. Considering most of our circuit was room temperature or hotter (the light source was too hot to touch after two hours of trials) we can safely say that we're going to have some non-negligible dark current. Photon detectors and CCDs are reguarly surrounded in liquid nitrogen to cool them in an attempt to prevent dark current [4]. Also, just because the room we are in is dark to our eyes does not necessarily mean there is no ambient light, though we don't know where else it could have come from.

To try and understand and combat the effect of dark current, we took many trials of measuring the current with the light source and ambient light sources on. Just in case, we left the slit on photoelectric emitter open, in case any ambient light was causing electron ejection. The results of these trials are replicated in Table 1. However, the results there represent dark current over the full range of voltages the voltage control could induce, while we were only interested in voltage ranging from -1 to 1 volt. So, by using the single data point of dark current taken around zero we have a dark current of 3.0×10^{-4} nA. This means that every current we measured was approximately 3.0×10^{-4} nA higher than it should have been. So to correct this, we simply shift the trend lines of our graphs to the left by this amount, as the slope should be unaffected if the change is uniform, as we assume. The stopping voltage graphs have negative slope, so this would result in a decrease of the actual stopping voltage when compared to the measured stopping voltage. The new shift can be found by simply adding the slope times the shift to the old y-intercept to find our corrected y-intercept. Then, we perform linear regression on these new values (represented in Figure 7) and perform the regression again.

However, performing this regression does not meaningfully change the slope of our results. Indeed, it appears the resulting graph actually has a negative slope. It certainly makes physical sense that the inclusion of dark current would reduce our measured value of h, as h is the amount of energy per unit frequency of light, and we are effectively removing some energy per frequency by cutting out the dark current. However, our value was already too small compared to the accepted value of h. Thus, we can safely conclude that dark current was not the primary cause of the divergence between our answer and the correct one, but this does not mean it was not our main source of systematic error.The divergence is more likely a result of the random error, primarily the aforementioned "eye-balling" of the linear error. It is also possible that the dark current we measured is not the dark current propagating in the circuit during the actual experiment. As mentioned, the light source produces large amounts of heat, which may have increased the temperature and dark current in the circuit, despite not being directly connected to it. During our dark current trials, we elected to keep this lamp off, but in retrospect we should have simply shutter the light source while keeping it on, to ensure we accounted for ambient heat.

Conclusion

The experiment as written is quite straightforward, but the execution required patience and many, many trials. There are not many different sources of measurable error in the experiment, so we can't rely on finesse or skill in the experiment. We just have to rely on a bulk of data to reduce error. In the end we only managed to find the stopping voltage for only 4 different colors of light. This is sample size is just too small to ensure the random error from our random selection of linear and non-linear points cancels out. If even one of our measurements was significantly off from the actual stopping voltage, the lack of samples means this would skew our measurement.

So we need more colors, but we run into another issue: we only have two other colors to use with the Mercury Cadmium lamp. One of those is a dark red, and we learned from a teaching assistant that the photo-emitter will not respond to red light. That leaves a blue color that we did not use, and the difference between 5 points and 4 is not that great, as messing up our fifth measurement would probably hurt more than help. However, we also noticed that the lamp emits at ultraviolet frequencies. Obviously, we can't see these beams, but we can sweep the reflective grating back and forth and watch for the response in the circuits current. When it spikes up, we can be sure that we have aligned a beam of ultraviolet light and go from there. We could also mark the angle of that particular ray of ultraviolet light, since there are many of them, and then move onto the next. With the addition of a blue light trial, that could effectively double the amount of data points and lead to a far better fit.

In lieu of a more appropriate sections for this concern, the

meaning of the y-intercept on Figure 6, showing the stopping voltage as a function of frequency. It may seem obvious from the Theory section and Equation 5 in particular that the y-intercept of this graph is none other the work function divided by the elementary charge e. However, upon performing the multiplication and unit conversion necessary, we find that the work function of our metal is 10^{-15} eV. We know that work function of most metals varies between 2 and 5 eV[5], so this result is absolutely ludicrous. The discrepancy arises from the difference between what theory predicts or y-intercept should be and what our y-intercept actually represents. Namely, in the regression itself, the y-intercept is nothing more than the stopping voltage required to prevent electrons excited by 0 hertz light from striking the anode, which naturally would be zero volts, as null frequency light would not excite electrons in any case. Viewed in this light, our result makes sense, and provides an excellent example of the care that one must take in interpreting the results of linear regressions. If one wished it, an estimate of the work function could be acquired by simply plugging in the stopping voltage and light frequency for a given point into Equation 5.

Citations

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