Discrete Optimization Specialization: Workshop 4
Composition

1 Introduction

Liu Bei is seriously smitten by Sun Shangxiang, the sister of Sun Quan. In order for Liu Bei to prove his devotion to her she demands a musical composition where each note is different, and the differences between each note are also different.

Composition - `composition.mzn`

Given notes numbered 1 to n, arrange them in a sequence so that the differences between adjacent notes are all different (the differences are all between 1 and $n-1$).

The data for the problem is given as

```
n = <number of notes>;
```

You should output a list of the notes in order. For example given the data

```
n = 8;
```

you should output the order of the notes, e.g.

```
order = [5, 2, 6, 4, 3, 8, 1, 7];
```

Notice the differences between notes [3, 4, 2, 1, 5, 7, 6] are all different.

Try your model on various sizes of n. How large can you go? Try different solvers.

1.1 Extensions

As an extra challenge, make the output pretty, showing the sequence of numbers and on the line below the differences between them, e.g. the output for the above solution should look like

```
5 2 6 4 3 8 1 7
 3 4 2 1 5 7 6
```

For the solution for $n = 13$ of the form [13, 1, 12, 2, 11, 3, 10, 4, 9, 5, 8, 6, 7] the output should be

```
13   1  12   2  11   3  10   4   9   5   8   6   7
  12  11  10   9   8   7   6   5   4   3   2   1
```

Make sure the output looks good for $2 \leq n \leq 99$.

2 Composition Dual - `compositiondual.mzn`

Let us think about the problem a little further. One of the differences between the notes has to be $n-1$. That means that the position of note 1 in the sequence has to be adjacent to the position of note n. In order to express this constraint you will need to have variables $posn[i]$ giving the position of note i, as opposed to the order of the notes. Thus the constraint can be expressed as: $abs(posn[1] - posn[n]) = 1$.

You can use the `inverse` global constraint to relate the two views of *order* and *posn* on the problem.

Since the differences also are a permutation of the numbers from 1 to $n-1$ we also know something about the position of the difference $n-1$. The difference $n-1$ must occur in between the position of the 1 and the position of the n in the sequence.

If we number the positions of the differences *dposn* from 1 to $n-1$ we can enforce the position of the $n-1$-st difference to be the minimum of the position of 1 and the position of n in the sequence. We can encode this as: $dposn[n-1] = min(posn[1], posn[n])$.

Again the $diffs$ and *dposn* are related by an inverse constraint. Here are the values of all the values for the solution [5, 2, 6, 4, 3, 8, 1, 7].

```
order = [5, 2, 6, 4, 3, 8, 1, 7];
diffs = [3, 4, 2, 1, 5, 7, 6];
posn  = [7, 2, 5, 4, 1, 3, 8, 6];
dposn = [4, 3, 1, 2, 5, 7, 6];
```

Try this model for various sizes of n. How large can you go? Use the statistics to see the difference in search that the two models `composition.mzn` and `compositiondual.mzn` have!

3 Technical Requirements

For completing the workshop you will need MiniZinc 2.0 (`http://www.minizinc.org/2.0/`).