THE UNIVERSITY CLASS SCHEDULING PROBLEM

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ABSTRACT

In this thesis, we investigate different models of the University Class Scheduling Problem (UCSP) and their corresponding solutions. We construct a base model using linear programming based upon [1], along with extensions that build up to a model that resembles the environment of Cal Poly Pomona (CPP). Then, with the collection of our models, we consider examples using simulated data to illustrate their behavior. Finally, we use real-world data obtained from CPP's Mathematics and Statistics Department with our CPP model to obtain an optimal solution to the modeled problem.

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Chapter 1

Introduction

University enrollment has been increasing over the last decade. From 2000 to 2017, the United States has experienced a 27% increase in total undergraduate enrollment in postsecondary institutions [2]. In the next decade, this enrollment is projected to increase by another 3%. This increase has placed much stress on colleges and universities in terms of opening more classes for students. With the growing number of college students, we encounter situations such as students being held back from graduating on time because the courses they need to take have reached maximum capacity, or the classes are at inconvenient time slots. With a set time schedule and an increase in classes, universities face a daunting problem of scheduling classes at appropriate time slots that benefit both students and faculty.

Assignment and timetable optimization problems schedule a fixed number of individuals to a fixed number of resources. These problems are equipped with a set of constraints in which the optimal scheduling must be satisfied. The University Class Scheduling Problem (UCSP) is one of the many timetable optimization problems. The UCSP has been tackled by researchers given the current situations universities face. The challenges that the UCSP faces are formulated as constraints, for example, instructors' preferences, educational policies, and physical resources that extend beyond the students. This presents a challenge for optimizing the physical and human resources. In the UCSP, each assignment is of a specific instructor to a course they will teach, in a specific classroom, in a specific time module. Recently, the UCSP has been investigated using many different meta-heuristic algorithms such as the Genetic Algorithm [3] [4], Tabu Search [5], and Simulated Annealing [6]. Other popular algorithms tackle the UCSP using swarm intelligence, such as Ant Colony Optimization [7], Honey Bee Optimization [8], and Particle Swarm Optimization [7] [9] [10].

The UCSP can be formulated as a linear programming (LP) problem [11] and thus solved using cut generation for an LP relaxation algorithm. The relaxation of an LP problem allows for less restrictive constraints, such as removing integrality. The algorithm requires cuts which add linear inequality constraints to the problem. The addition of these cut constraints is to reduce the feasible region for the LP relaxation problem without eliminating any feasible solutions for the problem. These inequalities attempt to restrict the feasible region of the LP relaxations so that the solution is closer to integers. Hillier and Lieberman [12] describes LP relaxation and cutting planes more in depth. Various previous literature has considered the UCSP using linear programming. In [1], the formulation of their model shows their university having an additional constraint where the capacity of each classroom cannot be exceed. In [6], their model considers having different departments interchanging classrooms. In other words, each department has a designated number of classrooms for assigning courses, but a different department is also able to use those classrooms. In [3], their model considers optional lectures for students, and if the optional lectures are in conflict with regular lectures, then there is a penalty imposed on the objective function. In this thesis, following [1], we start with the base case of ensuring courses can be assigned to specific classrooms at specific time modules. We then consider extensions, including adding instructor preferences and ratings of particular courses they want to teach.

The remainder of this thesis is organized as follows. In Chapter 2, we describe the environment in which our models are set, and introduce a basic model and various extensions to the UCSP at Cal Poly Pomona. In Chapter 3, we simulate data and execute the various models described in Chapter 2 to illustrate their behavior. In Chapter 4 we apply our models to real-world data for the Mathematics and Statistics Department at Cal Poly Pomona. In Chapter 5, we summarize our findings and discuss possible extensions and future work.

Chapter 2

Mathematical Models

2.1 A Depiction of the Mixed Integer Linear Programming Problem

Each UCSP problem is subjective to the university's/college's environment. The circumstances of class scheduling structures how the constraints are contrived. In this thesis we consider the problem of assigning course-sections to specific classrooms and time modules. In this chapter, we present 7 different models using linear programming, with Model 1 becoming the foundation of our later models. Model 1: Basic Model is modeled after [1] where binary decision variables are used to indicate if a course-section is assigned to a classroom and time module. Furthermore, we use constraints that ensure each course-section must have an assignment and each classroom can fit up to one course-section at a time, as described in [1]. The objective of [1] is to optimize the room utilization relative to room capacity across all course-sections. This causes the linear programming to avoid situations where large enrollment course-sections are assigned to small classrooms, and vice versa. Our base model differs from [1] by imposing a different objective, which is to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F time modules vs. the number assigned to T/TH time modules. Our later models explore additional objective functions and incorporate the selection of a teacher as part of the assignment.

The sets of all course-sections, classrooms, time modules, and instructors are defined as $\mathcal{I}$, $\mathcal{J}$, $\mathcal{K}$, and $\mathcal{T}$ respectively. Because Cal Poly Pomona offers both an Applied Mathematics/Statistics and a Pure Math/Secondary Teaching option, major course-sections can be considered as either applied or pure, so we define $\mathcal{I}_{appl} \subset \mathcal{I}$ for applied math course-sections and $\mathcal{I}_{pure} \subset \mathcal{I}$ for pure math course-sections. Furthermore, we define $\mathcal{I}_3 \subset \mathcal{I}$ and $\mathcal{I}_4 \subset \mathcal{I}$ as the sets of 3-unit course-sections and 4-unit course-sections.

For the classrooms, we define two subsets since a classroom either contains a chalkboard or a whiteboard. In particular, let $\mathcal{J}_c \subset \mathcal{J}$ be the set of classrooms with a chalkboard and let $\mathcal{J}_w \subset \mathcal{J}$ be the set of classrooms with a whiteboard.

The time modules are set up similarly where we can separate the modules by number of class units, day, similar time modules, and time of day. Let $\mathcal{K}_3 \subset \mathcal{K}$ and $\mathcal{K}_4 \subset \mathcal{K}$ be the sets of time modules for 3- and 4-unit course-sections. Sets of Monday/Wednesday/Friday (M/W/F) and Tuesday/Thursday (T/TH) time modules are defined as $\mathcal{K}_{mwf} \subset \mathcal{K}$ and $\mathcal{K}_{tth} \subset \mathcal{K}$, respectively. Note that at Cal Poly Pomona there are also classes that are held on Monday/Wednesday, Monday/Friday, and Wednesday/Friday. We define sets of these time modules as $\mathcal{K}_{mw}$, $\mathcal{K}_{mf}$, and $\mathcal{K}_{wf}$, respectively. Then, for example, a Monday/Wednesday at 10am course-section conflicts with Monday/Friday, Wednesday/Friday, and Monday/Wednesday/Friday course-sections at 10am; we call such time modules similar. Thus we define the set of time modules $\mathcal{K}_{similar}(k) \subset \mathcal{K}$ as the set of time modules that are similar in the time of day to a given $k \in \mathcal{K}$. We let morning time modules be those occurring from 7:00am to 11:59am, afternoon time modules from 12:00pm to 4:59pm, and evening classes occur from 5:00pm to 9:00pm. Thus we define $\mathcal{K}_{mo} \subset \mathcal{K}$, $\mathcal{K}_{af} \subset \mathcal{K}$, and $\mathcal{K}_{ev} \subset \mathcal{K}$ for morning, afternoon, and evening time modules, respectively.

When extending our model to include various teacher preferences, we wish to similarly divide the teachers into subsets. For teachers that want to teach pure course-sections or applied course-sections, we define $\mathcal{T}_{pure} \subset \mathcal{T}$ and $\mathcal{T}_{appl} \subset \mathcal{T}$, respectively. A teacher is able to select whether they want to teach on a M/W, W/F, M/F, M/W/F or a T/TH schedule. Thus, we define $\mathcal{T}_{mw} \subset \mathcal{T}$,$\mathcal{T}_{wf} \subset \mathcal{T}$,$\mathcal{T}_{mf} \subset \mathcal{T}$,$\mathcal{T}_{mwf} \subset \mathcal{T}$, and $\mathcal{T}_{tth} \subset \mathcal{T}$, respectively. Furthermore, the choice of a whiteboard or a chalkboard is considered as a teachers preference. Thus we let $\mathcal{T}_w \subset \mathcal{T}$ be the set of all teachers wanting to teach in a whiteboard classroom and $\mathcal{T}_c \subset \mathcal{T}$ be the set of all teachers wanting to teach in a chalkboard classroom. Lastly, we include the selection of teaching at a certain time of the day. Thus we let $\mathcal{T}_{mo} \subset \mathcal{T}$, $\mathcal{T}_{af} \subset \mathcal{T}$, and $\mathcal{T}_{ev} \subset \mathcal{T}$ be the set of teachers wanting to teach in the morning, afternoon, or evening, respectively. We also define $I = |\mathcal{I}|$, $J = |\mathcal{J}|$, $K = |\mathcal{K}|$, and $T = |\mathcal{T}|$ as the total number of classes, classrooms, time modules, and teachers, respectively.

2.2 Linear Models

2.2.1 Model 1: Basic Model

Our base model depicts the Cal Poly Pomona system. We want to assign each course-section $i \in \mathcal{I}$ to a classroom $j \in \mathcal{J}$ at a time module $k \in \mathcal{K}$, with the goal of minimizing the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F time modules vs. the number assigned to T/TH time modules. Note that if the assignments are perfectly balanced, half of the course-sections $\left(\text{namely } \frac{I}{2}\right)$ would be assigned to M/W, W/F, M/F, or M/W/F and half to T/TH. The decision variables for our base model are:

$x_{ijk} \in \mathcal{X}'$: binary decision variable indicating if course-section i is assigned to classroom j at time module k

$W \in \mathbb{R}$: real number used to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F vs. T/TH

where

$$\mathcal{X}' = \{x_{ijk} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}\}.$$

Using these decision variables, we formulate the linear program for our base model as follows:

$$\min \quad W$$

subject to

$$\sum_{i \in \mathcal{I}} x_{ijk} \leq 1 \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{2.1}$$

$$\sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijk} = 1 \qquad \forall i \in \mathcal{I} \tag{2.2}$$

$$\sum_{i \in \mathcal{I}_3} \sum_{k \in \mathcal{K}_4} x_{ijk} = 0 \qquad \forall j \in \mathcal{J} \tag{2.3}$$

$$\sum_{i \in \mathcal{I}_4} \sum_{k \in \mathcal{K}_3} x_{ijk} = 0 \qquad \forall j \in \mathcal{J} \tag{2.4}$$

$$\sum_{i \in \mathcal{I}} \sum_{k' \in \mathcal{K}_{similar}(k)} x_{ijk'} \leq 1 \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{2.5}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mwf} \cup \mathcal{K}_{mw} \cup \mathcal{K}_{wf} \cup \mathcal{K}_{mf}} x_{ijk} - \frac{I}{2} \leq W \tag{2.6}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{tth}} x_{ijk} - \frac{I}{2} \leq W \tag{2.7}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} x_{ijk} = I \tag{2.8}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}. \tag{2.9}$$

In the objective function, we want to minimize W, which upper bounds the difference between the ideal value of $\frac{I}{2}$ and the number of course-sections assigned to M/W, W/F, M/F, or M/W/F time modules and the number assigned to T/TH time modules. Constraint (2.1) ensures that there is at most one course-section in a given classroom at a given time module. Constraint (2.2) ensures that each course-section must be assigned to some classroom at some time module. Constraint (2.3) ensures that no 3-unit course-section is assigned to a 4-unit time module. Similarly, constraint (2.4) ensures that no 4-unit course-section is assigned to a 3-unit time module. Constraint (2.5) ensures that if a course-section is scheduled in a given classroom at a given time module, then the given classroom cannot be used in a time module that is similar. Constraints (2.6) and (2.7) are used to ensure that W upper bounds the difference between the number of course-sections actually assigned to M/W, W/F, M/F, or M/W/F and T/TH time modules, respectively, and the ideal value of $\frac{I}{2}$. Note that if it is desirable to have a different fraction of courses assigned to M/W, W/F, M/F, or M/W/F vs. T/TH, the $\frac{I}{2}$ values in constraints (2.6) and (2.7) can be adjusted accordingly. Constraint (2.8) ensures that the total number of assigned course-sections equals the number of course-sections offered. Finally, constraint (2.9) ensures that each decision variable takes on a binary value.

2.2.2 Model 2: Basic Teacher Model

For this model and the following models, we extend the base model to not only assign each course-section to a given classroom and a given time module, but also to a given teacher. Using this foundation we consider a number of variants with different objectives and constraints. In our teacher models, a new subscript is introduced on appropriate decision variables to account for which teacher is assigned to teach each course-section. The decision variables for this initial, Basic Teacher Model are

$x_{ijkt} \in \mathcal{X}$: binary decision variable indicating if course-section i is assigned to classroom j at time module k with teacher t

$W \in \mathbb{R}$: real number used to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F vs. T/TH

where

$$\mathcal{X} = \{x_{ijkt} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}\}.$$

In our Basic Teacher Model, we simply wish to incorporate the assignment of a teacher to each course-section, in addition to a classroom and time module. In the Basic Model from Section 2.2.1, the constraints were necessary since they prevented physical properties from being broken, such as having two course-sections being assigned in a classroom at the same time. Since these restrictions still hold, the initial constraints for this teacher model are a rehash of constraints (2.1) - (2.9), updated with the new indexing that specifies the teacher. We retain the same objective function because we are interested in adding new decision variables that directly affect the constraints. Then our Basic Teacher Model is

$$\min \quad W$$

subject to

$$\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} x_{ijkt} \leq 1 \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{2.10}$$

$$\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} x_{ijkt} = 1 \qquad \forall i \in \mathcal{I} \tag{2.11}$$

$$\sum_{i\in\mathcal{I}_3}\sum_{k\in\mathcal{K}_4}\sum_{t\in\mathcal{T}} x_{ijkt} = 0 \qquad \forall j \in \mathcal{J} \tag{2.12}$$

$$\sum_{i\in\mathcal{I}_4}\sum_{k\in\mathcal{K}_3}\sum_{t\in\mathcal{T}} x_{ijkt} = 0 \qquad \forall j \in \mathcal{J} \tag{2.13}$$

$$\sum_{i\in\mathcal{I}}\sum_{k'\in\mathcal{K}_{similar}(k)}\sum_{t\in\mathcal{T}} x_{ijk't} \leq 1 \qquad \forall j \in \mathcal{J}, k \in \mathcal{K} \tag{2.14}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{mwf}\cup\mathcal{K}_{mw}\cup\mathcal{K}_{wf}\cup\mathcal{K}_{mf}}\sum_{t\in\mathcal{T}} x_{ijkt} - \frac{I}{2} \leq W \tag{2.15}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{tth}}\sum_{t\in\mathcal{T}} x_{ijkt} - \frac{I}{2} \leq W \tag{2.16}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} x_{ijkt} = I \tag{2.17}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k'\in\mathcal{K}_{similar}(k)} x_{ijkt} \leq 1 \qquad \forall k \in \mathcal{K}, t \in \mathcal{T} \tag{2.18}$$

$$x_{ijkt} \in \{0, 1\} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}. \tag{2.19}$$

Constraints (2.10) - (2.17) are the original constraints modified to also sum over the teachers. Constraint (2.18) is an added constraint ensuring that a teacher is not assigned more than one course-section in a given time module (or time modules similar to it). Constraint (2.19) ensures that each decision variable x_{ijkt} is a binary value such that a value of 1 indicates teacher t is assigned to teach course-section i in classroom j at time module k.

2.2.3 Model 3: Teacher Preference Model

In the Teacher Preference Model, we incorporate various preferences to be adhered to into the constraints. Some common teacher preferences are board preferences, class time, day of the week, teaching pure or applied classes, and minimum and maximum number of course-sections to teach. In this model we use the same objective function and decision variables as in the Basic Teacher Model.

These preferences are placed within a matrix, called a Teacher Preference Matrix (T^p), such that the rows represent the teachers and the columns represent the preferences. The first and second column of T^p are nonnegative integers for the minimum and maximum number of course-sections a teacher will teach, respectively. The third column describes the teacher preference of a classroom with a whiteboard, chalkboard, or no preference. More specifically, values T^p_{t3} of the third column are such that $T^p_{t3} \in \{0, 1, 2\}$ where 1 indicates the teacher prefers a whiteboard, 2 indicates the teacher prefers a chalkboard, and 0 indicates no preference. The fourth column of this matrix represents the time of day a teacher wants to teach, with entries T^p_{t4} such that $T^p_{t4} \in \{0, 1, 2, 3\}$ where 1 indicates morning time modules, 2 indicates afternoon time modules, 3 indicated evening time modules, and 0 indicates having no preference. The fifth column is the day of week teaching preference, where $T^p_{t5} \in \{0, 1, 2\}$, where 1 indicates a teacher prefers to teach in M/W, W/F, M/F, or M/W/F time modules, 2 indicates a teacher prefers to teach T/TH, and 0 indicates a teacher has no preference when he or she teaches. The sixth and final column we consider (additional preferences are easily added) also has values such that $T^p_{t6} \in \{0, 1, 2\}$, where 1 indicates that the teacher prefers teaching pure mathematics course-sections, 2 indicates that the teacher prefers teaching applied mathematics course-sections, and 0 indicates that the teacher has no preference in terms of what course-sections he or she teaches. Thus, the Teacher Preference Matrix can be used to define the sets in Table 2.1.

Table 2.1: Definition of sets of teachers with specific preferences.

$\mathcal{T}_w^p = \{t \in \mathcal{T} : T_{t3}^p = 1\}$	$\mathcal{T}_c^p = \{t \in \mathcal{T} : T_{t3}^p = 2\}$
$\mathcal{T}_{mo}^p = \{t \in \mathcal{T} : T_{t4}^p = 1\}$	$\mathcal{T}_{af}^p = \{t \in \mathcal{T} : T_{t4}^p = 2\}$
$\mathcal{T}_{ev}^p = \{t \in \mathcal{T} : T_{t4}^p = 3\}$	
$\mathcal{T}_{mwf}^p = \{t \in \mathcal{T} : T_{t5}^p = 1\}$	$\mathcal{T}_{tth}^p = \{t \in \mathcal{T} : T_{t5}^p = 2\}$
$\mathcal{T}_{pure}^p = \{t \in \mathcal{T} : T_{t6}^p = 1\}$	$\mathcal{T}_{appl}^p = \{t \in \mathcal{T} : T_{t6}^p = 2\}$

In addition to constraints (2.10) - (2.19) of the Basic Teacher Model, to satisfy the preferences we have additional constraints

$$-\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} x_{ijkt} \le -T_{t,1}^p \qquad \forall t \in \mathcal{T} \tag{2.20}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} x_{ijkt} \le T_{t,2}^p \qquad \forall t \in \mathcal{T} \tag{2.21}$$

$$\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}_w} x_{ijkt} = 0 \qquad \forall j \in \mathcal{J}_c \tag{2.22}$$

$$\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}_c} x_{ijkt} = 0 \qquad \forall j \in \mathcal{J}_w \tag{2.23}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{mo}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{af} \cup \mathcal{T}_{ev} \tag{2.24}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{af}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{mo} \cup \mathcal{T}_{ev} \tag{2.25}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\mathcal{K}_{ev}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{mo} \cup \mathcal{T}_{af} \tag{2.26}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{mwf}\cup\mathcal{K}_{mw}\cup\mathcal{K}_{wf}\cup\mathcal{K}_{mf}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{tth} \tag{2.27}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}_{tth}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{mwf} \tag{2.28}$$

$$\sum_{i \in \mathcal{I}_{pure}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{applied} \tag{2.29}$$

$$\sum_{i \in \mathcal{I}_{appl}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijkt} = 0 \qquad \forall t \in \mathcal{T}_{pure}. \tag{2.30}$$

Constraints (2.20) and (2.21) restrict each teacher to between their preferred minimum and maximum number of course-sections to teach, respectively. Constraints (2.22) and (2.23) ensure whiteboard-preferring teachers are not assigned to chalkboard rooms or vice versa. Constraint (2.24) ensures a teacher preferring afternoon or evening time modules is not assigned to a morning time module, and constraints (2.25) and (2.26) are similar for afternoon and evening time modules, respectively. Constraints (2.27) and (2.28) ensure teachers preferring T/TH are not assigned to M/W, W/F, M/F, or M/W/F time modules or vice versa. Finally, constraints (2.29) and (2.30) ensure teachers preferring applied course-sections are not assigned pure course-sections or vice versa.

2.2.4 Model 4: Teacher Difference Model

The objectives for the previous models were to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F vs. T/TH. The objective that we use here is to minimize the difference between the number of course-sections assigned to each teacher. In other words, we assume the ideal number of course-sections assigned to a teacher is the total number of course-sections to be assigned divided by the number of teachers. Thus we have an objective of evenly balancing the number of course-sections assigned to each teacher. We introduce new decision variables, $Q_t \in \mathcal{Q}$ such that Q_t measures the difference between the number of course-sections taught by teacher $t \in \mathcal{T}$ and the ideal number of course-sections they should teach, $\frac{I}{T}$. Thus, the decision variables for this model are

$x_{ijkt} \in \mathcal{X}$: binary decision variable indicating if course-section i is assigned to classroom j at time module k with teacher t

$Q_t \in \mathcal{Q}$: a real number used to minimize the difference between the number of course-sections assigned to teacher t and the number they would teach under balanced assignments between teachers

where

$$\mathcal{X} = \{x_{ijkt} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}\} \text{ and}$$

$$\mathcal{Q} = \{Q_t : t \in \mathcal{T}\}.$$

Then our new objective is

$$\min \quad \sum_{t \in \mathcal{T}} Q_t.$$

The Teacher Difference Model uses the same constraints as in the previous subsection except for those pertaining to W, so (2.10) - (2.14) and (2.17) - (2.30). The additional constraints that define our decision variables Q_t are:

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijkt} - \frac{I}{T} \leq Q_t \qquad \forall t \in \mathcal{T} \tag{2.31}$$

$$\frac{I}{T} - \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} x_{ijkt} \leq Q_t \qquad \forall t \in \mathcal{T}. \tag{2.32}$$

Constraints (2.31) and (2.32) ensure that Q_t upper bounds the absolute value of the difference between the total number of course-sections teacher t actually teaches and the ideal number they should teach.

2.2.5 Model 5: Teacher Satisfaction Model

The previous models all had objective functions that minimized a particular balance (e.g., between days of the week or between teachers). In this model we instead consider an objective function which aims to ensure that teachers are as satisfied as possible with the course-sections they are assigned.

To facilitate this objective, we introduce a Teacher Satisfaction Matrix (T^s) which is a $T \times I$ matrix such that each row represents a teacher and each column represents a course-section. Since we choose to minimize our objective function, the rating scale for course-section satisfaction is $0 - 5$, with 0 being the most desired and 5 being the least desired. For this model, we use the same decision variables as our Basic Teacher Model and we use constraints from our Basic Teacher Model (2.10) - (2.14) and (2.17) - (2.19), and Teacher Preference Model (2.20) - (2.28). We do not consider constraints (2.29) and (2.30) because we now use course-section-by-course-section preferences instead of simply pure vs. applied, and because these preferences are now accounted for in our objective function. Thus our objective is

$$\min \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} T^s_{ti}\, x_{ijkt}.$$

2.2.6 Model 6: Teacher Tricriteria Model

In the previous subsections, we have many different models with different objective functions. An alternate approach we are interested in is to use a combined objective function. The Teacher Tricriteria Model allows us to account for three objectives which we have explored before: the minimization between the assignment of M/W, W/F, M/F, or M/W/F vs. T/TH course-sections, the minimization of the difference between assigned and ideal teaching loads, and the maximization of teacher satisfaction. These three objectives can be weighed with nonnegative weight parameters ω_1, ω_2, and ω_3 such that $\omega_1 + \omega_2 + \omega_3 = 1$. Then our objective for the Teacher Tricriteria Model is

$$\min \quad \omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t \in \mathcal{T}} Q_t.$$

Notice that the second criteria of our tricriteria is multiplied by a factor of $\frac{1}{T}$. This factor allows the model to inherently scale the teacher preference portion of the objective function so that the weights are not skewed to this portion. The decision variables are the union of those in the Basic Teacher Model and the Teacher Difference Model, namely

$x_{ijkt} \in \mathcal{X}$: binary decision variable indicating if course-section i is assigned to classroom j at time module k with teacher t

$W \in \mathbb{R}$: real number used to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F vs. T/TH

$Q_t \in \mathcal{Q}$: a real number used to minimize the difference between the number of course-sections assigned to teacher t and the number they would teach under balanced assignments between teachers

where

$$\mathcal{X} = \{x_{ijkt} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}\} \text{ and}$$

$$\mathcal{Q} = \{Q_t : t \in \mathcal{T}\}.$$

As for the constraints, we have the Basic Teacher Model constraints (2.10) - (2.19), the Teacher Preference Model constraints (2.20) - (2.28), and the Teacher Difference Model constraints (2.31) - (2.32).

2.2.7 An Application to Unit-Based Preferences

In the previous models, we allowed our teachers to have a preferred minimum and maximum number of course-sections to teach. In other settings, teachers may have a minimum and maximum number of units to teach. For example, at Cal Poly Pomona, members of the lecturer pool have such requirements. Thus, we now introduce a new set of of constraints for the minimum and maximum number of units a teacher can be assigned. In order to keep track of the number of units for each class, we introduce a function, $I^u(i) = q$, such that course-section i is a q-unit course-section. For example, $I^u(2) = 3$ identifies course-section 2 as a 3-unit course-section. To accommodate this change, columns one and two of the Teacher Preference Matrix can be reformatted to account for the minimum and maximum number of units, instead of the minimum and maximum of course-sections. Thus, our new constraints become

$$-\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} x_{ijkt}\cdot I^u(i) \le -T^p_{t1} \qquad \forall t \in \mathcal{T} \tag{2.33}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}} x_{ijkt}\cdot I^u(i) \le T^p_{t2} \qquad \forall t \in \mathcal{T}. \tag{2.34}$$

Constraints (2.33) and (2.34) ensure that the number of units assigned to each teacher is between the specified minimum and maximum, and replaces constraints (2.20) and (2.21). This replacement of constraints is applicable to Models 3, 4, 5, 6, and 7, where the objective functions and other constraints need not be changed.

2.2.8 Model 7: Cal Poly Pomona Model

Setup and Decision Variables

In the Cal Poly Pomona setting, we begin with the the unit-based preferences of Section 2.2.7 applied to Model 6: Teacher Tricriteria Model from Section 2.2.6, which has decision variables

$x_{ijkt} \in \mathcal{X}$: binary decision variable indicating if course-section i is assigned to classroom j at time module k with teacher t

$W \in \mathbb{R}$: real number used to minimize the difference between the number of course-sections assigned to M/W, W/F, M/F, or M/W/F vs. T/TH

$Q_t \in \mathcal{Q}$: a real number used to minimize the difference between the number of course-sections assigned to teacher t and the number they would teach under balanced assignments between teachers

where

$$\mathcal{X} = \{x_{ijkt} : i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}, t \in \mathcal{T}\} and$$

$$\mathcal{Q} = \{Q_t : t \in \mathcal{T}\}.$$

For this model, we now extend Model 6 to include two new preference scales, namely for the days of the week and the time of day that each teacher prefers to teach. To do so, we will also introduce new decision variables

$D_t \in \mathcal{D}$: a real number used to measure the satisfaction in the days of the week teaching schedule preference for teacher t

$B_t \in \mathcal{B}$: a real number used to measure the satisfaction in the time of day teaching schedule preference for teacher t

where

$$\mathcal{D} = \{D_t : t \in \mathcal{T}\} \text{ and}$$

$$\mathcal{B} = \{B_t : t \in \mathcal{T}\}.$$

Days of the Week Preference

In this model, teachers may now indicate they prefer a more complicated teaching schedule than simply M/W, W/F, M/F, or M/W/F vs. T/TH. For example, a teacher may want to teach on a Monday/Tuesday/Wednesday/Thursday (M/T/W/TH) schedule. Rather than imposing constraints like (2.27) and (2.28) as was done in Model 6, this model handles these preferences using a scale, which is incorporated into the objective function, as was done for course-section preferences in Model 5 of Section 2.2.5. Thus, we have the additional goal of minimizing the teacher day preference. The combinations of days a teacher can rate are

- Monday/Wednesday,
- Wednesday/Friday,
- Monday/Friday,
- Monday/Wednesday/Friday,
- Tuesday/Thursday,
- Monday/Tuesday/Wednesday/Thursday,
- Tuesday/Wednesday/Thursday/Friday,
- Monday/Tuesday/Thursday/Friday, and
- Monday/Tuesday/Wednesday/Thursday/Friday.

This totals to 9 different combinations of day preferences, and each teacher can rate each combination on a scale of highest preference to lowest preference (0 - 3). To keep track of the teacher day preferences, we define another matrix T^d, called the Teacher Day Preference Matrix, which is a $T \times 9$ matrix, where each row of the matrix represents a teacher and the columns represent the 9 different teaching day combinations.

We would like to incorporate a measure of satisfaction with a teacher's teaching schedule in the objective function. Thus, for each teacher $t \in \mathcal{T}$, we define D_t, a measure of their satisfaction with their teaching schedule, as

$$\begin{aligned} D_t = T^d_{t,1}\, y_t^{mw} + T^d_{t,2}\, y_t^{wf} + T^d_{t,3}\, y_t^{mf} + T^d_{t,4}\, y_t^{mwf} + T^d_{t,5}\, y_t^{tth} + T^d_{t,6}\, y_t^{mtwth} + T^d_{t,7}\, y_t^{twthf} \\ + T^d_{t,8}\, y_t^{mtthf} + T^d_{t,9}\, y_t^{mtwthf} \qquad \forall t \in \mathcal{T} \end{aligned}$$

where y_t^{mw}, y_t^{wf}, y_t^{mf}, y_t^{mwf}, y_t^{tth}, y_t^{mtwth}, y_t^{twthf}, y_t^{mtthf}, and y_t^{mtwthf} are binary decision variables such that

$$y_t^{mw} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/W schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/W schedule} \end{cases}$$

$$y_t^{wf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a W/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a W/F schedule} \end{cases}$$

$$y_t^{mf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/F schedule} \end{cases}$$

$$y_t^{mwf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/W/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/W/F schedule} \end{cases}$$

$$y_t^{tth} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a T/TH schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a T/TH schedule} \end{cases}$$

$$y_t^{mtwth} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/T/W/TH schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/T/W/TH schedule} \end{cases}$$

$$y_t^{twthf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a T/W/TH/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a T/W/TH/F schedule} \end{cases}$$

$$y_t^{mtthf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/T/TH/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/T/TH/F schedule} \end{cases}$$

$$y_t^{mtwthf} = \begin{cases} 1, & \text{if teacher } t \text{ teaches on a M/T/W/TH/F schedule} \\ 0, & \text{if teacher } t \text{ does not teach on a M/T/W/TH/F schedule.} \end{cases}$$

Note that a given teacher t falls into only one teaching schedule, thus we require

$$y_t^{mw} + y_t^{wf} + y_t^{mf} + y_t^{mwf} + y_t^{tth} + y_t^{mtwth} + y_t^{twthf} + y_t^{mtthf} + y_t^{mtwthf} = 1 \quad \forall t \in \mathcal{T}. \tag{2.35}$$

To figure out to which teaching schedule each teacher is assigned, we can use additional binary decision variables $z_t^m, z_t^t, z_t^w, z_t^{th}$, and z_t^f that indicate whether or not they are teaching on that particular day. Thus, for Mondays, taking M as a large integer, we have the pair of constraints

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mw} \cup \mathcal{K}_{mf} \cup \mathcal{K}_{mwf}} x_{ijkt} \leq M z_t^m \qquad \forall t \in \mathcal{T} \tag{2.36}$$

$$z_t^m \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mw} \cup \mathcal{K}_{mf} \cup \mathcal{K}_{mwf}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.37}$$

where the first forces z_t^m to be 1 when necessary and the second forces z_t^m to be 0 when necessary. We have similar constraints for the rest of the weekdays, yielding constraints

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{tth}} x_{ijkt} \leq M z_t^t \qquad \forall t \in \mathcal{T} \tag{2.38}$$

$$z_t^t \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{tth}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.39}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mw} \cup \mathcal{K}_{wf} \cup \mathcal{K}_{mwf}} x_{ijkt} \leq M z_t^w \qquad \forall t \in \mathcal{T} \tag{2.40}$$

$$z_t^w \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mw} \cup \mathcal{K}_{wf} \cup \mathcal{K}_{mwf}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.41}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{tth}} x_{ijkt} \leq M z_t^{th} \qquad \forall t \in \mathcal{T} \tag{2.42}$$

$$z_t^{th} \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{tth}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.43}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mf} \cup \mathcal{K}_{wf} \cup \mathcal{K}_{mwf}} x_{ijkt} \leq M z_t^f \qquad \forall t \in \mathcal{T} \tag{2.44}$$

$$z_t^f \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mf} \cup \mathcal{K}_{wf} \cup \mathcal{K}_{mwf}} x_{ijkt} \qquad \forall t \in \mathcal{T}. \tag{2.45}$$

With these constraints, we now know for all teachers, on which single days they teach. We can now use the day of the week decision variables to define the binary decision variable that indicate into which teaching schedule each teacher falls using the constraints

$$z_t^m + (1 - z_t^t) + z_t^w + (1 - z_t^{th}) + (1 - z_t^f) - 4 \leq y_t^{mw} \qquad \forall t \in \mathcal{T} \tag{2.46}$$

$$(1 - z_t^m) + (1 - z_t^t) + z_t^w + (1 - z_t^{th}) + z_t^f - 4 \leq y_t^{wf} \qquad \forall t \in \mathcal{T} \tag{2.47}$$

$$z_t^m + (1 - z_t^t) + (1 - z_t^w) + (1 - z_t^{th}) + z_t^f - 4 \leq y_t^{mf} \qquad \forall t \in \mathcal{T} \tag{2.48}$$

$$z_t^m + (1 - z_t^t) + z_t^w + (1 - z_t^{th}) + z_t^f - 4 \leq y_t^{mwf} \qquad \forall t \in \mathcal{T} \tag{2.49}$$

$$(1 - z_t^m) + z_t^t + (1 - z_t^w) + z_t^{th} + (1 - z_t^f) - 4 \leq y_t^{tth} \qquad \forall t \in \mathcal{T} \tag{2.50}$$

$$z_t^m + z_t^t + z_t^w + z_t^{th} + (1 - z_t^f) - 4 \leq y_t^{mtwth} \qquad \forall t \in \mathcal{T} \qquad (2.51)$$

$$(1 - z_t^m) + z_t^t + z_t^w + z_t^{th} + z_t^f - 4 \leq y_t^{twthf} \qquad \forall t \in \mathcal{T} \qquad (2.52)$$

$$z_t^m + z_t^t + (1 - z_t^w) + z_t^{th} + z_t^f - 4 \leq y_t^{mtthf} \qquad \forall t \in \mathcal{T} \qquad (2.53)$$

$$z_t^m + z_t^t + z_t^w + z_t^{th} + z_t^f - 4 \leq y_t^{mtwthf} \qquad \forall t \in \mathcal{T}. \qquad (2.54)$$

Time of Day Preference

In initial experiments it was found that including teaching time preferences as hard constraints could lead to situations where no feasible solution exists. For example, this result can occur when every teacher has a desire to work in the same times as each other. Thus, in this model we remove constraints (2.24) through (2.26) where teachers must not teach in times they do not want, and instead have satisfaction values for the times they are able to teach, which can be incorporated into the objective function.

We define B_t to measure the satisfaction of a teacher t with their time of day scheduling as

$$B_t = T_{t,1}^b u_t^{mo} + T_{t,2}^b u_t^{af} + T_{t,3}^b u_t^{ev} + T_{t,4}^b u_t^{mo,af} + T_{t,5}^b u_t^{af,ev} + T_{t,6}^b u_t^{mo,ev} + T_{t,7}^b u_t^{mo,af,ev} \qquad \forall t \in \mathcal{T}$$

where T_t^b is called the Best Times Preference Matrix and is a matrix of size $T \times 7$, where each row represent a teacher, and the columns represent the time of day scheduling: mornings, afternoons, evenings, mornings/afternoons, afternoons/evenings, mornings/evenings, and mornings/afternoons/evenings. The decision variables u_t^{mo}, u_t^{af}, u_t^{ev}, $u_t^{mo,af}$, $u_t^{af,ev}$, $u_t^{mo,ev}$, and $u^{mo,af,ev}$ are binary variables such that

$$u_t^{mo} = \begin{cases} 1, & \text{if teacher } t \text{ has a morning teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have a morning teaching schedule} \end{cases}$$

$$u_t^{af} = \begin{cases} 1, & \text{if teacher } t \text{ has an afternoon teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have an afternoon teaching schedule} \end{cases}$$

$$u_t^{ev} = \begin{cases} 1, & \text{if teacher } t \text{ has an evening teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have an evening teaching schedule} \end{cases}$$

$$u_t^{mo,af} = \begin{cases} 1, & \text{if teacher } t \text{ has a morning/afternoon teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have a morning/afternoon teaching schedule} \end{cases}$$

$$u_t^{af,ev} = \begin{cases} 1, & \text{if teacher } t \text{ has an afternoon/evening teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have an afternoon/evening teaching schedule} \end{cases}$$

$$u_t^{mo,ev} = \begin{cases} 1, & \text{if teacher } t \text{ has a morning/evening teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have a morning/evening teaching schedule.} \end{cases}$$

$$u_t^{mo,af,ev} = \begin{cases} 1, & \text{if teacher } t \text{ has a morning/afternoon/evening teaching schedule} \\ 0, & \text{if teacher } t \text{ does not have a morning/afternoon/evening teaching schedule.} \end{cases}$$

Since each teacher must fall into exactly one time of day schedule, we have the constraint

$$u_t^{mo} + u_t^{af} + u_t^{ev} + u_t^{mo,af} + u_t^{af,ev} + u_t^{mo,ev} + u^{mo,af,ev} = 1 \qquad \forall t \in \mathcal{T}. \tag{2.55}$$

In order to see which time of day each teacher teaches in, we have the following constraints

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mo}} x_{ijkt} \leq M v_t^{mo} \qquad \forall t \in \mathcal{T} \tag{2.56}$$

$$v_t^{mo} \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{mo}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.57}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{af}} x_{ijkt} \leq M v_t^{af} \qquad \forall t \in \mathcal{T} \tag{2.58}$$

$$v_t^{af} \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{af}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.59}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{ev}} x_{ijkt} \leq M v_t^{ev} \qquad \forall t \in \mathcal{T} \tag{2.60}$$

$$v_t^{ev} \leq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}_{ev}} x_{ijkt} \qquad \forall t \in \mathcal{T} \tag{2.61}$$

where M is a very large number, and v_t^{mo}, v_t^{af}, and v_t^{ev} are binary decision variables such that

$$v_t^{mo} = \begin{cases} 1, & \text{if teacher } t \text{ teaches in the morning} \\ 0, & \text{if teacher } t \text{ does not teach in the morning} \end{cases}$$

$$v_t^{af} = \begin{cases} 1, & \text{if teacher } t \text{ teaches in the afternoon} \\ 0, & \text{if teacher } t \text{ does not teach in the afternoon} \end{cases}$$

$$v_t^{ev} = \begin{cases} 1, & \text{if teacher } t \text{ teaches in the evening} \\ 0, & \text{if teacher } t \text{ does not teach in the evening.} \end{cases}$$

Constraints (2.56), (2.58), and (2.60) mark the time of day in which teacher t is teaching and ensure the appropriate v_t^{mo}, v_t^{af}, and/or v_t^{ev} are set to 1, while constraints (2.57), (2.59), and (2.61) ensure that these binary variables are 0 when necessary.

The next set of constraints ensure that the appropriate u_t decision variable is set equal to 1 to indicate the correct time of day scheduling for teacher t:

$$v_t^{mo} + (1 - v_t^{af}) + (1 - v_t^{ev}) - 2 \leq u_t^{mo} \qquad \forall t \in \mathcal{T} \tag{2.62}$$

$$(1 - v_t^{mo}) + v_t^{af} + (1 - v_t^{ev}) - 2 \leq u_t^{af} \qquad \forall t \in \mathcal{T} \tag{2.63}$$

$$(1 - v_t^{mo}) + (1 - v_t^{af}) + v_t^{ev} - 2 \leq u_t^{ev} \qquad \forall t \in \mathcal{T} \tag{2.64}$$

$$v_t^{mo} + v_t^{af} + (1 - v_t^{ev}) - 2 \leq u_t^{mo,af} \qquad \forall t \in \mathcal{T} \tag{2.65}$$

$$(1 - v_t^{mo}) + v_t^{af} + v_t^{ev} - 2 \leq u_t^{af,ev} \qquad \forall t \in \mathcal{T} \tag{2.66}$$

$$v_t^{mo} + (1 - v_t^{af}) + v_t^{ev} - 2 \leq u_t^{mo,ev} \qquad \forall t \in \mathcal{T}. \tag{2.67}$$

$$v_t^{mo} + v_t^{af} + v_t^{ev} - 2 \leq u_t^{mo,af,ev} \qquad \forall t \in \mathcal{T}. \tag{2.68}$$

Final CPP Model

We build upon Model 6: Teacher Tricriteria by including D_t and B_t in the objective function to create a pentacriteria objective. For weights ω_i such that $\sum_{i=1}^{5} \omega_i = 1$, the objective becomes

$$\min \quad \omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} T_{ti}^{s} x_{ijkt} + \omega_3 \cdot \sum_{t \in \mathcal{T}} Q_t + \omega_4 \cdot \sum_{t \in \mathcal{T}} D_t + \omega_5 \cdot \sum_{t \in \mathcal{T}} B_t.$$

subject to Constraints (2.10) - (2.19), (2.22) - (2.23), (2.31) - (2.68), and binary condition constraints

$$y_t^{mw} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.69}$$

$$y_t^{wf} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.70}$$

$$y_t^{mf} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.71}$$

$$y_t^{mwf} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.72}$$

$$y_t^{tth} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.73}$$

$$y_t^{mtwth} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.74}$$

$$y_t^{twthf} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.75}$$

$$y_t^{mtwthf} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.76}$$

$$z_t^{m} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.77}$$

$$z_t^t \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.78}$$

$$z_t^w \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.79}$$

$$z_t^{th} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.80}$$

$$z_t^f \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.81}$$

$$u_t^{mo} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.82}$$

$$u_t^{af} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.83}$$

$$u_t^{ev} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.84}$$

$$u_t^{mo,af} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.85}$$

$$u_t^{af,ev} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.86}$$

$$u_t^{mo,ev} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.87}$$

$$u_t^{mo,af,ev} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.88}$$

$$v_t^{mo} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.89}$$

$$v_t^{af} \in \{0,1\} \qquad \forall t \in \mathcal{T} \tag{2.90}$$

$$v_t^{ev} \in \{0,1\} \qquad \forall t \in \mathcal{T}. \tag{2.91}$$

2.2.9 Application to a Year-Long Model

All of the previous models considered scheduling for a single term. Thus, an extension of these approaches is to apply our models to a full academic year. It is possible to create a year-long model, but doing so increases the number of decision variables, which impacts the computation times and memory usage. In particular, to model a full academic year we could double the number of time modules to incorporate two semesters. Thus, it is simpler to consider running the model twice, once for each semester, and format the number of course-sections in each semester accordingly. The objective would remain the same, where we can choose between the objective functions from Models 2 - 7.

2.3 Alternate Models

The models described in Section 2.2 represent models for Cal Poly Pomona's scheduling system yielding optimal solutions. For comparison purposes, we also proceed with the scheduling in two non-optimal, alternate fashions. In other words, each of these algorithms does not attempt to find an optimal solution, but only a feasible solution. The first model is an assignment using a naive method, and the second model uses a random method. Note that these models are based off of the Basic Teacher Model.

2.3.1 Model 8: Naive Method

The Naive Method for the first alternate model is to use an algorithm which assigns a course-section i to the first available teacher, classroom, and time module. This algorithm first assigns each course-section a classroom and teacher in an order starting with the first classroom and first teacher available, and increments to the next classroom and teacher available while maintaining the same time module. When no more teachers and/or classrooms are available in this time module, we proceed to the next available time module. If we run out of time modules, then this method fails to find a feasible solution. To account for the different time modules for 3-unit and 4-unit course-sections, we first run the second part of the algorithm for the 3-unit course-sections, then for the 4-unit course-sections (or vice versa). Pseudocode for this approach is given by Algorithm 1.

Algorithm 1: Naive Method

Set both CRcounter and TRcounter to 0;

for *each course-section* **do**

  increment CRcounter and TRcounter;

  if CRcounter *is bigger than number of classrooms* **then**

    set CRcounter to 1

  end

  if TRcounter *is bigger than number of teachers* **then**

    set TRcounter to 1

  end

  Assign CRcounter and TRcounter to current course-section

end

Set TMcounter to 1;

for *each course-section* **do**

  Let CSSameRoom be a vector of course-sections already assigned to the current course-sections' classroom;

  Let CSSameTeacher be a vector of course-sections already assigned to the current course-sections' teacher;

  if *length of* CSSameRoom *or* CSSameTeacher *is greater than 1* **then**

    Let TMSameRoom be a vector of the time modules similar to those assigned to the course-sections in CSSameRoom;

    Let TMSameTeaher be a vector of the time modules similar to those assigned to the course-sections in CSSameTeacher;

    while TMcounter *is similar to any in* TMSameRoom *or* TMcounter *is similar to any in* TMSameTeacher **do**

      Increment TMcounter by 1;

      if TMcounter *is bigger than number of time modules* **then**

        set TMcounter to 1

      end

    end

  end

  Assign TMcounter to the current course-section;

end

2.3.2 Model 9: Random Method

The Random Method is the second alternate model we consider. As is implied in the name of the method, the algorithm first selects a random teacher and classroom for each course-section. The algorithm then assigns a time module to each course-section by repeatedly generating random time modules until the one generated is feasible for the course-section's teacher and classroom. If the size of the union of the set of unavailable time modules associated with the classroom and set of unavailable time modules associated with the teacher equals the total number of time modules, then this method fails to find a feasible solution. Similarly to the Naive Method, we account for the different time modules for 3-unit and 4-unit course-sections by first running the second part of the algorithm for the 3-unit course-sections, then for the 4-unit course-sections (or vice versa). Pseudocode for this approach is given by Algorithm 2.

Algorithm 2: Random Method

```
for each course-section do
    Assign a random value of a classroom and teacher to the current course-section
end
for each course-section do
    Assign TimeModule a random time module;
    Let CSSameRoom be a vector of course-sections already assigned to the current
      course-sections' classroom;
    Let CSSameTeacher be a vector of course-sections already assigned to the current
      course-sections' teacher;
    if length of CSSameRoom or CSSameTeacher is greater than 1 then
        Let TMSameRoom be a vector of the time modules similar to those assigned
          to the course-sections in CSSameRoom;
        Let TMSameTeaher be a vector of the time modules similar to those assigned
          to the course-sections in CSSameTeacher;
        while TimeModule is similar to any in TMSameRoom or TimeModule is
          similar to any in TMSameTeacher do
            Assign TimeModule a random time module;
        end
    end
    Assign TimeModule to the current course-section;
end
```

Chapter 3

Experiments Using Simulated Data

With our models previously described in Chapter 2, we can now present a number of examples to demonstrate solutions for generated data. In particular, we present solutions from the models of Sections 2.2.1, 2.2.2, 2.2.3, 2.2.4, and 2.2.6 and, for comparison, the alternative models in Sections 2.3.1 and 2.3.2. We generate simulated data by first defining the number of course-sections $I = 29$, classrooms $J = 11$, time modules $K = 86$, and teachers $T = 10$, which is loosely modeled after data from a typical Cal Poly Pomona quarter in the Mathematics and Statistics Department.

A list of course-sections and classrooms can be found in Appendices A.1.1 and A.2, while lists of time modules can be found in Appendixes A.3.1 and A.3.2. We use course-sections $i \in \mathcal{I}_3 = \{1, 2, 3\}$ as the simulated 3-unit course-sections, while the 4-unit course-sections are $\mathcal{I}_4 = \mathcal{I}_3^c$. The chalkboard and whiteboard classrooms we use are $\mathcal{J}_c = \{1, 2, 3, 4, 5\}$ and $\mathcal{J}_w = \{6, 7, 8, 9, 10, 11\}$, respectively.

Furthermore, the applied math course-sections are $\mathcal{I}_{appl} = \{1, 2, \ldots, 14\}$ and the pure math course-sections are $\mathcal{I}_{pure} = \{15, 16, \ldots, 29\}$.

Throughout the examples, we use a single Teacher Preference Matrix and a single Teacher Satisfaction Matrix, which are both randomly generated (with the restriction that entries in column 1 of T^p must be less than or equal to their corresponding column 2 entries) and are given by

$$T^p = \begin{bmatrix} 2 & 4 & 1 & 1 & 2 & 1 \\ 3 & 5 & 0 & 2 & 0 & 0 \\ 4 & 4 & 1 & 0 & 0 & 2 \\ 2 & 3 & 0 & 2 & 2 & 0 \\ 3 & 4 & 2 & 0 & 2 & 1 \\ 3 & 5 & 2 & 3 & 0 & 0 \\ 2 & 5 & 1 & 0 & 1 & 2 \\ 1 & 3 & 0 & 2 & 2 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 3 & 5 & 0 & 2 & 1 & 2 \end{bmatrix}$$

Table 3.1: Explanation of columns of a Teacher Preference Matrix.

Min	Max	Board	Time of Day	Days of Week	Pure/Applied
		0 = no pref	0 = no pref	0 = no pref	0 = no pref
		1 = whiteboard	1 = morning	1 = MWF	1 = pure
		2 = chalkboard	2 = afternoon	2 = TTH	2 = applied
			3 = evening		
			4 = morn + aft		
			5 = aft + eve		
			6 = morn + eve		

and

$$T^s = \begin{bmatrix} 4&3&5&2&4&5&3&5&5&5&4&4&5&5&4&5&5&2&2&5&5&2&5&2&5&5&5&4&3 \\ 3&5&2&5&2&5&5&5&5&5&5&5&2&5&4&5&5&5&3&5&5&5&5&5&2&5&2&5&5 \\ 2&2&2&4&2&4&3&4&3&4&2&4&2&4&3&4&4&4&3&3&2&4&4&4&2&4&2&2&2 \\ 4&3&5&4&2&5&3&5&5&5&2&2&4&5&3&5&5&5&3&4&4&4&5&5&4&5&4&2&3 \\ 5&5&2&5&5&5&5&2&5&5&5&5&3&4&5&5&5&5&5&5&5&5&5&5&3&4&3&5&5 \\ 3&5&5&5&5&5&4&5&5&5&5&5&2&5&5&5&5&5&4&4&3&5&5&5&2&5&2&5&5 \\ 5&3&5&4&5&5&2&3&5&5&4&3&5&5&5&5&5&5&5&5&5&4&5&5&5&5&5&4&3 \\ 5&5&3&5&5&5&3&3&5&3&5&4&4&2&5&3&5&5&5&5&5&5&5&5&4&2&4&5&5 \\ 4&2&5&4&5&5&2&2&5&5&3&2&4&4&4&5&5&5&4&4&4&4&5&5&4&4&4&3&2 \\ 3&2&4&5&4&4&2&5&5&5&2&2&2&5&4&5&5&5&2&5&5&5&5&5&2&5&2&2&2 \end{bmatrix}.$$

For convenience, Table 3 recaps the meaning of the columns of a Teacher Preference Matrix.

The simulated data show the teachers that want to use chalkboards and whiteboards are $\mathcal{T}_c = \{5, 6\}$ and $\mathcal{T}_w = \{1, 3, 7, 9\}$, while the teachers that want to teach in the morning, afternoon, and evening are $\mathcal{T}_{mo} = \{1\}$, $\mathcal{T}_{af} = \{2, 4, 8, 10\}$, and $\mathcal{T}_{ev} = \{6\}$, respectively. The teachers that want to teach on a M/W, W/F, M/F, or a M/W/F and T/TH are $\mathcal{T}_{mwf} = \{7, 9, 10\}$ and $\mathcal{T}_{tth} = \{1, 4, 5, 8\}$, while the teachers that want to teach pure and applied course-sections are $\mathcal{T}_{pure} = \{1, 5\}$ and $\mathcal{T}_{appl} = \{3, 7, 10\}$, respectively.

The LP-based models were implemented in MATLAB using the `intlinprog` function, which solves integer linear programming problems. The alternate models described in Section 2.3 do not use `intlinprog`, but rather use a direct implementation of the algorithm to attempt to generate a feasible solution.

3.1 Example of Model 1: Basic Model

In Model 1 from Section 2.2.1, the objective is to minimize the difference between the number of course-sections taught on M/W/F versus T/TH. Note that this model does not use the Teacher Preferences and Teacher Satisfaction Matrices. The resulting assignment of the course-sections can be visualized in Table 3.9 and Table 3.10. The optimal objective function value from this model is 0.5, indicating that the number of M/W, W/F, M/F, or M/W/F course-sections and the number of T/TH course-sections are both as close as possible to the balanced assignments value of $\frac{I}{2} = \frac{29}{2} = 14.5$. In particular, the number of course-sections that are scheduled as either M/W, W/F, M/F, or M/W/F is 14 and for T/TH this value is 15. Table 3.2 below displays the non-zero optimal decision variable values that were used to compute the objective function value.

Table 3.2: Optimal Decision Variable Values for the Example of Model 1.

$x_{1,10,23} = 1$	$x_{7,6,81} = 1$	$x_{13,3,67} = 1$	$x_{19,4,69} = 1$	$x_{25,9,69} = 1$
$x_{2,2,23} = 1$	$x_{8,3,82} = 1$	$x_{14,9,80} = 1$	$x_{20,8,80} = 1$	$x_{26,6,80} = 1$
$x_{3,5,38} = 1$	$x_{9,3,79} = 1$	$x_{15,8,69} = 1$	$x_{21,8,81} = 1$	$x_{27,10,80} = 1$
$x_{4,6,69} = 1$	$x_{10,2,69} = 1$	$x_{16,5,81} = 1$	$x_{22,2,82} = 1$	$x_{28,6,67} = 1$
$x_{5,10,79} = 1$	$x_{11,9,86} = 1$	$x_{17,10,82} = 1$	$x_{23,10,69} = 1$	$x_{29,5,69} = 1$
$x_{6,9,67} = 1$	$x_{12,4,67} = 1$	$x_{18,3,69} = 1$	$x_{24,9,79} = 1$	

3.2 Example of Model 2: Basic Teacher Model

In Model 2 from Section 2.2.2, we are assigning teachers to course-sections, but do not consider preference and thus do not use the Teacher Preference or Satisfaction Matrices. The resulting assignment of the course-sections can be visualized in Table 3.11 and Table 3.12. The optimal objective function value is 0.5, again indicating that the number of M/W, W/F, M/F, or M/W/F course-sections and the number of T/TH course-sections are both as close as possible to the balanced assignments value of $\frac{I}{2} = \frac{29}{2} = 14.5$. In particular, the number of course-sections that are scheduled as either M/W, W/F, M/F, or M/W/F is 15 and for T/TH this value is 14. Table 3.3 below displays the non-zero optimal decision variable values that were used to compute the objective function value.

Table 3.3: Optimal Decision Variable Values for the Example of Model 2.

$x_{1,8,35,9} = 1$	$x_{7,11,68,10} = 1$	$x_{13,4,70,6} = 1$	$x_{19,1,80,6} = 1$	$x_{25,8,47,8} = 1$
$x_{2,5,8,3} = 1$	$x_{8,4,83,1} = 1$	$x_{14,8,83,7} = 1$	$x_{20,7,70,1} = 1$	$x_{26,10,80,1} = 1$
$x_{3,6,39,2} = 1$	$x_{9,2,70,5} = 1$	$x_{15,10,63,3} = 1$	$x_{21,5,80,3} = 1$	$x_{27,9,70,8} = 1$
$x_{4,5,49,9} = 1$	$x_{10,11,70,2} = 1$	$x_{16,1,79,1} = 1$	$x_{22,9,80,7} = 1$	$x_{28,7,80,8} = 1$
$x_{5,3,46,7} = 1$	$x_{11,4,80,10} = 1$	$x_{17,10,86,5} = 1$	$x_{23,10,47,9} = 1$	$x_{29,6,70,10} = 1$
$x_{6,11,80,4} = 1$	$x_{12,8,70,9} = 1$	$x_{18,2,80,5} = 1$	$x_{24,8,80,9} = 1$	

3.3 Example of Model 3: Teacher Preference Model

In Model 3 from Section 2.2.3 we again consider M/W, W/F, M/F, or M/W/F versus T/TH but now include teacher preferences, which means that we need to use the Teacher Preference Matrix. The resulting assignment of the course-sections can be visualized in Table 3.13 and Table 3.14. The objective function value is also 0.5, indicating the same imbalance of one course-section between M/W, W/F, M/F, or M/W/F versus T/TH scheduling. The number of M/W, W/F, M/F, or M/W/F and T/TH course-sections are 15 and 14, respectively. Note that since an optimal solution was found, all teacher preferences were satisfied. Table 3.4 below displays the non-zero optimal decision variable values that were used to compute the objective function value.

Table 3.4: Optimal Decision Variable Value for the Example of Model 3.

$x_{1,8,42,4} = 1$	$x_{7,9,79,3} = 1$	$x_{13,8,72,10} = 1$	$x_{19,9,81,8} = 1$	$x_{25,10,79,1} = 1$
$x_{2,1,36,6} = 1$	$x_{8,7,69,7} = 1$	$x_{14,11,74,10} = 1$	$x_{20,3,80,5} = 1$	$x_{26,7,81,2} = 1$
$x_{3,3,43,6} = 1$	$x_{9,9,74,3} = 1$	$x_{15,7,67,9} = 1$	$x_{21,4,86,5} = 1$	$x_{27,3,81,5} = 1$
$x_{4,10,60,2} = 1$	$x_{10,10,67,3} = 1$	$x_{16,10,74,2} = 1$	$x_{22,1,79,5} = 1$	$x_{2810,60,9} = 1$
$x_{5,6,81,3} = 1$	$x_{11,11,67,7} = 1$	$x_{17,10,81,4} = 1$	$x_{23,8,69,9} = 1$	$x_{29,10,80,1} = 1$
$x_{6,3,86,6} = 1$	$x_{12,9,60,10} = 1$	$x_{18,10,72,9} = 1$	$x_{24,3,72,2} = 1$	

3.4 Example of Model 4: Teacher Difference Model

In Model 4 of Section 2.2.4 we have a different objective function, where the objective is to minimize the difference between the number of assigned course-sections for all teachers versus the ideal number of course-sections a teacher should teach under a balanced assignment. When we run the model, we achieve a minimum objective function value of 5.4 which is obtained from

$$\sum_{t=1}^{10} Q_t = 0.9 + 0.1 + 1.1 + 0.9 + 1.1 + 0.1 + 0.1 + 0.9 + 0.1 + 0.1 = 5.4.$$

Table 3.15 and Table 3.16 help to visualize the optimal solution for the scheduling. Table 3.5 below displays the non-zero optimal decision variable values that were used to compute the objective function value. From this table we see that teachers 1, 4, and 8 are assigned 2 course-sections each, teachers 2, 6, 7, 9, and 10 are assigned 3 course-sections each, and teachers 3 and 5 are assigned 4 course-sections each, which corroborates the objective function value since the ideal number of course-sections per teacher is $\frac{I}{T} = 2.9$. We also notice that by changing the objective, we did not minimize the difference of course-sections assigned to a M/W, W/F, M/F, or M/W/F vs. a T/TH schedule as we have 16 and 13 assigned, respectively.

Table 3.5: Optimal Decision Variable Values for the Example of Model 4.

$x_{1,10,42,4} = 1$	$x_{7,8,80,3} = 1$	$x_{13,6,85,3} = 1$	$x_{19,2,81,8} = 1$	$x_{25,5,81,5} = 1$
$x_{2,2,33,6} = 1$	$x_{8,1,72,10} = 1$	$x_{14,5,60,10} = 1$	$x_{20,6,80,1} = 1$	$x_{26,4,79,5} = 1$
$x_{3,11,42,8} = 1$	$x_{9,9,72,7} = 1$	$x_{15,6,72,9} = 1$	$x_{21,2,71,2} = 1$	$x_{27,1,80,5} = 1$
$x_{4,6,74,10} = 1$	$x_{10,11,60,7} = 1$	$x_{16,9,81,2} = 1$	$x_{22,8,79,1} = 1$	$x_{28,10,74,9} = 1$
$x_{5,9,46,3} = 1$	$x_{11,8,72,3} = 1$	$x_{17,10,60,9} = 1$	$x_{23,2,85,6} = 1$	$x_{29,9,74,2} = 1$
$x_{6,7,74,7} = 1$	$x_{12,7,81,4} = 1$	$x_{18,2,78,6} = 1$	$x_{24,11,78,9} = 1$	

3.5 Example of Model 6: Teacher Tricriteria Model

The objective in Model 6 from Section 2.2.6 is a combination of objectives described in Models 3, 4, and 5 of Section 2.2. This model uses both the Teacher Preference Matrix for the model's constraints, and Teacher Satisfaction Matrix for the objective along with equal weights of $\omega_1 = \omega_2 = \omega_3 = \frac{1}{3}$. The resulting assignment of the course-sections can be visualized in Table 3.17 and Table 3.18. From the two tables we see that the number of M/W, W/F, M/F, or M/W/F and T/TH course-sections are 14 and 15, respectively. The value of the objective function is $4.7\bar{6}$, which can be calculated from its 3 components by

$$\begin{aligned}
&\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t \\
&= \frac{1}{3} \cdot 0.5 + \frac{1}{3 \cdot 10} \cdot 84 + \frac{1}{3} \cdot 5.4 \\
&= 4.7\bar{6}.
\end{aligned}$$

Table 3.6 below displays the non-zero optimal decision variable values that were used to compute the objective function value.

Table 3.6: Optimal Decision Variable Values for the Example of Model 6.

$x_{1,10,42,2}$=1	$x_{7,11,69,7}$=1	$x_{13,5,86,6}$=1	$x_{19,10,74,10}$=1	$x_{25,5,78,6}$=1
$x_{2,9,42,4}$=1	$x_{8,5,83,5}$=1	$x_{14,4,86,5}$=1	$x_{20,6,80,3}$=1	$x_{26,5,80,5}$=1
$x_{3,11,42,8}$=1	$x_{9,9,75,3}$=1	$x_{15,11,81,4}$=1	$x_{21,9,73,3}$=1	$x_{27,9,81,2}$=1
$x_{4,11,78,7}$=1	$x_{10,1,83,5}$=1	$x_{16,10,81,8}$=1	$x_{22,10,78,9}$=1	$x_{28,10,72,10}$=1
$x_{5,8,74,2}$=1	$x_{11,10,70,9}$=1	$x_{17,11,86,3}$=1	$x_{23,6,49,7}$=1	$x_{29,11,73,9}$=1
$x_{6,11,60,10}$=1	$x_{12,11,75,9}$=1	$x_{18,11,80,1}$=1	$x_{24,11,79,1}$=1	

3.6 Example of Model 8: Naive Method

For the Naive Method from Section 2.3.1, we do not use the Teacher Preferences Matrix or the Teacher Satisfaction Matrix. Instead, we simply want to illustrate that the Naive Method produces a feasible solution. Tables 3.19, 3.20, 3.21, and 3.22 demonstrate this result. For the Naive Method there is no objective function to minimize, and thus the method could produce optimal or suboptimal solutions in different settings. Given a resulting schedule, we can, however, calculate the value of the tricriteria objective function with equal weights to be $9.5\bar{6}$ from

$$\begin{aligned} &\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t \\ &= \frac{1}{3}\cdot 14.5 + \frac{1}{3\cdot 10}\cdot 124 + \frac{1}{3}\cdot 1.8 \\ &= 9.5\bar{6} \end{aligned}$$

We notice that this value is larger than the corresponding optimal value of $4.7\bar{6}$ from Model 6 in Section 3.5. In fact, if we consider the objective of minimizing Q_t we have a value of 1.8 versus 5.4 in the example of Model 4: Teacher Difference, despite Model 4 being a linear program. This is due to the fact that the Naive

Method violates hard constraints by assigning teachers to classrooms they may not prefer, decreasing the objective value of $\sum_{t\in\mathcal{T}} Q_t$, but effectively increasing the overall arching objective function value. By having such assignments, we calculate there are 6 classroom board preference violations for the naive model. Even with violations in this model, the objective function value is larger than that of Model 6. Table 3.7 indicates which decision variables were selected for the feasible schedule from the Naive Method.

Table 3.7: Decision Variable Values for the Example of Model 8.

$x_{1,1,1,1}$=1	$x_{7,7,46,7}$=1	$x_{13,2,47,3}$=1	$x_{19,8,47,9}$=1	$x_{25,3,48,5}$=1
$x_{2,2,1,2}$=1	$x_{8,8,46,8}$=1	$x_{14,3,47,4}$=1	$x_{20,9,47,10}$=1	$x_{26,4,48,6}$=1
$x_{3,3,1,3}$=1	$x_{9,9,46,9}$=1	$x_{15,4,47,5}$=1	$x_{21,10,48,1}$=1	$x_{27,5,48,7}$=1
$x_{4,4,46,4}$=1	$x_{10,10,46,10}$=1	$x_{16,5,47,26}$=1	$x_{22,11,48,2}$=1	$x_{28,6,48,8}$=1
$x_{5,5,46,5}$=1	$x_{11,11,47,1}$=1	$x_{17,6,47,7}$=1	$x_{23,1,48,3}$=1	$x_{29,7,48,9}$=1
$x_{6,6,46,6}$=1	$x_{12,1,47,2}$=1	$x_{18,7,47,8}$=1	$x_{24,2,48,4}$=1	

3.7 Example of Model 9: Random Method

For the Random Method from Section 2.3.2 we generate random assignments and, like the previous example, we do not use the Teacher Preference Matrix or the Teacher Satisfaction Matrix. Tables 3.23, 3.24, and 3.25 demonstrate the resulting feasible solution. Calculating the value of the tricriteria objective function with equal weights for this solution yields $11.0\bar{6}$ as given by

$$\begin{aligned}
&\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t \\
&= \frac{1}{3}\cdot 6.5 + \frac{1}{3\cdot 10}\cdot 119 + \frac{1}{3}\cdot 14.8
\end{aligned}$$

$$= 11.0\bar{6}.$$

Since this method is random, we can examine its behavior on average. When running the method one-hundred times, we achieve a mean of 10.9483, a median of 10.9000, and a mode of 10.7667 for the objective function value. From these results we see that the particular result of $11.0\bar{6}$ was slightly higher than the average and that, on average, the random method performs slightly worse than the naive method. Additionally, we again note that this value is larger in comparison to the optimal value from Model 6. Similar to the Model 8: Naive Method, we calculate the number of classroom board preference violations to be 8, and this model produces a higher value of the objective function. Table 3.8 indicates which decision variables were selected for the feasible schedule.

Table 3.8: Decision Variable Values for the Example of Model 9.

$x_{1,9,41,2}$=1	$x_{7,8,55,6}$=1	$x_{13,3,54,5}$=1	$x_{19,2,51,8}$=1	$x_{25,10,67,5}$=1
$x_{2,11,25,7}$=1	$x_{8,11,72,6}$=1	$x_{14,5,84,1}$=1	$x_{20,11,78,9}$=1	$x_{26,4,56,5}$=1
$x_{3,2,44,3}$=1	$x_{9,1,78,4}$=1	$x_{15,4,60,5}$=1	$x_{21,9,69,5}$=1	$x_{27,4,80,6}$=1
$x_{4,9,53,4}$=1	$x_{10,5,60,7}$=1	$x_{16,6,60,2}$=1	$x_{22,5,86,6}$=1	$x_{28,10,54,3}$=1
$x_{5,5,50,2}$=1	$x_{11,11,75,8}$=1	$x_{17,9,83,2}$=1	$x_{23,10,63,1}$=1	$x_{29,3,56,4}$=1
$x_{6,5,68,2}$=1	$x_{12,5,78,6}$=1	$x_{18,8,52,5}$=1	$x_{24,1,64,10}$=1	

Table 3.9: Schedule for Model 1 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Course 2 – 3 7:00 - 8:05 AM 3-1616	Course 2 – 2 8:00 - 9:50 AM 3-1623	Course 2 – 3 7:00 - 8:05 AM 3-1616	Course 2 – 2 8:00 - 9:50 AM 3-1623	Course 2 – 3 7:00 - 8:05 AM 3-1616
Course 4 – 3 7:00 - 8:05 AM 8-247	Course 3 – 3 8:00 - 9:50 AM 8-246	Course 4 – 3 7:00 - 8:05 AM 8-247	Course 3 – 3 8:00 - 9:50 AM 8-246	Course 4 – 3 7:00 - 8:05 AM 8-247
Course 5 – 1 7:00 - 8:05 AM 8-246	Course 8 – 3 8:00 - 9:50 AM 3-1616	Course 5 – 1 7:00 - 8:05 AM 8-246	Course 8 – 3 8:00 - 9:50 AM 3-1616	Course 5 – 1 7:00 - 8:05 AM 8-246
Course 10 – 1 7:00 - 8:05 AM 8-248	Course 1 – 3 8:30 - 9:45 AM 8-249	Course 10 – 1 7:00 - 8:05 AM 8-248	Course 1 – 3 8:30 - 9:45 AM 8-249	Course 10 – 1 7:00 - 8:05 AM 8-248
Course 1 – 1 8:00 – 8:50 AM 3-1623	Course 5 – 2 10:00 - 11:50 AM 3-1616	Course 1 – 1 8:00 – 8:50 AM 3-1623	Course 5 – 2 10:00 - 11:50 AM 3-1616	Course 1 – 1 8:00 – 8:50 AM 3-1623
Course 1 – 2 8:00 – 8:50 AM 8-210	Course 7 – 2 10:00 - 11:50 AM 8-260	Course 1 – 2 8:00 – 8:50 AM 8-210	Course 7 – 2 10:00 - 11:50 AM 8-260	Course 1 – 2 8:00 – 8:50 AM 8-210
Course 2 – 1 9:30 - 10:35 AM 8-248	Course 9 – 2 10:00 - 11:50 AM 8-248	Course 2 – 1 9:30 - 10:35 AM 8-248	Course 9 – 2 10:00 - 11:50 AM 8-248	Course 2 – 1 9:30 - 10:35 AM 8-248
Course 4 – 1 9:30 - 10:35 AM 8-210	Course 9 – 3 10:00 - 11:50 AM 3-1623	Course 4 – 1 9:30 - 10:35 AM 8-210	Course 9 – 3 10:00 - 11:50 AM 3-1623	Course 4 – 1 9:30 - 10:35 AM 8-210

Table 3.10: Schedule for Model 1 using simulated data. (cont.)

Course 5 – 3 9:30 - 10:35 AM 8-260	Course 3 – 1 1:00 - 2:50 PM 8-248	Course 5 – 3 9:30 - 10:35 AM 8-260	Course 3 – 1 1:00 - 2:50 PM 8-248	Course 5 – 3 9:30 - 10:35 AM 8-260
Course 6 – 3 9:30 - 10:35 AM 8-246	Course 6 – 1 1:00 - 2:50 PM 8-249	Course 6 – 3 9:30 - 10:35 AM 8-246	Course 6 – 1 1:00 - 2:50 PM 8-249	Course 6 – 3 9:30 - 10:35 AM 8-246
Course 7 – 1 9:30 - 10:35 AM 8-247	Course 7 – 3 1:00 - 2:50 PM 8-260	Course 7 – 1 9:30 - 10:35 AM 8-247	Course 7 – 3 1:00 - 2:50 PM 8-260	Course 7 – 1 9:30 - 10:35 AM 8-247
Course 8 – 2 9:30 - 10:35 AM 3-1623	Course 3 – 2 3:00 - 4:50 PM 8-246	Course 8 – 2 9:30 - 10:35 AM 3-1623	Course 3 – 2 3:00 - 4:50 PM 8-246	Course 8 – 2 9:30 - 10:35 AM 3-1623
Course 9 – 1 9:30 - 10:35 AM 3-1616	Course 4 – 2 3:00 - 4:50 PM 3-1616	Course 9 – 1 9:30 - 10:35 AM 3-1616	Course 4 – 2 3:00 - 4:50 PM 3-1616	Course 9 – 1 9:30 - 10:35 AM 3-1616
Course 10 – 2 9:30 - 10:35 AM 8-249	Course 6 – 2 3:00 - 4:50 PM 3-1623	Course 10 – 2 9:30 - 10:35 AM 8-249	Course 6 – 2 3:00 - 4:50 PM 3-1623	Course 10 – 2 9:30 - 10:35 AM 8-249
	Course 8 – 1 3:00 - 4:50 PM 8-210		Course 8 – 1 3:00 - 4:50 PM 8-210	

Table 3.11: Schedule for Model 2 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 10 Course 3 – 1 8:15 - 9:20 AM 3-1637	Teacher 1 Course 6 – 1 8:00 - 9:50 AM 8-156	Teacher 10 Course 3 – 1 8:15 - 9:20 AM 3-1637	Teacher 1 Course 6 – 1 8:00 - 9:50 AM 8-156	Teacher 10 Course 3 – 1 8:15 - 9:20 AM 3-1637
Teacher 5 Course 3 – 3 10:45 - 11:50 AM 8-210	Teacher 2 Course 1 – 3 10:00 - 11:15 AM 8-248	Teacher 5 Course 3 – 3 10:45 - 11:50 AM 8-210	Teacher 2 Course 1 – 3 10:00 - 11:15 AM 8-248	Teacher 5 Course 3 – 3 10:45 - 11:50 AM 8-210
Teacher 2 Course 4 – 1 10:45 - 11:50 AM 3-1637	Teacher 4 Course 2 – 3 10:00 - 11:50 AM 3-1637	Teacher 2 Course 4 – 1 10:45 - 11:50 AM 3-1637	Teacher 4 Course 2 – 3 10:00 - 11:50 AM 3-1637	Teacher 2 Course 4 – 1 10:45 - 11:50 AM 3-1637
Teacher 9 Course 4 – 3 10:45 - 11:50 AM 8-260	Teacher 10 Course 4 – 2 10:00 - 11:50 AM 8-247	Teacher 9 Course 4 – 3 10:45 - 11:50 AM 8-260	Teacher 10 Course 4 – 2 10:00 - 11:50 AM 8-247	Teacher 9 Course 4 – 3 10:45 - 11:50 AM 8-260
Teacher 6 Course 5 – 1 10:45 - 11:50 AM 8-247	Teacher 5 Course 6 – 3 10:00 - 11:50 AM 8-210	Teacher 6 Course 5 – 1 10:45 - 11:50 AM 8-247	Teacher 5 Course 6 – 3 10:00 - 11:50 AM 8-210	Teacher 6 Course 5 – 1 10:45 - 11:50 AM 8-247
Teacher 1 Course 7 – 2 10:45 - 11:50 AM 8-250	Teacher 6 Course 7 – 1 10:00 - 11:50 AM 8-156	Teacher 1 Course 7 – 2 10:45 - 11:50 AM 8-250	Teacher 6 Course 7 – 1 10:00 - 11:50 AM 8-156	Teacher 1 Course 7 – 2 10:45 - 11:50 AM 8-250
Teacher 8 Course 9 – 3 10:45 - 11:50 AM 3-1616	Teacher 3 Course 7 – 3 10:00 - 11:50 AM 8-249	Teacher 8 Course 9 – 3 10:45 - 11:50 AM 3-1616	Teacher 3 Course 7 – 3 10:00 - 11:50 AM 8-249	Teacher 8 Course 9 – 3 10:45 - 11:50 AM 3-1616
Teacher 10 Course 10 – 2 10:45 - 11:50 AM 8-248	Teacher 7 Course 8 – 1 10:00 - 11:50 AM 3-1616	Teacher 10 Course 10 – 2 10:45 - 11:50 AM 8-248	Teacher 7 Course 8 – 1 10:00 - 11:50 AM 3-1616	Teacher 10 Course 10 – 2 10:45 - 11:50 AM 8-248

Table 3.12: Schedule for Model 2 using simulated data. (cont.)

Teacher 7 Course 2 – 2 11:00 - 12:50 PM 8-246	Teacher 9 Course 8 – 3 10:00 - 11:50 AM 8-260	Teacher 7 Course 2 – 2 11:00 - 12:50 PM 8-246	Teacher 9 Course 8 – 3 10:00 - 11:50 AM 8-260	
	Teacher 1 Course 9 – 2 10:00 - 11:50 AM 3-1623	Teacher 3 Course 1 – 2 11:30 - 12:45 PM 8-249	Teacher 1 Course 9 – 2 10:00 - 11:50 AM 3-1623	Teacher 3 Course 1 – 2 11:30 - 12:45 PM 8-249
Teacher 9 Course 8 – 2 1:00 - 2:50 PM 3-1623	Teacher 8 Course 10 – 1 10:00 - 11:50 AM 8-250	Teacher 9 Course 8 – 2 1:00 - 2:50 PM 3-1623	Teacher 8 Course 10 – 1 10:00 - 11:50 AM 8-250	
Teacher 8 Course 9 – 1 1:00 - 2:50 PM 8-260	Teacher 1 Course 3 – 2 5:00 - 6:50 PM 8-247	Teacher 8 Course 9 – 1 1:00 - 2:50 PM 8-260	Teacher 1 Course 3 – 2 5:00 - 6:50 PM 8-247	
Teacher 9 Course 2 – 1 5:00 - 6:50 PM 8-249	Teacher 7 Course 5 – 2 5:00 - 6:50 PM 8-260	Teacher 9 Course 2 – 1 5:00 - 6:50 PM 8-249	Teacher 7 Course 5 – 2 5:00 - 6:50 PM 8-260	
Teacher 3 Course 5 – 3 5:00 - 6:50 PM 3-1623	Teacher 5 Course 6 – 2 8:00 - 9:50 PM 3-1623		Teacher 5 Course 6 – 2 8:00 - 9:50 PM 3-1623	Teacher 3 Course 5 – 3 5:00 - 6:50 PM 3-1623
Teacher 9 Course 1 – 1 8:00 – 8:50 PM 8-260		Teacher 9 Course 1 – 1 8:00 – 8:50 PM 8-260		Teacher 9 Course 1 – 1 8:00 – 8:50 PM 8-260

Table 3.13: Schedule for Model 3 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 3 Course 4 – 1 7:00 - 8:05 AM 3-1623	Teacher 3 Course 3 – 1 8:00 - 9:50 AM 3-1616	Teacher 3 Course 4 – 1 7:00 - 8:05 AM 3-1623	Teacher 3 Course 3 – 1 8:00 - 9:50 AM 3-1616	Teacher 3 Course 4 – 1 7:00 - 8:05 AM 3-1623
Teacher 7 Course 4 – 2 7:00 - 8:05 AM 3-1637	Teacher 5 Course 8 – 1 8:00 - 9:50 AM 8-156	Teacher 7 Course 4 – 2 7:00 - 8:05 AM 3-1637	Teacher 5 Course 8 – 1 8:00 - 9:50 AM 8-156	Teacher 7 Course 4 – 2 7:00 - 8:05 AM 3-1637
Teacher 9 Course 5 – 3 7:00 - 8:05 AM 8-250	Teacher 1 Course 9 – 1 8:00 - 9:50 AM 3-1623	Teacher 9 Course 5 – 3 7:00 - 8:05 AM 8-250	Teacher 1 Course 9 – 1 8:00 - 9:50 AM 3-1623	Teacher 9 Course 5 – 3 7:00 - 8:05 AM 8-250
Teacher 7 Course 3 – 2 9:30 - 10:35 AM 8-250	Teacher 5 Course 7 – 2 10:00 - 11:50 AM 8-246	Teacher 7 Course 3 – 2 9:30 - 10:35 AM 8-250	Teacher 5 Course 7 – 2 10:00 - 11:50 AM 8-246	Teacher 7 Course 3 – 2 9:30 - 10:35 AM 8-250
Teacher 9 Course 8 – 2 9:30 - 10:35 AM 8-260	Teacher 1 Course 10 – 2 10:00 - 11:50 AM 3-1623	Teacher 9 Course 8 – 2 9:30 - 10:35 AM 8-260	Teacher 1 Course 10 – 2 10:00 - 11:50 AM 3-1623	Teacher 9 Course 8 – 2 9:30 - 10:35 AM 8-260
Teacher 2 Course 2 – 1 11:00 - 12:50 PM 3-1623	Teacher 3 Course 2 – 2 1:00 - 2:50 PM 8-248		Teacher 3 Course 2 – 2 1:00 - 2:50 PM 8-248	Teacher 2 Course 2 – 1 11:00 - 12:50 PM 3-1623
Teacher 10 Course 4 – 3 11:00 - 12:50 PM 3-1616	Teacher 4 Course 6 – 2 1:00 - 2:50 PM 3-1623		Teacher 4 Course 6 – 2 1:00 - 2:50 PM 3-1623	Teacher 10 Course 4 – 3 11:00 - 12:50 PM 3-1616
Teacher 9 Course 10 – 1 11:00 - 12:50 PM 3-1637	Teacher 8 Course 7 – 1 1:00 - 2:50 PM 3-1616		Teacher 8 Course 7 – 1 1:00 - 2:50 PM 3-1616	Teacher 9 Course 10 – 1 11:00 - 12:50 PM 3-1637

Table 3.14: Schedule for Model 3 using simulated data. (cont.)

Teacher 10 Course 5 – 1 1:15 - 2:20 PM 8-260	Teacher 2 Course 9 – 2 1:00 - 2:50 PM 8-250	Teacher 10 Course 5 – 1 1:15 - 2:20 PM 8-260	Teacher 2 Course 9 – 2 1:00 - 2:50 PM 8-250	Teacher 10 Course 5 – 1 1:15 - 2:20 PM 8-260
Teacher 9 Course 6 – 3 1:15 - 2:20 PM 3-1623	Teacher 5 Course 9 – 3 1:00 - 2:50 PM 8-246	Teacher 9 Course 6 – 3 1:15 - 2:20 PM 3-1623	Teacher 5 Course 9 – 3 1:00 - 2:50 PM 8-246	Teacher 9 Course 6 – 3 1:15 - 2:20 PM 3-1623
Teacher 2 Course 8 – 3 1:15 - 2:20 PM 8-246	Teacher 4 Course 1 – 1 4:00 - 5:15 PM 8-260	Teacher 2 Course 8 – 3 1:15 - 2:20 PM 8-246	Teacher 4 Course 1 – 1 4:00 - 5:15 PM 8-260	Teacher 2 Course 8 – 3 1:15 - 2:20 PM 8-246
Teacher 3 Course 3 – 3 3:45 - 4:50 PM 3-1616	Teacher 6 Course 1 – 3 5:30 - 6:45 PM 8-246	Teacher 3 Course 3 – 3 3:45 - 4:50 PM 3-1616	Teacher 6 Course 1 – 3 5:30 - 6:45 PM 8-246	Teacher 3 Course 3 – 3 3:45 - 4:50 PM 3-1616
Teacher 10 Course 5 – 2 3:45 - 4:50 PM 3-1637	Teacher 6 Course 2 – 3 8:00 - 9:50 PM 8-246	Teacher 10 Course 5 – 2 3:45 - 4:50 PM 3-1637	Teacher 6 Course 2 – 3 8:00 - 9:50 PM 8-246	Teacher 10 Course 5 – 2 3:45 - 4:50 PM 3-1637
Teacher 2 Course 6 – 1 3:45 - 4:50 PM 3-1623	Teacher 5 Course 7 – 3 8:00 - 9:50 PM 8-247	Teacher 2 Course 6 – 1 3:45 - 4:50 PM 3-1623	Teacher 5 Course 7 – 3 8:00 - 9:50 PM 8-247	Teacher 2 Course 6 – 1 3:45 - 4:50 PM 3-1623
Teacher 6 Course 1 – 2 9:00 – 9:50 PM 8-156		Teacher 6 Course 1 – 2 9:00 – 9:50 PM 8-156		Teacher 6 Course 1 – 2 9:00 – 9:50 PM 8-156

Table 3.15: Schedule for Model 4 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 3 Course 2 – 2 11:00 - 12:50 PM 3-1616	Teacher 1 Course 8 – 1 8:00 - 9:50 AM 8-260	Teacher 3 Course 2 – 2 11:00 - 12:50 PM 3-1616	Teacher 1 Course 8 – 1 8:00 - 9:50 AM 8-260	
Teacher 7 Course 4 – 1 11:00 - 12:50 PM 3-1637	Teacher 5 Course 9 – 2 8:00 - 9:50 AM 8-247		Teacher 5 Course 9 – 2 8:00 - 9:50 AM 8-247	Teacher 7 Course 4 – 1 11:00 - 12:50 PM 3-1637
Teacher 10 Course 5 – 2 11:00 - 12:50 PM 8-249	Teacher 3 Course 3 – 1 10:00 - 11:50 AM 8-260		Teacher 3 Course 3 – 1 10:00 - 11:50 AM 8-260	Teacher 10 Course 5 – 2 11:00 - 12:50 PM 8-249
Teacher 9 Course 6 – 2 11:00 - 12:50 PM 3-1623	Teacher 1 Course 7 – 2 10:00 - 11:50 AM 8-248		Teacher 1 Course 7 – 2 10:00 - 11:50 AM 8-248	Teacher 9 Course 6 – 2 11:00 - 12:50 PM 3-1623
Teacher 2 Course 7 – 3 12:00 - 1:05 PM 8-210	Teacher 5 Course 9 – 3 10:00 - 11:50 AM 8-156	Teacher 2 Course 7 – 3 12:00 - 1:05 PM 8-210	Teacher 5 Course 9 – 3 10:00 - 11:50 AM 8-156	Teacher 2 Course 7 – 3 12:00 - 1:05 PM 8-210
Teacher 10 Course 3 – 2 1:15 - 2:20 PM 8-156	Teacher 4 Course 4 – 3 1:00 - 2:50 PM 8-250	Teacher 10 Course 3 – 2 1:15 - 2:20 PM 8-156	Teacher 4 Course 4 – 3 1:00 - 2:50 PM 8-250	Teacher 10 Course 3 – 2 1:15 - 2:20 PM 8-156
Teacher 7 Course 3 – 3 1:15 - 2:20 PM 3-1616	Teacher 2 Course 6 – 1 1:00 - 2:50 PM 3-1616	Teacher 7 Course 3 – 3 1:15 - 2:20 PM 3-1616	Teacher 2 Course 6 – 1 1:00 - 2:50 PM 3-1616	Teacher 7 Course 3 – 3 1:15 - 2:20 PM 3-1616
Teacher 3 Course 4 – 2 1:15 - 2:20 PM 8-260	Teacher 8 Course 7 – 1 1:00 - 2:50 PM 8-210	Teacher 3 Course 4 – 2 1:15 - 2:20 PM 8-260	Teacher 8 Course 7 – 1 1:00 - 2:50 PM 8-210	Teacher 3 Course 4 – 2 1:15 - 2:20 PM 8-260

Table 3.16: Schedule for Model 4 using simulated data. (cont.)

Teacher 9 Course 5 – 3 1:15 - 2:20 PM 8-248	Teacher 5 Course 9 – 1 1:00 - 2:50 PM 8-249	Teacher 9 Course 5 – 3 1:15 - 2:20 PM 8-248	Teacher 5 Course 9 – 1 1:00 - 2:50 PM 8-249	Teacher 9 Course 5 – 3 1:15 - 2:20 PM 8-248
Teacher 10 Course 2 – 1 3:45 - 4:50 PM 8-248	Teacher 4 Course 1 – 1 4:00 - 5:15 PM 3-1623	Teacher 10 Course 2 – 1 3:45 - 4:50 PM 8-248	Teacher 4 Course 1 – 1 4:00 - 5:15 PM 3-1623	Teacher 10 Course 2 – 1 3:45 - 4:50 PM 8-248
Teacher 7 Course 2 – 3 3:45 - 4:50 PM 8-250	Teacher 8 Course 1 – 3 4:00 - 5:15 PM 3-1637	Teacher 7 Course 2 – 3 3:45 - 4:50 PM 8-250	Teacher 8 Course 1 – 3 4:00 - 5:15 PM 3-1637	Teacher 7 Course 2 – 3 3:45 - 4:50 PM 8-250
Teacher 9 Course 10 – 1 3:45 - 4:50 PM 3-1623	Teacher 3 Course 5 – 1 7:00 - 8:50 PM 8-248	Teacher 9 Course 10 – 1 3:45 - 4:50 PM 3-1623	Teacher 3 Course 5 – 1 7:00 - 8:50 PM 8-248	Teacher 9 Course 10 – 1 3:45 - 4:50 PM 3-1623
Teacher 2 Course 10 – 2 3:45 - 4:50 PM 3-1616	Teacher 6 Course 8 – 2 7:00 - 8:50 PM 8-210	Teacher 2 Course 10 – 2 3:45 - 4:50 PM 3-1616	Teacher 6 Course 8 – 2 7:00 - 8:50 PM 8-210	Teacher 2 Course 10 – 2 3:45 - 4:50 PM 3-1616
Teacher 6 Course 1 – 2 6:00 – 6:50 PM 8-210		Teacher 6 Course 1 – 2 6:00 – 6:50 PM 8-210		Teacher 6 Course 1 – 2 6:00 – 6:50 PM 8-210
Teacher 6 Course 6 – 3 8:45 - 9:50 PM 8-210		Teacher 6 Course 6 – 3 8:45 - 9:50 PM 8-210		Teacher 6 Course 6 – 3 8:45 - 9:50 PM 8-210
Teacher 9 Course 8 – 3 8:45 - 9:50 PM 3-1637		Teacher 9 Course 8 – 3 8:45 - 9:50 PM 3-1637		Teacher 9 Course 8 – 3 8:45 - 9:50 PM 3-1637

Table 3.17: Schedule for Model 6 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 7 Course 3 – 1 9:30 - 10:35 AM 3-1637	Teacher 1 Course 8 – 3 8:00 - 9:50 AM 3-1637	Teacher 7 Course 3 – 1 9:30 - 10:35 AM 3-1637	Teacher 1 Course 8 – 3 8:00 - 9:50 AM 3-1637	Teacher 7 Course 3 – 1 9:30 - 10:35 AM 3-1637
Teacher 9 Course 4 – 2 10:45 - 11:50 AM 3-1623	Teacher 1 Course 6 – 3 10:00 - 11:50 AM 3-1637	Teacher 9 Course 4 – 2 10:45 - 11:50 AM 3-1623	Teacher 1 Course 6 – 3 10:00 - 11:50 AM 3-1637	Teacher 9 Course 4 – 2 10:45 - 11:50 AM 3-1623
Teacher 10 Course 2 – 3 11:00 - 12:50 PM 3-1637	Teacher 3 Course 7 – 2 10:00 - 11:50 AM 8-248		Teacher 3 Course 7 – 2 10:00 - 11:50 AM 8-248	Teacher 10 Course 2 – 3 11:00 - 12:50 PM 3-1637
Teacher 10 Course 10 – 1 1:15 - 2:20 PM 3-1623	Teacher 5 Course 9 – 2 10:00 - 11:50 AM 8-249	Teacher 10 Course 10 – 1 1:15 - 2:20 PM 3-1623	Teacher 5 Course 9 – 2 10:00 - 11:50 AM 8-249	Teacher 10 Course 10 – 1 1:15 - 2:20 PM 3-1623
Teacher 3 Course 7 – 3 2:30 - 3:35 PM 3-1616	Teacher 4 Course 5 – 3 1:00 - 2:50 PM 3-1637	Teacher 3 Course 7 – 3 2:30 - 3:35 PM 3-1616	Teacher 4 Course 5 – 3 1:00 - 2:50 PM 3-1637	Teacher 3 Course 7 – 3 2:30 - 3:35 PM 3-1616
Teacher 9 Course 10 – 2 2:30 - 3:35 PM 3-1637	Teacher 8 Course 6 – 1 1:00 - 2:50 PM 3-1623	Teacher 9 Course 10 – 2 2:30 - 3:35 PM 3-1637	Teacher 8 Course 6 – 1 1:00 - 2:50 PM 3-1623	Teacher 9 Course 10 – 2 2:30 - 3:35 PM 3-1637
Teacher 2 Course 2 – 2 3:45 - 4:50 PM 8-260	Teacher 2 Course 9 – 3 1:00 - 2:50 PM 3-1616	Teacher 2 Course 2 – 2 3:45 - 4:50 PM 8-260	Teacher 2 Course 9 – 3 1:00 - 2:50 PM 3-1616	Teacher 2 Course 2 – 2 3:45 - 4:50 PM 8-260
Teacher 10 Course 7 – 1 3:45 - 4:50 PM 3-1623	Teacher 2 Course 1 – 1 4:00 - 5:15 PM 3-1623	Teacher 10 Course 7 – 1 3:45 - 4:50 PM 3-1623	Teacher 2 Course 1 – 1 4:00 - 5:15 PM 3-1623	Teacher 10 Course 7 – 1 3:45 - 4:50 PM 3-1623

Table 3.18: Schedule for Model 6 using simulated data. (cont.)

Teacher 3 Course 3 – 3 5:00 - 6:05 PM 3-1616	Teacher 4 Course 1 – 2 4:00 - 5:15 PM 3-1616	Teacher 3 Course 3 – 3 5:00 - 6:05 PM 3-1616	Teacher 4 Course 1 – 2 4:00 - 5:15 PM 3-1616	Teacher 3 Course 3 – 3 5:00 - 6:05 PM 3-1616
Teacher 9 Course 4 – 3 5:00 - 6:05 PM 3-1637	Teacher 8 Course 1 – 3 4:00 - 5:15 PM 3-1637	Teacher 9 Course 4 – 3 5:00 - 6:05 PM 3-1637	Teacher 8 Course 1 – 3 4:00 - 5:15 PM 3-1637	Teacher 9 Course 4 – 3 5:00 - 6:05 PM 3-1637
Teacher 7 Course 8 – 2 5:00 - 6:50 PM 8-248	Teacher 5 Course 3 – 2 5:00 - 6:50 PM 8-249	Teacher 7 Course 8 – 2 5:00 - 6:50 PM 8-248	Teacher 5 Course 3 – 2 5:00 - 6:50 PM 8-249	
Teacher 7 Course 2 – 1 8:45 - 9:50 PM 3-1637	Teacher 6 Course 4 – 1 5:00 - 6:50 PM 8-156	Teacher 7 Course 2 – 1 8:45 - 9:50 PM 3-1637	Teacher 6 Course 4 – 1 5:00 - 6:50 PM 8-156	Teacher 7 Course 2 – 1 8:45 - 9:50 PM 3-1637
Teacher 9 Course 8 – 1 8:45 - 9:50 PM 3-1623	Teacher 6 Course 5 – 1 8:00 - 9:50 PM 8-249	Teacher 9 Course 8 – 1 8:45 - 9:50 PM 3-1623	Teacher 6 Course 5 – 1 8:00 - 9:50 PM 8-249	Teacher 9 Course 8 – 1 8:45 - 9:50 PM 3-1623
Teacher 6 Course 9 – 1 8:45 - 9:50 PM 8-249	Teacher 5 Course 5 – 2 8:00 - 9:50 PM 8-247	Teacher 6 Course 9 – 1 8:45 - 9:50 PM 8-249	Teacher 5 Course 5 – 2 8:00 - 9:50 PM 8-247	Teacher 6 Course 9 – 1 8:45 - 9:50 PM 8-249
	Teacher 3 Course 6 – 2 8:00 - 9:50 PM 3-1637		Teacher 3 Course 6 – 2 8:00 - 9:50 PM 3-1637	

Table 3.19: Schedule for Model 8 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 4 Course 2 – 1 11:00 - 12:50 PM 8-247		Teacher 4 Course 2 – 1 11:00 - 12:50 PM 8-247		
Teacher 5 Course 2 – 2 11:00 - 12:50 PM 8-249		Teacher 5 Course 2 – 2 11:00 - 12:50 PM 8-249		
Teacher 6 Course 2 – 3 11:00 - 12:50 PM 8-248		Teacher 6 Course 2 – 3 11:00 - 12:50 PM 8-248		
Teacher 7 Course 3 – 1 11:00 - 12:50 PM 8-250		Teacher 7 Course 3 – 1 11:00 - 12:50 PM 8-250		
Teacher 8 Course 3 – 2 11:00 - 12:50 PM 8-260		Teacher 8 Course 3 – 2 11:00 - 12:50 PM 8-260		
Teacher 9 Course 3 – 3 11:00 - 12:50 PM 3-1616		Teacher 9 Course 3 – 3 11:00 - 12:50 PM 3-1616		
Teacher 10 Course 4 – 1 11:00 - 12:50 PM 3-1623		Teacher 10 Course 4 – 1 11:00 - 12:50 PM 3-1623		
Teacher 1 Course 1 – 1 11:30 - 12:45 PM 8-156		Teacher 1 Course 1 – 1 11:30 - 12:45 PM 8-156		

Table 3.20: Schedule for Model 8 using simulated data. (cont.)

Teacher 2 Course 1 – 2 11:30 - 12:45 PM 8-210		Teacher 2 Course 1 – 2 11:30 - 12:45 PM 8-210		
Teacher 3 Course 1 – 3 11:30 - 12:45 PM 8-246		Teacher 3 Course 1 – 3 11:30 - 12:45 PM 8-246		
Teacher 1 Course 4 – 2 1:00 - 2:50 PM 3-1637		Teacher 1 Course 4 – 2 1:00 - 2:50 PM 3-1637		
Teacher 2 Course 4 – 3 1:00 - 2:50 PM 8-156		Teacher 2 Course 4 – 3 1:00 - 2:50 PM 8-156		
Teacher 3 Course 5 – 1 1:00 - 2:50 PM 8-210		Teacher 3 Course 5 – 1 1:00 - 2:50 PM 8-210		
Teacher 4 Course 5 – 2 1:00 - 2:50 PM 8-246		Teacher 4 Course 5 – 2 1:00 - 2:50 PM 8-246		
Teacher 5 Course 5 – 3 1:00 - 2:50 PM 8-247		Teacher 5 Course 5 – 3 1:00 - 2:50 PM 8-247		
Teacher 6 Course 6 – 1 1:00 - 2:50 PM 8-249		Teacher 6 Course 6 – 1 1:00 - 2:50 PM 8-249		

Table 3.21: Schedule for Model 8 using simulated data. (cont.)

Teacher 7 Course 6 – 2 1:00 - 2:50 PM 8-248		Teacher 7 Course 6 – 2 1:00 - 2:50 PM 8-248		
Teacher 8 Course 6 – 3 1:00 - 2:50 PM 8-250		Teacher 8 Course 6 – 3 1:00 - 2:50 PM 8-250		
Teacher 9 Course 7 – 1 1:00 - 2:50 PM 8-260		Teacher 9 Course 7 – 1 1:00 - 2:50 PM 8-260		
Teacher 10 Course 7 – 2 1:00 - 2:50 PM 3-1616		Teacher 10 Course 7 – 2 1:00 - 2:50 PM 3-1616		
Teacher 1 Course 7 – 3 3:00 - 4:50 PM 3-1623		Teacher 1 Course 7 – 3 3:00 - 4:50 PM 3-1623		
Teacher 2 Course 8 – 1 3:00 - 4:50 PM 3-1637		Teacher 2 Course 8 – 1 3:00 - 4:50 PM 3-1637		
Teacher 3 Course 8 – 2 3:00 - 4:50 PM 8-156		Teacher 3 Course 8 – 2 3:00 - 4:50 PM 8-156		
Teacher 4 Course 8 – 3 3:00 - 4:50 PM 8-210		Teacher 4 Course 8 – 3 3:00 - 4:50 PM 8-210		

Table 3.22: Schedule for Model 8 using simulated data. (cont)

Teacher 5 Course 9 – 1 3:00 - 4:50 PM 8-246		Teacher 5 Course 9 – 1 3:00 - 4:50 PM 8-246		
Teacher 6 Course 9 – 2 3:00 - 4:50 PM 8-247		Teacher 6 Course 9 – 2 3:00 - 4:50 PM 8-247		
Teacher 7 Course 9 – 3 3:00 - 4:50 PM 8-249		Teacher 7 Course 9 – 3 3:00 - 4:50 PM 8-249		
Teacher 8 Course 10 – 1 3:00 - 4:50 PM 8-248		Teacher 8 Course 10 – 1 3:00 - 4:50 PM 8-248		
Teacher 9 Course 10 – 2 3:00 - 4:50 PM 8-250		Teacher 9 Course 10 – 2 3:00 - 4:50 PM 8-250		

Table 3.23: Schedule for Model 9 using simulated data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 5 Course 9 – 1 7:00 - 8:05 AM 3-1623	Teacher 6 Course 9 – 3 10:00 - 11:50 AM 8-247	Teacher 5 Course 9 – 1 7:00 - 8:05 AM 3-1623	Teacher 6 Course 9 – 3 10:00 - 11:50 AM 8-247	Teacher 5 Course 9 – 1 7:00 - 8:05 AM 3-1623
Teacher 2 Course 2 – 3 8:15 - 9:20 AM 8-249	Teacher 2 Course 1 – 1 2:30 - 3:45 PM 3-1616	Teacher 2 Course 2 – 3 8:15 - 9:20 AM 8-249	Teacher 2 Course 1 – 1 2:30 - 3:45 PM 3-1616	Teacher 2 Course 2 – 3 8:15 - 9:20 AM 8-249
Teacher 5 Course 7 – 3 9:30 - 10:35 AM 3-1616	Teacher 2 Course 6 – 2 5:00 - 6:50 PM 3-1616	Teacher 5 Course 7 – 3 9:30 - 10:35 AM 3-1616	Teacher 2 Course 6 – 2 5:00 - 6:50 PM 3-1616	Teacher 5 Course 7 – 3 9:30 - 10:35 AM 3-1616
Teacher 7 Course 1 – 2 10:00 – 10:50 AM 3-1637	Teacher 1 Course 5 – 2 6:00 - 7:50 PM 8-249	Teacher 7 Course 1 – 2 10:00 – 10:50 AM 3-1637	Teacher 1 Course 5 – 2 6:00 - 7:50 PM 8-249	Teacher 7 Course 1 – 2 10:00 – 10:50 AM 3-1637
	Teacher 3 Course 1 – 3 7:00 - 8:15 PM 8-210	Teacher 4 Course 2 – 1 11:00 - 12:50 PM 3-1616	Teacher 3 Course 1 – 3 7:00 - 8:15 PM 8-210	Teacher 4 Course 2 – 1 11:00 - 12:50 PM 3-1616
Teacher 7 Course 4 – 1 11:00 - 12:50 PM 8-249	Teacher 6 Course 8 – 1 8:00 - 9:50 PM 8-249		Teacher 6 Course 8 – 1 8:00 - 9:50 PM 8-249	Teacher 7 Course 4 – 1 11:00 - 12:50 PM 8-249
Teacher 5 Course 5 – 3 11:00 - 12:50 PM 8-247				Teacher 5 Course 5 – 3 11:00 - 12:50 PM 8-247
Teacher 2 Course 6 – 1 11:00 - 12:50 PM 8-248				Teacher 2 Course 6 – 1 11:00 - 12:50 PM 8-248

Table 3.24: Schedule for Model 9 using simulated data. (cont.)

		Teacher 5 Course 5 – 1 1:00 - 2:50 PM 8-246		Teacher 5 Course 5 – 1 1:00 - 2:50 PM 8-246
		Teacher 3 Course 10 – 1 1:00 - 2:50 PM 3-1623		Teacher 3 Course 10 – 1 1:00 - 2:50 PM 3-1623
Teacher 6 Course 3 – 2 1:15 - 2:20 PM 3-1637		Teacher 6 Course 3 – 2 1:15 - 2:20 PM 3-1637		Teacher 6 Course 3 – 2 1:15 - 2:20 PM 3-1637
		Teacher 6 Course 3 – 1 3:00 - 4:50 PM 8-260		Teacher 6 Course 3 – 1 3:00 - 4:50 PM 8-260
Teacher 8 Course 4 – 2 5:00 - 6:05 PM 3-1637		Teacher 8 Course 4 – 2 5:00 - 6:05 PM 3-1637		Teacher 8 Course 4 – 2 5:00 - 6:05 PM 3-1637
		Teacher 5 Course 9 – 2 5:00 - 6:50 PM 8-247		Teacher 5 Course 9 – 2 5:00 - 6:50 PM 8-247
		Teacher 4 Course 10 – 2 5:00 - 6:50 PM 8-246		Teacher 4 Course 10 – 2 5:00 - 6:50 PM 8-246
Teacher 1 Course 8 – 2 5:00 - 6:50 PM 3-1623				Teacher 1 Course 8 – 2 5:00 - 6:50 PM 3-1623

Table 3.25: Schedule for Model 9 using simulated data. (cont.)

Teacher 2 Course 2 – 2 6:00 - 7:50 PM 8-249		Teacher 2 Course 2 – 2 6:00 - 7:50 PM 8-249		
Teacher 10 Course 8 – 3 6:00 - 7:50 PM 8-156				Teacher 10 Course 8 – 3 6:00 - 7:50 PM 8-156
Teacher 8 Course 7 – 1 7:00 - 8:50 PM 8-210		Teacher 8 Course 7 – 1 7:00 - 8:50 PM 8-210		
Teacher 5 Course 6 – 3 8:00 - 9:50 PM 8-260		Teacher 5 Course 6 – 3 8:00 - 9:50 PM 8-260		
Teacher 4 Course 3 – 3 8:45 - 9:50 PM 8-156		Teacher 4 Course 3 – 3 8:45 - 9:50 PM 8-156		Teacher 4 Course 3 – 3 8:45 - 9:50 PM 8-156
Teacher 6 Course 4 – 3 8:45 - 9:50 PM 8-249		Teacher 6 Course 4 – 3 8:45 - 9:50 PM 8-249		Teacher 6 Course 4 – 3 8:45 - 9:50 PM 8-249
Teacher 9 Course 7 – 2 8:45 - 9:50 PM 3-1637		Teacher 9 Course 7 – 2 8:45 - 9:50 PM 3-1637		Teacher 9 Course 7 – 2 8:45 - 9:50 PM 3-1637

Chapter 4

Experiment Using CPP Data

4.1 Setup for the CPP Setting

Models 1 - 6 of Section 2.2 depict generalized models of the UCSP that can be modified to specific universities. Model 7 is our final model, which we present as a representative model for Cal Poly Pomona. To examine the behavior of this model, we consider data depicting the CPP setting. We use the same number of classrooms and time modules as in Chapter 3, namely $J = 11$ and $K = 86$, as these choices were based upon CPP, and can be found again in Appendices A.2, A.3.1, and A.3.2. The preference data for the experiment in this chapter are gathered from Cal Poly Pomona's Mathematics and Statistics Department. In the data provided to the department, teachers specify a range of times of day they prefer to teach. We adapt these preference to a scale for use in the Best Times Preference Matrix. For example, if a teacher has selected to teach in the morning, then they are given a satisfaction of 0 being the best for teaching in the morning. A value of 1 is given to other combinations involving the morning, such as mornings/afternoons and mornings/evenings. The combination of afternoon/evening which doesn't include morning is given the value of 3, being the least desired. From the data, we extract a total of $T = 34$ teachers and $I = 101$ course-sections, with 54 course-sections in the fall semester and 48 course-sections in the spring semester. We will be using the spring semester data which is comprised of 48 course-sections. In the CPP data, there are 42 named courses, but the data presents 48 course-sections, meaning that some courses have multiple sections. The 48 course-sections can be found in Appendix A.1.2.

While MATLAB sufficed as a tool for the experiments in Chapter 3, it is not designed for the type of large-scale problem presented here with the large number of course-sections and teachers. Thus we instead consider a reduced problem of using $T = 20$ teachers for the $I = 48$ course-sections for the spring semester. The preference data for this reduced problem can be found in Appendix B. Additionally, to assist with memory restrictions and computational speed, we use Gurobi, a mathematical programming solver, to solve our LP problem via its MATLAB interface.

4.2 Results and Discussion for the CPP Setting

With our implementation of Model 7, we achieve an optimal solution which is illustrated in Table 4.1, Table 4.2, and Table 4.3. We notice that the number of course-sections assigned to a M/W, W/F, M/F, or M/W/F time module and the number of course-sections assigned to a T/TH time module are both 24. We also notice that 1 teacher (Teacher 13) is assigned no course-sections, 9 teachers (Teachers 3, 6, 7, 8, 9, 14, 16, 19, and 20) are assigned 2 course-sections each, and

10 teachers (Teachers 1, 2, 4, 5, 10, 11, 12, 15, 17, and 18) are assigned 3 course-sections each. Since the ideal number of course-sections per teacher is $\frac{I}{T} = \frac{48}{20} = 2.4$, we obtain $\sum_{t\in\mathcal{T}} Q_t = 12$. In the optimal assignment, every teacher is given their most preferred weekly teaching schedule along with their preferred time of day to teach. This yields $\sum_{t\in\mathcal{T}} D_t = \sum_{t\in\mathcal{T}} B_t = 0$. We can see the objective function value evaluated as

$$\begin{aligned}
&\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t + \omega_4 \cdot \sum_{t\in\mathcal{T}} D_t + \omega_5 \cdot \sum_{t\in\mathcal{T}} B_t \\
&= \frac{1}{5} \cdot 0 + \frac{1}{5 \cdot 20} \cdot 19 + \frac{1}{5} \cdot 12 + \frac{1}{5} \cdot 0 + \frac{1}{5} \cdot 0 \\
&= 2.5900.
\end{aligned}$$

As points of comparison, we also apply Model 8: Naive Method and Model 9: Random Method to the CPP data. Since Model 7: CPP Method ensures that teachers will be assigned in their desired classroom preference, there will be no violations of the sort, i.e. a teacher who wishes to teach in a whiteboard classroom is ensured to teach in a whiteboard classroom, and similarly with chalkboard classroom preferences. The Naive and Random Methods make no such guarantee. Thus, in addition to considering the objective function value, we can measure the number of classroom type violations the Naive and Random Methods produce.

The result of the Naive Method can be visualized in Tables 4.4 - 4.9 and is seen through the objective function as

$$\begin{aligned}
&\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t + \omega_4 \cdot \sum_{t\in\mathcal{T}} D_t + \omega_5 \cdot \sum_{t\in\mathcal{T}} B_t \\
&= \frac{1}{5} \cdot 24 + \frac{1}{5 \cdot 20} \cdot 191 + \frac{1}{5} \cdot 9.6 + \frac{1}{5} \cdot 30 + \frac{1}{5} \cdot 29 \\
&= 20.4300
\end{aligned}$$

with 17 classroom preference violations.

Since all course-sections were assigned on either M/W, W/F, M/F, or M/W/F, the value of W is maximized, which shows this method is inefficient. However, the Naive Method is able to efficiently assign course-sections equally amongst teachers, resulting in a lower value of $\sum_{t\in\mathcal{T}} Q_t = 9.6$ than even the CPP Model.

Since the Niave Method does not take preferences into consideration, the values of our objectives for D_t and B_t are significantly higher versus the CPP Model.

For the Random method, the feasible schedule found can be visualized in Tables 4.10 - 4.14 and we have an objective function value of

$$\begin{aligned}
&\omega_1 \cdot W + \frac{\omega_2}{T} \cdot \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} T^s_{ti} x_{ijkt} + \omega_3 \cdot \sum_{t\in\mathcal{T}} Q_t + \omega_4 \cdot \sum_{t\in\mathcal{T}} D_t + \omega_5 \cdot \sum_{t\in\mathcal{T}} B_t \\
&\quad = \frac{1}{5} \cdot 9 + \frac{1}{5 \cdot 20} \cdot 169 + \frac{1}{5} \cdot 23.2000 + \frac{1}{5} \cdot 27 + \frac{1}{5} \cdot 25 \\
&\quad = 18.5300
\end{aligned}$$

with 20 classrooms preference violations. We notice in the results that there are more M/W, W/F, M/F, or M/W/F course-sections than T/TH course-sections, which results in a larger W value of 9. Since this method finds assignments randomly, the number of course-sections assigned to a M/W, W/F, M/F, or M/W/F vs. T/TH would be porportional to the number of time modules on M/W, W/F, M/F, or M/W/F vs. T/TH. Additionally for the Random Method, we see a larger value for $\sum_{t\in\mathcal{T}} Q_t$ which is somewhat peculiar since we have a uniformly random method, thus on average each teacher should teach close to the ideal number of course-sections to teach; however, this need not be the case for every run of the method.

Furthermore, the nature of the method may produce large values for the time of day and weekly teaching preference, thus $\sum_{t\in\mathcal{T}} D_t$ and $\sum_{t\in\mathcal{T}} B_t$ may be larger than the CPP Model. Similarly to our examples with the Random Method in Section 3, we can calculate measures of centers to analyze the behavior. For 100 iterations of the Random Method using CPP Data, we have a mean, median, and mode of the objective function value of 20.0388, 20.1800, and 19.9800, respectively. We can also examine the mininum objective value as 16.8300 and 23.1700 for the maximum.

Comparatively, we see that having a linear programming model allows for better results. The optimal solution from Model 7 has objective function value 2.5900 compared to 20.4300 for the Naive Method and 18.5300 for the Random Method, both of which also produced classroom type violations.

Table 4.1: Schedule for Model 7 using CPP data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 12 Course 29 – 1 7:00 – 7:50 AM 3-1637	Teacher 2 Course 4 – 1 7:00 - 8:15 AM 3-1637	Teacher 12 Course 29 – 1 7:00 – 7:50 AM 3-1637	Teacher 2 Course 4 – 1 7:00 - 8:15 AM 3-1637	Teacher 12 Course 29 – 1 7:00 – 7:50 AM 3-1637
Teacher 4 Course 5 – 3 7:00 - 8:05 AM 8-250	Teacher 11 Course 22 – 1 7:00 - 8:15 AM 8-248	Teacher 4 Course 5 – 3 7:00 - 8:05 AM 8-250	Teacher 11 Course 22 – 1 7:00 - 8:15 AM 8-248	Teacher 4 Course 5 – 3 7:00 - 8:05 AM 8-250
Teacher 19 Course 3 – 6 8:15 - 9:20 AM 3-1616	Teacher 17 Course 3 – 2 8:00 - 9:50 AM 8-210	Teacher 19 Course 3 – 6 8:15 - 9:20 AM 3-1616	Teacher 17 Course 3 – 2 8:00 - 9:50 AM 8-210	Teacher 19 Course 3 – 6 8:15 - 9:20 AM 3-1616
Teacher 4 Course 17 – 1 9:30 - 10:35 AM 8-260	Teacher 16 Course 15 – 2 8:00 - 9:50 AM 8-156	Teacher 4 Course 17 – 1 9:30 - 10:35 AM 8-260	Teacher 16 Course 15 – 2 8:00 - 9:50 AM 8-156	Teacher 4 Course 17 – 1 9:30 - 10:35 AM 8-260
Teacher 18 Course 25 – 2 11:00 - 12:50 PM 8-260	Teacher 1 Course 25 – 1 8:00 - 9:50 AM 3-1623	Teacher 18 Course 25 – 2 11:00 - 12:50 PM 8-260	Teacher 1 Course 25 – 1 8:00 - 9:50 AM 3-1623	
Teacher 10 Course 11 – 2 11:30 - 12:45 PM 3-1637	Teacher 14 Course 37 – 1 8:30 - 9:45 AM 8-247	Teacher 10 Course 11 – 2 11:30 - 12:45 PM 3-1637	Teacher 14 Course 37 – 1 8:30 - 9:45 AM 8-247	
Teacher 3 Course 14 – 1 11:30 - 12:45 PM 8-250	Teacher 16 Course 16 – 1 10:00 - 11:15 AM 8-247	Teacher 3 Course 14 – 1 11:30 - 12:45 PM 8-250	Teacher 16 Course 16 – 1 10:00 - 11:15 AM 8-247	
Teacher 5 Course 26 – 1 12:00 – 12:50 PM 3-1616	Teacher 7 Course 18 – 1 10:00 - 11:15 AM 8-249	Teacher 5 Course 26 – 1 12:00 – 12:50 PM 3-1616	Teacher 7 Course 18 – 1 10:00 - 11:15 AM 8-249	Teacher 5 Course 26 – 1 12:00 – 12:50 PM 3-1616

Table 4.2: Schedule for Model 7 using CPP data. (cont.)

Teacher 6 Course 40 – 1 1:00 - 2:15 PM 8-210	Teacher 2 Course 5 – 2 10:00 - 11:50 AM 8-260	Teacher 6 Course 40 – 1 1:00 - 2:15 PM 8-210	Teacher 2 Course 5 – 2 10:00 - 11:50 AM 8-260	
Teacher 20 Course 3 – 3 1:15 - 2:20 PM 8-249	Teacher 11 Course 6 – 1 10:00 - 11:50 AM 3-1616	Teacher 20 Course 3 – 3 1:15 - 2:20 PM 8-249	Teacher 11 Course 6 – 1 10:00 - 11:50 AM 3-1616	Teacher 20 Course 3 – 3 1:15 - 2:20 PM 8-249
Teacher 5 Course 6 – 2 1:15 - 2:20 PM 3-1637	Teacher 15 Course 15 – 3 10:00 - 11:50 AM 3-1623	Teacher 5 Course 6 – 2 1:15 - 2:20 PM 3-1637	Teacher 15 Course 15 – 3 10:00 - 11:50 AM 3-1623	Teacher 5 Course 6 – 2 1:15 - 2:20 PM 3-1637
Teacher 4 Course 17 – 2 1:15 - 2:20 PM 8-248	Teacher 1 Course 25 – 3 10:00 - 11:50 AM 8-248	Teacher 4 Course 17 – 2 1:15 - 2:20 PM 8-248	Teacher 1 Course 25 – 3 10:00 - 11:50 AM 8-248	Teacher 4 Course 17 – 2 1:15 - 2:20 PM 8-248
Teacher 18 Course 26 – 2 2:30 - 3:45 PM 3-1616	Teacher 15 Course 8 – 1 1:00 - 2:15 PM 8-260	Teacher 18 Course 26 – 2 2:30 - 3:45 PM 3-1616	Teacher 15 Course 8 – 1 1:00 - 2:15 PM 8-260	
Teacher 20 Course 3 – 5 3:00 - 4:50 PM 8-210	Teacher 3 Course 24 – 1 1:00 - 2:15 PM 3-1637	Teacher 20 Course 3 – 5 3:00 - 4:50 PM 8-210	Teacher 3 Course 24 – 1 1:00 - 2:15 PM 3-1637	
	Teacher 2 Course 5 – 1 1:00 - 2:50 PM 8-250	Teacher 10 Course 20 – 1 3:00 - 4:50 PM 3-1637	Teacher 2 Course 5 – 1 1:00 - 2:50 PM 8-250	Teacher 10 Course 20 – 1 3:00 - 4:50 PM 3-1637
Teacher 5 Course 11 – 1 5:00 – 5:50 PM 8-250	Teacher 8 Course 20 – 3 3:00 - 4:50 PM 8-248	Teacher 5 Course 11 – 1 5:00 – 5:50 PM 8-250	Teacher 8 Course 20 – 3 3:00 - 4:50 PM 8-248	Teacher 5 Course 11 – 1 5:00 – 5:50 PM 8-250

Table 4.3: Schedule for Model 7 using CPP data. (cont.)

Teacher 9 Course 10 – 1 5:30 - 6:45 PM 8-246	Teacher 17 Course 3 – 7 5:00 - 6:50 PM 8-247	Teacher 9 Course 10 – 1 5:30 - 6:45 PM 8-246	Teacher 17 Course 3 – 7 5:00 - 6:50 PM 8-247	
Teacher 18 Course 30 – 1 5:30 - 6:45 PM 8-260	Teacher 7 Course 3 – 8 5:00 - 6:50 PM 8-246	Teacher 18 Course 30 – 1 5:30 - 6:45 PM 8-260	Teacher 7 Course 3 – 8 5:00 - 6:50 PM 8-246	
Teacher 12 Course 7 – 1 5:30 - 6:45 PM 3-1616	Teacher 1 Course 26 – 4 5:30 - 6:45 PM 8-250		Teacher 1 Course 26 – 4 5:30 - 6:45 PM 8-250	Teacher 12 Course 7 – 1 5:30 - 6:45 PM 3-1616
Teacher 6 Course 2 – 1 8:00 - 9:50 PM 8-156	Teacher 17 Course 3 – 1 8:00 - 9:50 PM 8-249	Teacher 6 Course 2 – 1 8:00 - 9:50 PM 8-156	Teacher 17 Course 3 – 1 8:00 - 9:50 PM 8-249	
Teacher 11 Course 6 – 3 8:00 - 9:50 PM 3-1616	Teacher 19 Course 3 – 4 8:00 - 9:50 PM 3-1616	Teacher 11 Course 6 – 3 8:00 - 9:50 PM 3-1616	Teacher 19 Course 3 – 4 8:00 - 9:50 PM 3-1616	
Teacher 10 Course 20 – 2 8:00 - 9:50 PM 8-260	Teacher 15 Course 15 – 1 8:00 - 9:50 PM 8-250		Teacher 15 Course 15 – 1 8:00 - 9:50 PM 8-250	Teacher 10 Course 20 – 2 8:00 - 9:50 PM 8-260
Teacher 9 Course 33 – 1 8:30 - 9:45 PM 8-246	Teacher 8 Course 26 – 3 8:30 - 9:45 PM 3-1637	Teacher 9 Course 33 – 1 8:30 - 9:45 PM 8-246	Teacher 8 Course 26 – 3 8:30 - 9:45 PM 3-1637	
Teacher 12 Course 41 – 1 8:30 - 9:45 PM 8-250	Teacher 14 Course 38 – 1 8:30 - 9:45 PM 8-246	Teacher 12 Course 41 – 1 8:30 - 9:45 PM 8-250	Teacher 14 Course 38 – 1 8:30 - 9:45 PM 8-246	

Table 4.4: Schedule for Model 8 using CPP data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 1 Course 2 – 1 11:00 - 12:50 PM 8-156		Teacher 1 Course 2 – 1 11:00 - 12:50 PM 8-156		
Teacher 2 Course 3 – 1 11:00 - 12:50 PM 8-210		Teacher 2 Course 3 – 1 11:00 - 12:50 PM 8-210		
Teacher 3 Course 3 – 2 11:00 - 12:50 PM 8-246		Teacher 3 Course 3 – 2 11:00 - 12:50 PM 8-246		
Teacher 4 Course 3 – 3 11:00 - 12:50 PM 8-247		Teacher 4 Course 3 – 3 11:00 - 12:50 PM 8-247		
Teacher 5 Course 3 – 4 11:00 - 12:50 PM 8-249		Teacher 5 Course 3 – 4 11:00 - 12:50 PM 8-249		
Teacher 6 Course 3 – 5 11:00 - 12:50 PM 8-248		Teacher 6 Course 3 – 5 11:00 - 12:50 PM 8-248		
Teacher 7 Course 3 – 6 11:00 - 12:50 PM 8-250		Teacher 7 Course 3 – 6 11:00 - 12:50 PM 8-250		
Teacher 8 Course 3 – 7 11:00 - 12:50 PM 8-260		Teacher 8 Course 3 – 7 11:00 - 12:50 PM 8-260		

Table 4.5: Schedule for Model 8 using CPP data. (cont.)

Teacher 9 Course 3 – 8 11:00 - 12:50 PM 3-1616		Teacher 9 Course 3 – 8 11:00 - 12:50 PM 3-1616		
Teacher 10 Course 5 – 1 11:00 - 12:50 PM 3-1623		Teacher 10 Course 5 – 1 11:00 - 12:50 PM 3-1623		
Teacher 11 Course 5 – 2 11:00 - 12:50 PM 3-1637		Teacher 11 Course 5 – 2 11:00 - 12:50 PM 3-1637		
Teacher 12 Course 5 – 3 1:00 - 2:50 PM 8-156		Teacher 12 Course 5 – 3 1:00 - 2:50 PM 8-156		
Teacher 13 Course 6 – 1 1:00 - 2:50 PM 8-210		Teacher 13 Course 6 – 1 1:00 - 2:50 PM 8-210		
Teacher 14 Course 6 – 2 1:00 - 2:50 PM 8-246		Teacher 14 Course 6 – 2 1:00 - 2:50 PM 8-246		
Teacher 15 Course 6 – 3 1:00 - 2:50 PM 8-247		Teacher 15 Course 6 – 3 1:00 - 2:50 PM 8-247		
Teacher 16 Course 15 – 1 1:00 - 2:50 PM 8-249		Teacher 16 Course 15 – 1 1:00 - 2:50 PM 8-249		

Table 4.6: Schedule for Model 8 using CPP data. (cont.)

Teacher 17 Course 15 – 2 1:00 - 2:50 PM 8-248		Teacher 17 Course 15 – 2 1:00 - 2:50 PM 8-248		
Teacher 18 Course 15 – 3 1:00 - 2:50 PM 8-250		Teacher 18 Course 15 – 3 1:00 - 2:50 PM 8-250		
Teacher 19 Course 17 – 1 1:00 - 2:50 PM 8-260		Teacher 19 Course 17 – 1 1:00 - 2:50 PM 8-260		
Teacher 20 Course 17 – 2 1:00 - 2:50 PM 3-1616		Teacher 20 Course 17 – 2 1:00 - 2:50 PM 3-1616		
Teacher 1 Course 20 – 1 1:00 - 2:50 PM 3-1623		Teacher 1 Course 20 – 1 1:00 - 2:50 PM 3-1623		
Teacher 2 Course 20 – 2 1:00 - 2:50 PM 3-1637		Teacher 2 Course 20 – 2 1:00 - 2:50 PM 3-1637		
Teacher 3 Course 20 – 3 3:00 - 4:50 PM 8-156		Teacher 3 Course 20 – 3 3:00 - 4:50 PM 8-156		
Teacher 4 Course 25 – 1 3:00 - 4:50 PM 8-210		Teacher 4 Course 25 – 1 3:00 - 4:50 PM 8-210		

Table 4.7: Schedule for Model 8 using CPP data. (cont.)

Teacher 5 Course 25 – 2 3:00 - 4:50 PM 8-246		Teacher 5 Course 25 – 2 3:00 - 4:50 PM 8-246		
Teacher 6 Course 25 – 3 3:00 - 4:50 PM 8-247		Teacher 6 Course 25 – 3 3:00 - 4:50 PM 8-247		
Teacher 7 Course 4 – 1 4:00 - 5:15 PM 8-249		Teacher 7 Course 4 – 1 4:00 - 5:15 PM 8-249		
Teacher 8 Course 7 – 1 4:00 - 5:15 PM 8-248		Teacher 8 Course 7 – 1 4:00 - 5:15 PM 8-248		
Teacher 9 Course 8 – 1 4:00 - 5:15 PM 8-250		Teacher 9 Course 8 – 1 4:00 - 5:15 PM 8-250		
Teacher 10 Course 10 – 1 4:00 - 5:15 PM 8-260		Teacher 10 Course 10 – 1 4:00 - 5:15 PM 8-260		
Teacher 11 Course 11 – 1 4:00 - 5:15 PM 3-1616		Teacher 11 Course 11 – 1 4:00 - 5:15 PM 3-1616		
Teacher 12 Course 11 – 2 4:00 - 5:15 PM 3-1623		Teacher 12 Course 11 – 2 4:00 - 5:15 PM 3-1623		

Table 4.8: Schedule for Model 8 using CPP data. (cont.)

Teacher 13 Course 14 – 1 4:00 - 5:15 PM 3-1637		Teacher 13 Course 14 – 1 4:00 - 5:15 PM 3-1637		
Teacher 14 Course 16 – 1 5:30 - 6:45 PM 8-156		Teacher 14 Course 16 – 1 5:30 - 6:45 PM 8-156		
Teacher 15 Course 18 – 1 5:30 - 6:45 PM 8-210		Teacher 15 Course 18 – 1 5:30 - 6:45 PM 8-210		
Teacher 16 Course 22 – 1 5:30 - 6:45 PM 8-246		Teacher 16 Course 22 – 1 5:30 - 6:45 PM 8-246		
Teacher 17 Course 24 – 1 5:30 - 6:45 PM 8-247		Teacher 17 Course 24 – 1 5:30 - 6:45 PM 8-247		
Teacher 18 Course 26 – 1 5:30 - 6:45 PM 8-249		Teacher 18 Course 26 – 1 5:30 - 6:45 PM 8-249		
Teacher 19 Course 26 – 2 5:30 - 6:45 PM 8-248		Teacher 19 Course 26 – 2 5:30 - 6:45 PM 8-248		
Teacher 20 Course 26 – 3 5:30 - 6:45 PM 8-250		Teacher 20 Course 26 – 3 5:30 - 6:45 PM 8-250		

Table 4.9: Schedule for Model 8 using CPP data. (cont.)

Teacher 1 Course 26 – 4 5:30 - 6:45 PM 8-260		Teacher 1 Course 26 – 4 5:30 - 6:45 PM 8-260		
Teacher 2 Course 29 – 1 5:30 - 6:45 PM 3-1616		Teacher 2 Course 29 – 1 5:30 - 6:45 PM 3-1616		
Teacher 3 Course 30 – 1 5:30 - 6:45 PM 3-1623		Teacher 3 Course 30 – 1 5:30 - 6:45 PM 3-1623		
Teacher 4 Course 33 – 1 5:30 - 6:45 PM 3-1637		Teacher 4 Course 33 – 1 5:30 - 6:45 PM 3-1637		
Teacher 5 Course 37 – 1 7:00 - 8:15 PM 8-156		Teacher 5 Course 37 – 1 7:00 - 8:15 PM 8-156		
Teacher 6 Course 38 – 1 7:00 - 8:15 PM 8-210		Teacher 6 Course 38 – 1 7:00 - 8:15 PM 8-210		
Teacher 7 Course 40 – 1 7:00 - 8:15 PM 8-246		Teacher 7 Course 40 – 1 7:00 - 8:15 PM 8-246		
Teacher 8 Course 41 – 1 7:00 - 8:15 PM 8-247		Teacher 8 Course 41 – 1 7:00 - 8:15 PM 8-247		

Table 4.10: Schedule for Model 9 using CPP data.

Monday	Tuesday	Wednesday	Thursday	Friday
Teacher 2 Course 33 – 1 8:00 – 8:50 AM 8-250	Teacher 14 Course 6 – 1 8:00 - 9:50 AM 8-249	Teacher 2 Course 33 – 1 8:00 – 8:50 AM 8-250	Teacher 14 Course 6 – 1 8:00 - 9:50 AM 8-249	Teacher 2 Course 33 – 1 8:00 – 8:50 AM 8-250
Teacher 11 Course 37 – 1 8:00 – 8:50 AM 8-156	Teacher 3 Course 14 – 1 10:00 - 11:15 AM 8-210	Teacher 11 Course 37 – 1 8:00 – 8:50 AM 8-156	Teacher 3 Course 14 – 1 10:00 - 11:15 AM 8-210	Teacher 11 Course 37 – 1 8:00 – 8:50 AM 8-156
Teacher 17 Course 16 – 1 9:00 – 9:50 AM 8-246	Teacher 11 Course 10 – 1 1:00 - 2:15 PM 8-246	Teacher 17 Course 16 – 1 9:00 – 9:50 AM 8-246	Teacher 11 Course 10 – 1 1:00 - 2:15 PM 8-246	Teacher 17 Course 16 – 1 9:00 – 9:50 AM 8-246
Teacher 4 Course 3 – 3 9:30 - 10:35 AM 3-1637	Teacher 9 Course 20 – 2 1:00 - 2:50 PM 8-248	Teacher 4 Course 3 – 3 9:30 - 10:35 AM 3-1637	Teacher 9 Course 20 – 2 1:00 - 2:50 PM 8-248	Teacher 4 Course 3 – 3 9:30 - 10:35 AM 3-1637
Teacher 14 Course 15 – 2 11:00 - 12:50 PM 3-1623	Teacher 16 Course 29 – 1 2:30 - 3:45 PM 8-247	Teacher 14 Course 15 – 2 11:00 - 12:50 PM 3-1623	Teacher 16 Course 29 – 1 2:30 - 3:45 PM 8-247	
	Teacher 3 Course 25 – 3 3:00 - 4:50 PM 8-210	Teacher 20 Course 3 – 4 11:00 - 12:50 PM 3-1637	Teacher 3 Course 25 – 3 3:00 - 4:50 PM 8-210	Teacher 20 Course 3 – 4 11:00 - 12:50 PM 3-1637
	Teacher 9 Course 17 – 1 4:00 - 5:15 PM 8-156	Teacher 6 Course 25 – 1 11:00 - 12:50 PM 3-1616	Teacher 9 Course 17 – 1 4:00 - 5:15 PM 8-156	Teacher 6 Course 25 – 1 11:00 - 12:50 PM 3-1616

Table 4.11: Schedule for Model 9 using CPP data. (cont.)

Teacher 18 Course 11 – 1 12:00 – 12:50 PM 8-260	Teacher 14 Course 5 – 2 5:00 - 6:50 PM 3-1637	Teacher 18 Course 11 – 1 12:00 – 12:50 PM 8-260	Teacher 14 Course 5 – 2 5:00 - 6:50 PM 3-1637	Teacher 18 Course 11 – 1 12:00 – 12:50 PM 8-260
Teacher 17 Course 3 – 5 12:00 - 1:05 PM 8-248	Teacher 8 Course 30 – 1 5:30 - 6:45 PM 3-1616	Teacher 17 Course 3 – 5 12:00 - 1:05 PM 8-248	Teacher 8 Course 30 – 1 5:30 - 6:45 PM 3-1616	Teacher 17 Course 3 – 5 12:00 - 1:05 PM 8-248
	Teacher 12 Course 40 – 1 5:30 - 6:45 PM 8-210	Teacher 13 Course 26 – 1 1:00 - 2:15 PM 8-246	Teacher 12 Course 40 – 1 5:30 - 6:45 PM 8-210	Teacher 13 Course 26 – 1 1:00 - 2:15 PM 8-246
Teacher 19 Course 38 – 1 1:00 - 2:15 PM 3-1616	Teacher 1 Course 41 – 1 7:00 - 8:15 PM 8-248		Teacher 1 Course 41 – 1 7:00 - 8:15 PM 8-248	Teacher 19 Course 38 – 1 1:00 - 2:15 PM 3-1616
Teacher 2 Course 15 – 1 1:00 - 2:50 PM 8-156	Teacher 20 Course 15 – 3 7:00 - 8:50 PM 8-247		Teacher 20 Course 15 – 3 7:00 - 8:50 PM 8-247	Teacher 2 Course 15 – 1 1:00 - 2:50 PM 8-156
Teacher 9 Course 3 – 6 2:30 - 3:35 PM 8-210	Teacher 6 Course 6 – 3 8:00 - 9:50 PM 8-156	Teacher 9 Course 3 – 6 2:30 - 3:35 PM 8-210	Teacher 6 Course 6 – 3 8:00 - 9:50 PM 8-156	Teacher 9 Course 3 – 6 2:30 - 3:35 PM 8-210
Teacher 19 Course 26 – 4 2:30 - 3:45 PM 8-250	Teacher 14 Course 3 – 8 8:30 - 9:45 PM 3-1637	Teacher 19 Course 26 – 4 2:30 - 3:45 PM 8-250	Teacher 14 Course 3 – 8 8:30 - 9:45 PM 3-1637	

Table 4.12: Schedule for Model 9 using CPP data. (cont.)

Teacher 15 Course 5 – 3 3:00 - 4:50 PM 3-1616	Teacher 15 Course 20 – 3 8:30 - 9:45 PM 8-260	Teacher 15 Course 5 – 3 3:00 - 4:50 PM 3-1616	Teacher 15 Course 20 – 3 8:30 - 9:45 PM 8-260	
Teacher 16 Course 3 – 7 3:00 - 4:50 PM 3-1637				Teacher 16 Course 3 – 7 3:00 - 4:50 PM 3-1637
Teacher 19 Course 22 – 1 4:00 - 5:15 PM 8-246		Teacher 19 Course 22 – 1 4:00 - 5:15 PM 8-246		
		Teacher 4 Course 24 – 1 4:00 - 5:15 PM 8-247		Teacher 4 Course 24 – 1 4:00 - 5:15 PM 8-247
		Teacher 8 Course 26 – 2 4:00 - 5:15 PM 8-248		Teacher 8 Course 26 – 2 4:00 - 5:15 PM 8-248
Teacher 6 Course 3 – 1 5:00 - 6:50 PM 8-210		Teacher 6 Course 3 – 1 5:00 - 6:50 PM 8-210		
		Teacher 20 Course 4 – 1 5:30 - 6:45 PM 8-248		Teacher 20 Course 4 – 1 5:30 - 6:45 PM 8-248

Table 4.13: Schedule for Model 9 using CPP data. (cont.)

Teacher 17 Course 18 – 1 5:30 - 6:45 PM 8-246				Teacher 17 Course 18 – 1 5:30 - 6:45 PM 8-246
Teacher 12 Course 26 – 3 5:30 - 6:45 PM 3-1623				Teacher 12 Course 26 – 3 5:30 - 6:45 PM 3-1623
		Teacher 13 Course 2 – 1 6:00 - 7:50 PM 3-1637		Teacher 13 Course 2 – 1 6:00 - 7:50 PM 3-1637
Teacher 16 Course 17 – 2 6:15 - 7:20 PM 8-249		Teacher 16 Course 17 – 2 6:15 - 7:20 PM 8-249		Teacher 16 Course 17 – 2 6:15 - 7:20 PM 8-249
Teacher 20 Course 3 – 2 7:30 - 8:35 PM 8-250		Teacher 20 Course 3 – 2 7:30 - 8:35 PM 8-250		Teacher 20 Course 3 – 2 7:30 - 8:35 PM 8-250
Teacher 17 Course 5 – 1 8:00 - 9:50 PM 8-156		Teacher 17 Course 5 – 1 8:00 - 9:50 PM 8-156		
Teacher 15 Course 6 – 2 8:00 - 9:50 PM 8-210				Teacher 15 Course 6 – 2 8:00 - 9:50 PM 8-210

Table 4.14: Schedule for Model 9 using CPP data. (cont.)

Teacher 12 Course 7 – 1 8:30 - 9:45 PM 8-247		Teacher 12 Course 7 – 1 8:30 - 9:45 PM 8-247		
Teacher 16 Course 8 – 1 8:30 - 9:45 PM 8-246		Teacher 16 Course 8 – 1 8:30 - 9:45 PM 8-246		
Teacher 11 Course 11 – 2 8:30 - 9:45 PM 3-1637		Teacher 11 Course 11 – 2 8:30 - 9:45 PM 3-1637		
Teacher 4 Course 20 – 1 8:45 - 9:50 PM 3-1616		Teacher 4 Course 20 – 1 8:45 - 9:50 PM 3-1616		Teacher 4 Course 20 – 1 8:45 - 9:50 PM 3-1616
Teacher 14 Course 25 – 2 8:45 - 9:50 PM 8-260		Teacher 14 Course 25 – 2 8:45 - 9:50 PM 8-260		Teacher 14 Course 25 – 2 8:45 - 9:50 PM 8-260

Chapter 5

Conclusion

In this thesis, we investigate the University Class Scheduling Problem in the Cal Poly Pomona setting. We use a mixed integer linear programming problem to construct linear models with the objective of finding an optimal schedule given a number of course-sections, classrooms, time modules, and teachers. We first begin with a basic model that is described in [1], which finds an optimal scheduling for each course-section assigned to a classroom and a time module.

Model 1: Basic Model follows [1] in the use of binary decision variables and constraints (2.1) and (2.2); however, the other constraints and objective function in [1] are constructed to resemble a generic university setting. For Models 1, 2, and 3 we instead define the objective to be to minimize the difference of assigned course-sections between M/W, W/F, M/F, or M/W/F vs. T/TH time modules. Model 1 solves this basic problem. We then proceed by incorporating teachers into our model so that each course-section is assigned to a teacher in a classroom at a time module, which will be Model 2. In Model 3, we allow our teachers to specify preferences which form hard constraints so that the teachers are not assigned course-sections, classrooms, or time modules that they are not willing to teach.

We change the objective function in Model 4 to have the goal of equally distributing the course-sections across the teachers. Model 5 presents a new setting where teachers provide a satisfaction ranking for each course-section and we use a new objective function where we want to minimize the total teacher dissatisfaction with the course-sections assigned to them. By combining Models 3, 4, and 5, we create a tricriteria objective function for Model 6 in which all three previous objectives are incorporated into a weighted average.

In Model 7, we extend our models to represent the CPP environment. We update the model to treat day of the week and time of day preferences as scores as part of the objective function, leading to a pentacriteria objective function.

We also present alternative models that use a naive algorithm and a random algorithm to find a feasible, but not necessarily optimal, solution.

Implementing our models in MATLAB, we obtain results for simulated data. By separately calculating objective function values for the naive and random methods, we show the results of our LP models are optimal whereas those of the alternate methods are not. By obtaining real-world data from CPP's Mathematics and Statistics Department, we are able to apply our CPP model to these data to find and analyze an optimal solution and compare it with the sub-optimal results found by the alternate naive and random methods.

There are a few extensions that we can conduct in the future. First, similar to how we construct preferences for teachers, we could also perhaps apply this preference idea to students, thus creating more constraints and updating the objective function. This would allow our models to interact with students, and provide an optimal solution that tends to student needs. Additionally, the CPP Model is constructed such that the classroom preference constraint is a hard constraint, meaning that the model must generate optimal solutions in which teachers are scheduled in their preferred classrooms. An alternative would be to include the classroom preference in the objective function. Finally, while Model 7 is a model that resembles the CPP environment, we can extend it to other universities, each of which may have a different set of requirements.

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Appendices

Appendix A

Course-Sections, Classrooms, and Time Modules

A.1 Course-Sections

A.1.1 Simulated Course-Sections

1	Course 1 – 1	11	Course 4 – 2	21	Course 7 – 3
2	Course 1 – 2	12	Course 4 – 3	22	Course 8 – 1
3	Course 1 – 3	13	Course 5 – 1	23	Course 8 – 2
4	Course 2 – 1	14	Course 5 – 2	24	Course 8 – 3
5	Course 2 – 2	15	Course 5 – 3	25	Course 9 – 1
6	Course 2 – 3	16	Course 6 – 1	26	Course 9 – 2
7	Course 3 – 1	17	Course 6 – 2	27	Course 9 – 3
8	Course 3 – 2	18	Course 6 – 3	28	Course 10 – 1
9	Course 3 – 3	19	Course 7 – 1	29	Course 10 – 2
10	Course 4 – 1	20	Course 7 – 2		

A.1.2 Spring Semester Course-Sections

1	Course 2 – 1	17	Course 7 – 1	33	Course 22 – 1
2	Course 3 – 1	18	Course 8 – 1	34	Course 24 – 1
3	Course 3 – 2	19	Course 10 – 1	35	Course 25 – 1
4	Course 3 – 3	20	Course 11 – 1	36	Course 25 – 2
5	Course 3 – 4	21	Course 11 – 2	37	Course 25 – 3
6	Course 3 – 5	22	Course 14 – 1	38	Course 26 – 1
7	Course 3 – 6	23	Course 15 – 1	39	Course 26 – 2
8	Course 3 – 7	24	Course 15 – 2	40	Course 26 – 3
9	Course 3 – 8	25	Course 15 – 3	41	Course 26 – 4
10	Course 4 – 1	26	Course 16 – 1	42	Course 29 – 1
11	Course 5 – 1	27	Course 17 – 1	43	Course 30 – 1
12	Course 5 – 2	28	Course 17 – 2	44	Course 33 – 1
13	Course 5 – 3	29	Course 18 – 1	45	Course 37 – 1
14	Course 6 – 1	30	Course 20 – 1	46	Course 38 – 1
15	Course 6 – 2	31	Course 20 – 2	47	Course 40 – 1
16	Course 6 – 3	32	Course 20 – 3	48	Course 41 – 1

A.2 Classrooms

1	8-156
2	8-210
3	8-246
4	8-247
5	8-249
6	8-248
7	8-250
8	8-260
9	3-1616
10	3-1623
11	3-1637

A.3 Time Modules

A.3.1 3-unit Time Modules

MW		WF		MF		MWF		TTH	
1	11:30 - 12:45 PM	8	11:30 - 12:45 PM	15	11:30 - 12:45 PM	22	7:00 – 7:50 AM	37	7:00 - 8:15 AM
2	1:00 - 2:15 PM	9	1:00 - 2:15 PM	16	1:00 - 2:15 PM	23	8:00 – 8:50 AM	38	8:30 - 9:45 AM
3	2:30 - 3:45 PM	10	2:30 - 3:45 PM	17	2:30 - 3:45 PM	24	9:00 – 9:50 AM	39	10:00 - 11:15 AM
4	4:00 - 5:15 PM	11	4:00 - 5:15 PM	18	4:00 - 5:15 PM	25	10:00 – 10:50 AM	40	1:00 - 2:15 PM
5	5:30 - 6:45 PM	12	5:30 - 6:45 PM	19	5:30 - 6:45 PM	26	11:00 – 11:50 AM	41	2:30 - 3:45 PM
6	7:00 - 8:15 PM	13	7:00 - 8:15 PM	20	7:00 - 8:15 PM	27	12:00 – 12:50 PM	42	4:00 - 5:15 PM
7	8:30 - 9:45 PM	14	8:30 - 9:45 PM	21	8:30 - 9:45 PM	28	1:00 – 1:50 PM	43	5:30 - 6:45 PM
						29	2:00 – 2:50 PM	44	7:00 - 8:15 PM
						30	3:00 – 3:50 PM	45	8:30 - 9:45 PM
						31	4:00 – 4:50 PM		
						32	5:00 – 5:50 PM		
						33	6:00 – 6:50 PM		
						34	7:00 – 7:50 PM		
						35	8:00 – 8:50 PM		
						36	9:00 – 9:50 PM		

A.3.2 4-unit Time Modules

MW		WF		MF		MWF		TTH	
46	11:00 - 12:50 PM	53	11:00 - 12:50 PM	60	11:00 - 12:50 PM	67	7:00 - 8:05 AM	79	8:00 - 9:50 AM
47	1:00 - 2:50 PM	54	1:00 - 2:50 PM	61	1:00 - 2:50 PM	68	8:15 - 9:20 AM	80	10:00 - 11:50 AM
48	3:00 - 4:50 PM	55	3:00 - 4:50 PM	62	3:00 - 4:50 PM	69	9:30 - 10:35 AM	81	1:00 - 2:50 PM
49	5:00 - 6:50 PM	56	5:00 - 6:50 PM	63	5:00 - 6:50 PM	70	10:45 - 11:50 AM	82	3:00 - 4:50 PM
50	6:00 - 7:50 PM	57	6:00 - 7:50 PM	64	6:00 - 7:50 PM	71	12:00 - 1:05 PM	83	5:00 - 6:50 PM
51	7:00 - 8:50 PM	58	7:00 - 8:50 PM	65	7:00 - 8:50 PM	72	1:15 - 2:20 PM	84	6:00 - 7:50 PM
52	8:00 - 9:50 PM	59	8:00 - 9:50 PM	66	8:00 - 9:50 PM	73	2:30 - 3:35 PM	85	7:00 - 8:50 PM
						74	3:45 - 4:50 PM	86	8:00 - 9:50 PM
						75	5:00 - 6:05 PM		
						76	6:15 - 7:20 PM		
						77	7:30 - 8:35 PM		
						78	8:45 - 9:50 PM		

Appendix B

CPP Data

B.1 Units and Classroom Preferences Data

Teacher	Spring Units	Board Preference Whiteboard (1), Chalkboard (2), No Pref (0)
1	12	1
2	12	1
3	12	1
4	12	1
5	12	1
6	8	0
7	8	0
8	12	0
9	12	0
10	12	1
11	12	1
12	12	1
13	0	1
14	15	2
15	12	1
16	12	2
17	12	2
18	12	1
19	15	1
20	8	2

B.2 Time of Day Preferences Data

Mo (1), Af (2), Ev (3)	1	2	3	4	5	6	7
Mo/Af (4), Af/Ev (5),	Mo	Af	Ev	Mo/Af	Af/Ev	Mo/Ev	Mo/Af/Ev
Mo/Ev, No Pref (0)	Great (0), Good (1), Okay (2), Never (3)						
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
5	3	1	1	2	0	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
5	3	1	1	2	0	2	2
4	1	1	3	0	2	2	2
3	2	2	0	3	1	1	2
0	0	0	0	0	0	0	0
4	1	1	3	0	2	2	2
5	3	1	1	2	0	2	2
4	1	1	3	0	2	2	2
4	1	1	3	0	2	2	2
5	3	1	1	2	0	2	2
1	0	3	3	2	3	2	2

B.3 Weekly Teaching Preference Data Part 1

Teacher	1 MW	2 WF	3 MF	4 MWF	5 TTh	6 MTWTh
1	1	1	1	3	0	3
2	2	2	2	2	0	3
3	1	1	1	1	1	0
4	1	1	1	0	1	3
5	2	1	1	0	2	2
6	0	1	1	3	1	3
7	1	2	2	3	0	1
8	3	3	3	3	0	3
9	0	1	1	2	0	2
10	3	2	2	0	3	3
11	1	1	1	1	1	0
12	1	1	1	0	1	0
13	2	2	2	3	0	3
14	0	1	1	3	0	3
15	3	3	3	3	0	3
16	0	1	1	2	0	2
17	3	3	3	3	0	3
18	0	1	1	1	0	2
19	3	3	3	3	3	1
20	3	3	3	0	3	3

B.4 Weekly Teaching Preference Data Part 2

Teacher	7 TWThF	8 MTThF	9 MTWThF
1	3	3	3
2	3	3	3
3	2	2	3
4	3	3	3
5	2	2	2
6	3	3	3
7	2	2	3
8	3	3	3
9	2	2	3
10	3	3	3
11	2	2	2
12	2	2	1
13	3	3	3
14	3	3	3
15	3	3	3
16	2	2	3
17	3	3	3
18	2	2	3
19	2	2	0
20	3	3	3

B.5 Teacher Course Preference Data Part 1

Teacher	Course													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1														
2	1	1		0	0	2	2	2	1	1	2	2	2	2
3												0	0	0
4					0	0	0							
5				1	1	0	1	1	1		0			
6	0	0												
7			0	2	1	1	1	2	2	2	2	2	2	2
8				2	2	0	1	2	2	2	2	2	2	2
9	1	1		1	1	2	2	2	1	1	2	2	2	2
10			1			0	1		1	2	0			
11			1			0							0	0
12			2	2	1	0	0	2	2	2	2	1	0	0
13														
14														
15					2			0						
16			1		0									
17			0			1								
18														
19			1											
20			0											
Units	4	4	4	3	4	4	3	3	4	3	3	3	3	3
Sections	0	1	8	1	3	3	1	1	0	1	2	0	0	1

B.6 Teacher Course Preference Data Part 2

Teacher	Course													
	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1											0	0		
2	2	2	2	2	2	2	2	2	2	2				
3										1				
4	1	1	0		1	1								
5	2		2		0	1			2			0		
6														
7	2	2	0	0	0	0	0	0	2	2				
8	2	2	2	2	2	0	2	2	2	2	2	0	2	1
9	2	2	2	2	2	2	2	2	2	2				
10	1				0	0								
11			1				0	0						
12	2	2	2	2	2	2	2	2	2	1	1	1	0	0
13												2		
14														
15	0	1							2					
16	0	0			0	2			1					
17			2			0								
18											0	0	2	1
19											2	2	2	2
20														
Units	4	3	4	3	3	4	3	3	3	3	4	3	4	3
Sections	3	1	2	1	0	3	0	1	0	1	3	4	0	0

B.7 Teacher Course Preference Data Part 3

Teacher	Course													
	29	30	31	32	33	34	35	36	37	38	39	40	41	42
1														
2			0	1	1									
3														
4							0			0				
5							0			2				
6														
7														
8	1	2				2	1	2	2	2	1	1	2	2
9			0	0	0									
10							0			1				
11														
12	0	1				0	0	2	2	2	1	1	0	0
13														
14														
15								2	2					
16														
17							2				2	2		
18	1	0											2	0
19	2	2												
20														
Units	3	3	4	3	3	4	3	3	3	3	3	3	3	3
Sections	1	1	0	0	1	0	0	0	1	1	0	1	1	0