Discrete Optimization Specialization: Workshop 6
Weighing an Elephant: Part 1

1 Introduction

Sun Quan has presented an elephant to Cao Cao, and Cao Cao would like to know how heavy the elephant is, but he has no scale large enough to weigh the elephant. He asked all his officials how to weigh the elephant but none could answer.

His son Cao Chong devises a solution. First we must move the elephant onto a boat and mark the line the water reaches on the boat. Then we must load equal sized stones onto the boat until the water reaches the same line. We can then determine the weight from the number of stones.

Weighing an Elephant: Part 1 - `elephant1.mzn`

Cao Chong's plan is sound but the first problem is to get the elephant onto the boat. It has no great desire to walk across the narrow pier from the bank to the boat. Cao Chong devises a plan to tempt the elephant over to the boat.

Cao Chong has G guards, W bowls of water and F bowls of food together with the elephant on the bank.

The elephant will move to the boat if (a) there is an equal number of bowls of food and water on the boat, (b) the boat has more bowls of food than those on the bank and (c) the boat has more bowls of water than those on the bank. Unfortunately the elephant will move back to the bank if (1) the elephant is on the boat, (2) there is an equal number of bowls of food and water on the bank, (3) the bank has more bowls of food than those on the boat and (4) the bank has more bowls of water than those on the boat.

The elephant will eat if there is only food left where it is and it doesn't move. Similarly the elephant will drink if there is only water left where it is and it doesn't move. If the elephant either eats or drinks then the weighing will be wrong since it will include the weight of the food or drink. Hence eating and drinking can't be allowed.

Each guard can carry one bowl of food or water in each step: from bank to boat, or boat to bank. All the movements in the same step have to be in the same direction, either from bank to boat, or boat to bank.

The aim is to get the elephant onto the boat, with no food or water left in the boat in the least number of steps, and no more than a given T time steps. Then the boats waterline can be marked. Note that no water or food should be moved after the end step (when the elephant's weight is marked on the boat).

2 Data Format Specification

The input data declarations and principle decisions are

```
int: T; % maximum time allowed;
set of int: TIME = 1..T;
```

```
int: G; % number of guards
int: F;
int: W;

array[TIME] of var -F..F: move_food;      % move food +ve bank to boat, -ve boat to bank
array[TIME] of var -W..W: move_water;     % move water +ve bank to boat, -ve boat to bank
var TIME: end;                            % end time;
```

The decisions are, in every time step, how many bowls of food are moved and in which direction, and how many bowls of water are moved and in which direction. Note that there is no possibility of moving food in both directions, and so we can just count the total moved in one direction; similarly for water.

For input data (`elephant1_0.dzn`)

```
T = 10;
G = 3;
F = 5;
W = 5;
```

a solution might be

```
move_food = [0, 3, -2, -1, 0, 0, 0, 0, 0, 0];
move_water = [3, 0, -1, -2, 0, 0, 0, 0, 0, 0];
end = 4;
```

which moves 3 water from boat to bank, then 3 food from boat to bank. The elephant will then move to the boat. Then we move 2 food and 1 water back to the bank, and then the remaining 1 food and 2 water back to the bank. The elephant is now on the boat alone (and note: the elephant will try to move back to the bank, but before that we will have marked the boat's waterline).

Note that there may not always be a solution. For example for input data

```
T = 8;
G = 2;
F = 3;
W = 3;
```

the answer should be

```
=====UNSATISFIABLE=====
```

since there is no possible plan. A plan that moves 2 food to the boat and then 2 water to the boat (or vice versa) will get the elephant to the boat. But then moving 2 food or 2 water back will result in the elephant eating the remaining bowls on the boat, while moving 1 food and 1 water back will get the elephant to move back to the bank.

3 Technical Requirements

For completing the workshop you will need MiniZinc 2.0 (`http://www.minizinc.org/2.0/`).